Linear Quadratic and Exponential Models

Name: ____________	Date: ____________	Score: ______ / 34

Quick Review

Three families show up over and over in modeling, and they look very different in a data table:

Linear ($y = mx + b$): constant *first differences* for equally-spaced x. Each step adds the same amount.

Quadratic ($y = ax^2 + bx + c$): constant *second differences*. First differences themselves climb (or fall) by a constant amount per step.

Exponential ($y = a \cdot b^x$): constant *ratio* between consecutive y-values. Each step multiplies by the same factor.

The quick test. Take a table with equally-spaced x. Compute first differences in y. Constant? Linear. If not, compute second differences. Constant? Quadratic. If not, compute *ratios* of consecutive y. Constant? Exponential. Otherwise it's something else (cubic, logarithmic, mixed).

Long-run behavior. For large positive x, an exponential growth function eventually overtakes *any* polynomial — linear, quadratic, even x^{100}. The multiplicative growth wins by enough to dominate any additive or polynomial growth, given enough time.

Common slips. A constant percentage growth ("5% per year") is exponential, *not* linear — the dollar amount added each year keeps rising. And confusing $y = kx$ (linear) with $y = k \cdot 1^x$ (constant): even if $b = 1$ is technically exponential, it's flat.

PRACTICE

Identify the model that fits each table or description.

1. $f(x) = 4x + 7$; family ________
2. $g(x) = 5 \cdot 2^x$; family ________
3. $h(x) = 3x^2 + 1$; family ________
4. Identify the family of the table below. ________

x	0	1	2	3
y	4	8	16	32

5. Identify the family of the table below. ________

x	0	1	2	3	4
y	1	3	7	13	21

6. Constant first differences signal which family? ________
7. Constant second differences signal which family? ________
8. Constant ratios of consecutive y-values signal which family? ________
9. Which grows fastest for large positive x: $100x$, x^2, 2^x, or 1000? ________
10. First differences $6, 6, 6, 6$. Family? ________
11. Ratios $3, 3, 3, 3$ for $y = 3, 9, 27, 81$. Family? ________
12. Population doubles every hour, starting at 500. Model for $P(t)$? ________
13. Bank account: 5% annual, compounded yearly. Linear or exponential? ________
14. Car depreciates 15% per year from $\$25{,}000$. Value after 4 years? ________
15. Identify the family of $f(x) = 7$. ________
16. From a table with y-values $3, 5, 7, 9, 11$: ________
17. From a table with y-values $1, 2, 4, 8, 16$: family and ratio. ________
18. Same starting value: linear adds 10/yr; exponential grows 10%/yr. Long run, which wins? ________

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19. Is $y = 2x^3$ linear, quadratic, exponential, or none of these? ________

20. Common ratio for $y = 3, 9, 27, 81$? ________

Word Problems

21. Classify each table as linear, quadratic, or exponential. ________

Table A

x	0	1	2	3
y	5	8	11	14

Table B

x	0	1	2	3
y	5	10	20	40

22. A car depreciates by 15% per year, starting at $\$25{,}000$. Which family fits this, and what is the car's value after 4 years (round to the nearest dollar)? ____

23. For sufficiently large x, rank these growth functions from slowest to fastest: $k(x) = 1000$, $h(x) = 100x$, $g(x) = x^2$, $f(x) = 2^x$. Justify by computing each at $x = 20$. ____

24. A bacteria culture doubles every hour starting from 500 bacteria. Write a function $P(t)$ for the population after t hours, identify the family, and find $P(5)$. ____

Additional Practice

25. Solve $x^2 - 5x + 6 = 0$. ________

26. Solve $x^2 = 49$. ________

27. Find the vertex of $y = (x - 3)^2 - 4$. ________

28. Find the axis of symmetry of $y = x^2 + 6x + 1$. ________

29. Factor $x^2 + 7x + 10$. ________

30. Find the discriminant of $x^2 - 4x + 8 = 0$. ________

31. Solve $2x^2 - 8 = 0$. ________

32. Write roots -1 and 6 as a quadratic. ________

33. Find the y-intercept of $y = x^2 - 3x - 10$. ________

34. Find zeros of $y = (x - 4)(x + 2)$. ________

Answer Keys

1. linear
2. exponential
3. quadratic
4. exponential
5. quadratic
6. linear
7. quadratic
8. exponential
9. 2^x
10. linear
11. exponential
12. $P(t) = 500 \cdot 2^t$
13. exponential
14. $\$13{,}050$
15. constant
16. linear, slope 2
17. exponential, ratio 2
18. exponential
19. none (it's cubic)
20. 3
21. A: linear; B: exponential
22. exponential; $\$13{,}050$
23. $k < h < g < f$ (by growth rate, for large x)
24. $P(t) = 500 \cdot 2^t$ (exponential); $P(5) = 16{,}000$

Additional Practice Answers

25. $x = 2,\ 3$
26. $x = -7,\ 7$
27. $(3, -4)$
28. $x = -3$
29. $(x+5)(x+2)$
30. -16
31. $x = -2,\ 2$
32. $(x+1)(x-6)$
33. $(0, -10)$
34. $x = 4,\ -2$

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. It fits $y = mx + b$ with slope $m = 4$ and intercept $b = 7$. The variable appears to the first power only, so it is linear — equal x-steps add a constant 4 each time.
2. The variable x is in the exponent and the base 2 is constant — the signature of an exponential function. With $b = 2 > 1$ it is growth, multiplying by 2 each step.
3. The highest power of x is 2, so this is a degree-2 (quadratic) function with a parabola graph. Its second differences are constant, unlike linear or exponential.
4. Start with the key idea: Ratios: $8/4 = 2$, $16/8 = 2$, $32/16 = 2$. Constant ratio of 2. So $y = 4 \cdot 2^x$. That gives a quick check on the answer.
5. First differences: $2, 4, 6, 8$ (not constant). Second differences: $2, 2, 2$ (constant). Quadratic.
6. If equally-spaced x-values produce y-values that change by the same amount each step, you are adding a fixed increment — the hallmark of a linear function, and that common difference is the slope.
7. When the first differences themselves change by a constant amount (so the second differences are constant), the data fits a degree-2 rule. That is the test for a quadratic.
8. A constant ratio means each step multiplies by the same factor rather than adding a fixed amount. That repeated multiplier is the base b in $a \cdot b^x$, so the family is exponential.
9. Test a large input like $x = 20$: $2^{20} = 1{,}048{,}576$, far above $100(20) = 2000$, $20^2 = 400$, and the constant 1000. Exponential growth eventually overtakes any polynomial because it keeps multiplying instead of adding.
10. The y-values increase by the same 6 at each equal x-step, so the first differences are constant — that means linear, with slope 6.
11. Each output is 3 times the previous one ($9/3 = 3$, $27/9 = 3$, $81/27 = 3$), so the constant ratio of 3 marks it exponential. A model is $y = 3^x$ indexed from $x = 1$.
12. Doubling each hour means multiplying by a constant factor of 2 per unit time, so it is exponential with base $b = 2$. The starting count 500 becomes the coefficient a, giving $P(t) = 500 \cdot 2^t$.
13. A fixed *percent* per year means the balance is multiplied by $1 + 0.05 = 1.05$ each year, not increased by a fixed dollar amount. Repeated multiplication is exponential, not linear.
14. Losing 15% leaves a factor of $1 - 0.15 = 0.85$ each year, so $V(t) = 25000(0.85)^t$. After 4 years, $V(4) = 25000(0.85)^4 = 25000(0.5220) \approx \$13{,}050$. A constant-percent loss like this is exponential decay.
15. There is no x in the rule, so every input gives the same output 7. The graph is a flat horizontal line, which is a constant function.
16. The first differences are $5-3, 7-5, 9-7, 11-9 = 2, 2, 2, 2$ — all equal — so the data is linear with slope 2. Starting from $x = 0$, the rule is $y = 2x + 3$.
17. Consecutive ratios are $2/1 = 2$, $4/2 = 2$, $8/4 = 2$, $16/8 = 2$ — a constant ratio of 2 — so the family is exponential. Starting from $x = 0$, that gives $y = 2^x$.
18. Eventually the percentage growth produces larger dollar increases than the fixed amount. Multiplicative beats additive in the long run.
19. The highest power of x is 3, so it is a cubic — outside our three families. You can confirm: first, second, and ratio tests would each fail to give a constant (it takes the *third* differences to settle down).
20. Divide each term by the one before it: $9/3 = 3$, $27/9 = 3$, $81/27 = 3$. The common ratio is 3, which is the base b in an exponential model.
21. For Table A, first differences are $3, 3, 3$ — constant. Linear with slope 3 and intercept 5: $y = 3x + 5$. For Table B, first differences are $5, 10, 20$ — *not* constant. Check ratios: $10/5 = 20/10 = 40/20 = 2$. Constant ratio of 2, so exponential: $y = 5 \cdot 2^x$. (Always check differences first, then ratios. The two tests never both succeed for the same nontrivial table.)
22. A constant percent change per unit time is the definition of exponential change. Each year, the car keeps $1 - 0.15 = 0.85$ of its value, so $V(t) = 25000(0.85)^t$. At $t = 4$: $V(4) = 25000(0.85)^4 = 25000(0.52200625) \approx 13050.16$. Rounded: $\$13{,}050$. (Linear depreciation would lose the same dollar amount each year, which isn't how percentage loss works.)
23. At $x = 20$: $k = 1000$, $h = 100(20) = 2000$, $g = 20^2 = 400$, $f = 2^{20} = 1{,}048{,}576$. Notice the surprise: at $x = 20$ the quadratic (400) is still below the constant (1000) and the linear (2000) — the crossings come later. By $x = 30$, $g = 900$ and $h = 3000$, still below; by $x = 50$, $g = 2500$ and $h = 5000$, getting close. The asymptotic ordering for very large x is constant $<$ linear $<$ quadratic $<$ exponential, with the exponential racing away from everything else.
24. Doubling each unit time is the signature of exponential growth with base 2. The initial value 500 is the coefficient: $P(t) = 500 \cdot 2^t$. At $t = 5$: $P(5) = 500 \cdot 32 = 16{,}000$. (Quick mental check: in five doublings, $500 \to 1000 \to 2000 \to 4000 \to 8000 \to 16{,}000$. Same number.)

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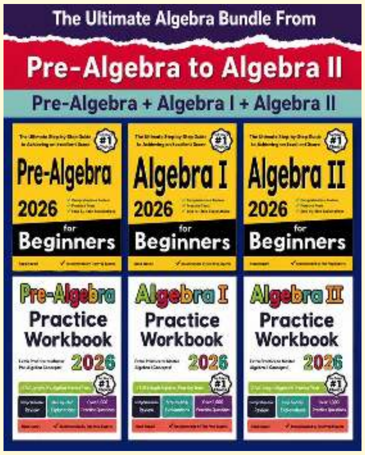

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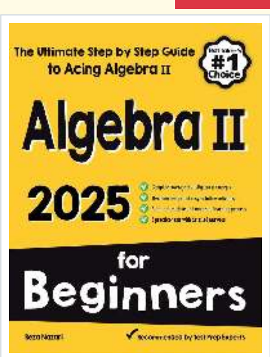

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