Linear Programming and Constraints

Name: ______	Date: ______	Score: ____ / 24

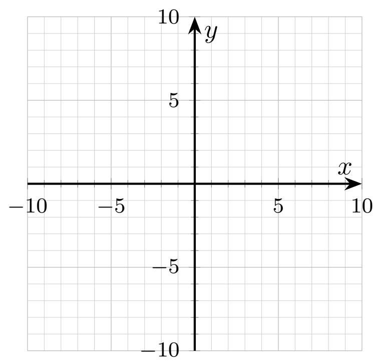
Quick Review

A **linear programming problem** asks you to optimize (maximize or minimize) a linear *objective function* subject to a system of linear *constraints* — inequalities that describe what's allowed. The set of points satisfying all constraints is called the **feasible region**, a polygon (often shaded) in the plane. The big idea: *the optimum of a linear objective over a closed bounded feasible region always lands at a corner (vertex) of the region*. So the recipe is short. (1) Translate the story into variables and constraints; (2) graph the constraints and find the feasible region; (3) identify the corner points; (4) plug each corner into the objective; (5) pick the highest (or lowest) value. Watch out for implicit constraints — $x \geq 0$ and $y \geq 0$ are usually unwritten but required (you can't make a negative number of products). And if the feasible region is empty, the problem has no solution. If it's unbounded, a maximum may not exist (but a minimum might).

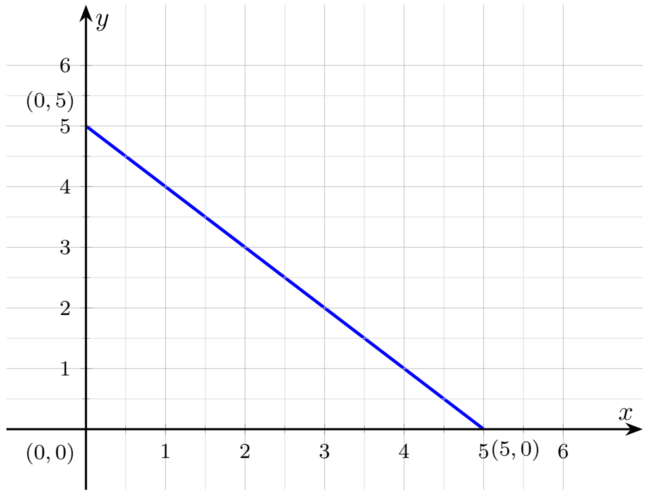
PRACTICE

Set up constraints, identify the feasible region's corners, and optimize the objective.

1. A constraint is an inequality the variables must satisfy. True or false? ________
2. In a linear program, the optimum of the objective always occurs at a corner of the feasible region. True or false? ________
3. Identify the corners of the feasible region defined by $x \geq 0, y \geq 0, x + y \leq 4$. ________



4. Maximize $P = 3x + 2y$ subject to $x + y \le 5$, $x \ge 0$, $y \ge 0$. The feasible region is shown below. Find the maximum value of P. ________



5. Minimize $C = 4x + 5y$ over $x \ge 0, y \ge 0, x + y \ge 6$. ________

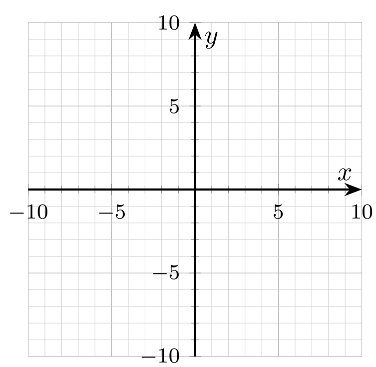
6. How many corners does the feasible region $x \ge 0, y \ge 0, 2x + y \le 8, x + 2y \le 8$ have? ________

7. Find the intersection of $2x + y = 8$ and $x + 2y = 8$. ________

8. Maximize $P = 5x + 4y$ over $x \ge 0, y \ge 0, x + y \le 10, 2x + y \le 12$. ________

9. Is $(3, 2)$ in the feasible region $\{x \ge 0, y \ge 0, x + y \le 6, 2x + y \le 7\}$? ________

10. Identify all axis-corners of $x \ge 0, y \ge 0, 3x + 4y \le 12$. ________



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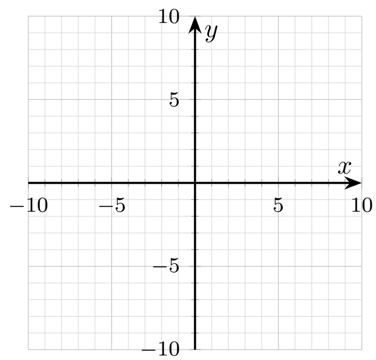
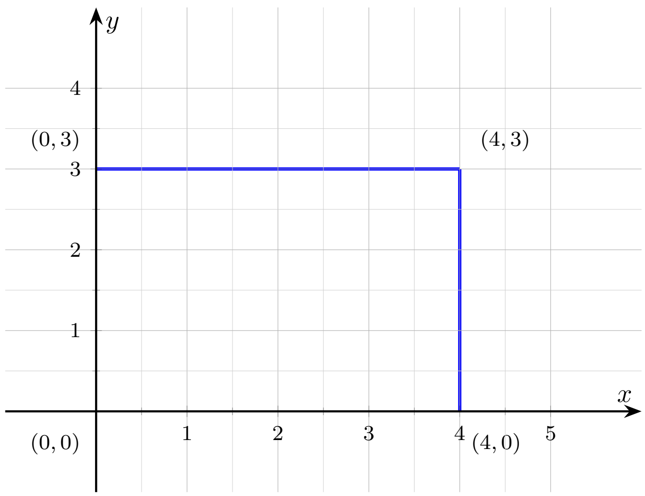
11. Sketch the feasible region: $x \geq 0, y \geq 0, x + y \leq 5, y \leq 3$. ________



12. Maximize $P = x + y$ subject to $x \leq 4$, $y \leq 3$, $x \geq 0$, $y \geq 0$. The feasible region is shown below. Find the maximum value of P. ________

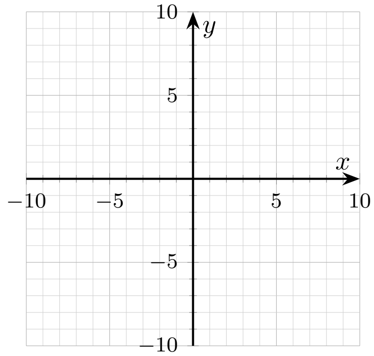


13. Minimize $C = 2x + 3y$ over $x + y \geq 4, x \geq 0, y \geq 0$. ________

14. Is the feasible region $\{x \geq 5, x \leq 2\}$ nonempty? ________

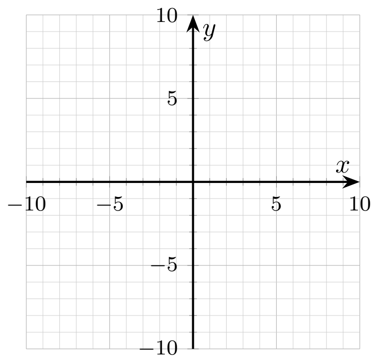
15. What word describes a region where the feasible set extends infinitely far in some direction? ________

16. Identify the corner of the feasible region $x \geq 0, y \geq 0, 2x + 3y \leq 12, x + y \leq 5$ that maximizes $P = 4x + 5y$. ________



17. In a linear program with feasible region bounded by $x \geq 0, y \geq 0, x + y \leq 6$, what is the minimum of $P = x + y$? ________

18. Sketch the feasible region: $x \geq 1, y \geq 0, x + y \leq 5$. ________



19. Maximize $P = 2x + 3y$ over $x \geq 0, y \geq 0, x + 2y \leq 8, 3x + 2y \leq 12$. ________

20. What's the minimum of $C = 3x + 2y$ over $x \geq 0, y \geq 0, x + y \geq 4$? ________

♦ Word Problems

21. A factory makes chairs (x) and tables (y). Each chair takes 2 hours of labor and each table 4 hours, with at most 40 labor-hours available per week. Chairs use 3 board-feet of wood; tables use 5 board-feet, with at most 60 board-feet available. Profit is $\$30$ per chair and $\$50$ per table. How many of each should be made to maximize profit? ____

22. A diet study requires at least 30 units of vitamin C and 40 units of vitamin D. Two foods are available: food A provides 3 units of C and 2 units of D per serving, food B provides 1 unit of C and 4 units of D per serving. Food A costs $\$2$ and food B costs $\$3$ per serving. Find the servings to meet requirements at minimum cost. ____

23. A bakery makes pies (p) and cakes (c). Each pie uses 2 cups of flour and 1 egg; each cake uses 3 cups of flour and 2 eggs. The bakery has 24 cups of flour and 14 eggs. Profit is $\$5$ per pie and $\$8$ per cake. Maximize profit. ____

24. A campaign budgets $\$2{,}000$ for two ad types: print ($\$200$ each) and online ($\$100$ each). The campaign needs at least 15 ads total. Each print ad reaches $1{,}000$ people; each online ad reaches 500. Maximize the total reach. ____

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Answer Keys

1. true
2. true (for closed bounded regions)
3. $(0,0),\ (4,0),\ (0,4)$
4. $P_{\max} = 15$ at $(5,0)$
5. $C_{\min} = 24$ at $(6,0)$
6. 4
7. $(\frac{8}{3}, \frac{8}{3})$
8. $P_{\max} = 42$ at $(2,8)$
9. no
10. $(0,0),\ (4,0),\ (0,3)$
11. 4 corners
12. $P_{\max} = 7$ at $(4,3)$
13. $C_{\min} = 8$ at $(4,0)$
14. no (empty)
15. unbounded
16. $P_{\max} = 22$ at $(3,2)$
17. $P_{\min} = 0$ at $(0,0)$
18. 3 corners
19. $P_{\max} = 13$ at $(2,3)$
20. $C_{\min} = 8$ at $(0,4)$
21. $x = 20,\ y = 0,\ P = \$600$
22. $x = 8,\ y = 6,\ C = \$34$
23. $p = 6,\ c = 4,\ P = \$62$
24. print 5, online 10, reach $10{,}000$

Step-by-Step Explanations

1. A careful way to see it: Yes. In a linear program, constraints carve out the feasible region. That gives a quick check on the answer.

2. True for any closed bounded feasible region. The linearity of the objective forces the extreme value to a vertex.

3. The three lines $x = 0$, $y = 0$, $x + y = 4$ enclose a triangle with corners at $(0,0)$, $(4,0)$, and $(0,4)$.

4. Corners: $(0,0)$, $(5,0)$, $(0,5)$. Plug each: $P(0,0) = 0$, $P(5,0) = 15$, $P(0,5) = 10$. Max at $(5,0)$.

5. With $x + y \geq 6$ and both nonneg, the boundary is the line $x + y = 6$ for $x, y \geq 0$, and the region extends outward. The cheapest corner on the boundary is $(6,0)$: $C = 24$. At $(0,6)$: $C = 30$. So min at $(6,0)$.

6. The four lines (two axes plus two slanted) bound a quadrilateral. Corners: $(0,0)$, $(4,0)$, intersection of the two slanted lines, and $(0,4)$ — four total.

7. Multiply the first by 2: $4x + 2y = 16$. Subtract the second: $3x = 8 \Rightarrow x = \frac{8}{3}$. Then $y = 8 - 2 \cdot \frac{8}{3} = \frac{8}{3}$.

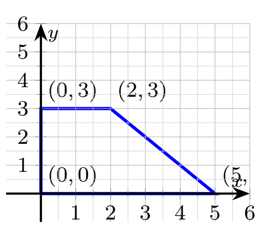
8. Corners: $(0,0)$, $(6,0)$, the intersection of $x + y = 10$ and $2x + y = 12$ (subtract: $x = 2$, so $y = 8$), and $(0,10)$. Plug each: $P(0,0) = 0$, $P(6,0) = 30$, $P(2,8) = 10 + 32 = 42$, $P(0,10) = 40$. Max 42 at $(2,8)$.

9. Check each constraint: $3 \geq 0$ and $2 \geq 0$ both hold; $3 + 2 = 5 \leq 6$ holds; but $2(3) + 2 = 8 \leq 7$ *fails*. The point isn't feasible. (Always check every constraint — one failure rules a point out.)

10. Keep the rule visible: $y = 0 \Rightarrow 3x \leq 12 \Rightarrow x \leq 4$. $x = 0 \Rightarrow 4y \leq 12 \Rightarrow y \leq 3$. Triangle corners: $(0,0)$, $(4,0)$, $(0,3)$. That gives a quick check on the answer.

11. One steady path is: Corners: $(0,0)$, $(5,0)$, intersection of $x + y = 5$ and $y = 3$: $(2,3)$, and $(0,3)$. That gives a quick check on the answer.

Answer graph



12. A rectangle with corners $(0,0), (4,0), (4,3), (0,3)$. Plug each into $P = x + y$: $0, 4, 7, 3$. Maximum is 7 at $(4,3)$.

13. Corner on the boundary $x + y = 4$: $(4,0)$ and $(0,4)$. $C(4,0) = 8$, $C(0,4) = 12$. Min at $(4,0)$. (The region extends to infinity, but those infinite directions only *increase* C.)

14. Keep the rule visible: No real x is both ≥ 5 and ≤ 2. The constraints contradict. That gives a quick check on the answer.

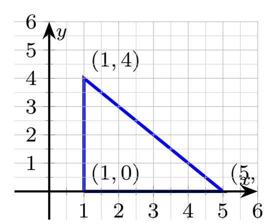
15. An unbounded region. A maximum may not exist for some objectives if the region is unbounded.

16. Find the corners. With $x, y \geq 0$: $(0,0)$; on the x-axis $(5,0)$ from $x + y = 5$ and $(6,0)$ from $2x + 3y = 12$ — the binding one is the smaller $(5,0)$. On the y-axis: $y \leq 5$ from $x + y$, $y \leq 4$ from $2x + 3y$ — binding $(0,4)$. Intersection of the slants: solve $x + y = 5$, $2x + 3y = 12$. From first $x = 5 - y$, plug: $2(5 - y) + 3y = 12 \Rightarrow y = 2$, $x = 3$. Plug all corners: $P(0,0) = 0$, $P(5,0) = 20$, $P(3,2) = 12 + 10 = 22$, $P(0,4) = 20$. Max 22 at $(3,2)$.

17. A careful way to see it: Corners: $(0,0), (6,0), (0,6)$. P values: $0, 6, 6$. Min is 0 at the origin. That gives a quick check on the answer.

18. Corners: intersection of $x = 1$ and $y = 0$ is $(1,0)$; intersection of $y = 0$ and $x + y = 5$ is $(5,0)$; intersection of $x = 1$ and $x + y = 5$ is $(1,4)$. Triangle.

Answer graph



19. Corners: $(0,0), (4,0), (0,4)$ and the intersection of $x + 2y = 8$ and $3x + 2y = 12$: subtract $\Rightarrow 2x = 4 \Rightarrow x = 2, y = 3$. Plug all: $P(0,0) = 0, P(4,0) = 8, P(0,4) = 12, P(2,3) = 4 + 9 = 13$. Max 13 at $(2,3)$.

20. Corners on the boundary $x + y = 4$ in the first quadrant: $(4,0)$ and $(0,4)$. $C(4,0) = 12$, $C(0,4) = 8$. Min is 8 at $(0,4)$.

21. Constraints: $2x + 4y \leq 40$ (labor), $3x + 5y \leq 60$ (wood), $x, y \geq 0$. The two slanted lines intersect at $(20,0)$: from labor $x = 20 - 2y$, plug into wood $3(20 - 2y) + 5y = 60 \Rightarrow -y = 0 \Rightarrow y = 0$, $x = 20$. Corners of the feasible region: $(0,0)$, $(20,0)$, $(0,10)$ (labor is tighter than wood on the y-axis). Profit: $P(0,0) = 0$, $P(20,0) = 600$, $P(0,10) = 500$. Max \$600 at $(20,0)$: 20 chairs, 0 tables.

22. Let x, y be servings of A,B. Constraints: $3x + y \geq 30$, $2x + 4y \geq 40$, $x, y \geq 0$. Corners in the first quadrant: on the x-axis, binding constraint gives $x = 10$ from $3x \geq 30$, but also $2x \geq 40 \Rightarrow x \geq 20$ — more restrictive: $(20,0)$. On the y-axis: $y \geq 30$ from C, $y \geq 10$ from D — more restrictive $(0,30)$. Intersection of $3x + y = 30$ and $2x + 4y = 40$: from first $y = 30 - 3x$. Plug: $2x + 4(30 - 3x) = 40 \Rightarrow -10x = -80 \Rightarrow x = 8$, $y = 6$. Check: $3(8) + 6 = 30$ ✓; $2(8) + 4(6) = 40$ ✓. Cost: $C(20,0) = 40, C(0,30) = 90$, $C(8,6) = 16 + 18 = 34$. Min is \$34 at $(8,6)$.

23. Constraints: $2p + 3c \leq 24$ (flour), $p + 2c \leq 14$ (eggs), $p, c \geq 0$. The two slants meet at $(6,4)$: from eggs $p = 14 - 2c$; plug into flour $2(14 - 2c) + 3c = 24 \Rightarrow -c = -4 \Rightarrow c = 4$, $p = 6$. Other corners on the axes: $(0,0)$, $(12,0)$ (flour is tighter than eggs on the p-axis), $(0,7)$ (eggs is tighter on the c-axis). Profit: $P(0,0) = 0, P(12,0) = 60, P(6,4) = 30 + 32 = 62, P(0,7) = 56$. Max \$62 at $(6,4)$: 6 pies and 4 cakes.

24. Let p, o. Constraints: $200p + 100o \leq 2000 \Rightarrow 2p + o \leq 20$; $p + o \geq 15$; $p, o \geq 0$. Find corners. Intersection of $2p + o = 20$ and $p + o = 15$: subtract $\Rightarrow p = 5$, $o = 10$. On p-axis: $p + 0 \geq 15 \Rightarrow p \geq 15$, and $2p \leq 20 \Rightarrow p \leq 10$ — contradiction, no corner on p-axis alone. On o-axis: $o \geq 15$ and $o \leq 20$ — $(0,15)$ to $(0,20)$. Reach: $R(5,10) = 5000 + 5000 = 10000$; $R(0,15) = 7500$; $R(0,20) = 10000$. Max 10,000 at either $(5,10)$ or $(0,20)$. (Tie at the maximum — both choices reach equally many people.)

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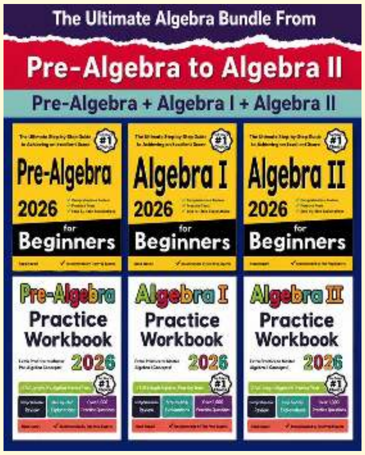

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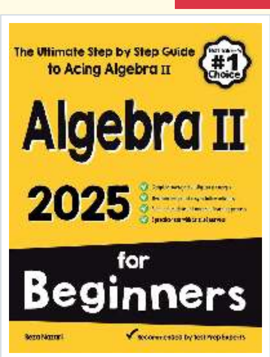

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