Law of Sines

Name: ______________________	Date: ______________	Score: _____ / 33

Quick Review

The Law of Sines works for *any* triangle, not just right triangles. It pairs each side with the sine of the angle across from it:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

Notation reminder. Sides a, b, c sit opposite angles A, B, C. So a is across from A, b is across from B, c is across from C. Always pair them that way.

When to use Law of Sines. You need at least one angle-with-its-opposite-side pair to start. That happens in:

AAS (angle, angle, side – not the included side);

ASA (find the third angle from $A + B + C = 180°$, then go);

SSA (the ambiguous case – read on).

The ambiguous case (SSA). When you're given two sides and a non-included angle, there can be *zero, one, or two* triangles. After computing $\sin B$, if the result is > 1, there's no triangle. If $= 1$, exactly one (a right triangle). If $0 < \sin B < 1$, you get two candidate angles – B and $180° - B$ – and you check whether each lets the three-angle sum stay below $180°$.

Find side from AAS: $\frac{a}{\sin A} = \frac{b}{\sin B} \Rightarrow b = \frac{a \sin B}{\sin A}$.

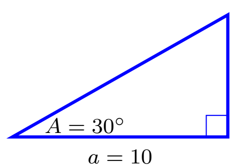
Find angle from ASS: $\frac{\sin A}{a} = \frac{\sin B}{b} \Rightarrow \sin B = \frac{b \sin A}{a}$. Then take $\arcsin$ and check the supplement.

Common slips. Pairing a side with the wrong angle (not the opposite one). Forgetting that all three angles sum to $180°$. In the ambiguous case, taking just the calculator answer and missing the supplementary solution.

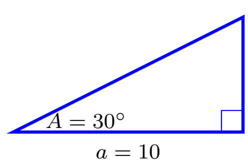
PRACTICE

Round answers to the nearest tenth unless told otherwise. Mark the ambiguous case when both triangles are valid.

1. In $\triangle ABC$, $a = 10$, $A = 30°$, $B = 60°$. Find b. The schematic below marks the known side $a = 10$ and angle $A = 30°$; the side b opposite B is left blank. ________



2. In $\triangle ABC$, $A = 50°$, $B = 70°$. Find C. ________
3. In $\triangle ABC$, $a = 8$, $A = 40°$, $b = 12$. Find $\sin B$ to two decimal places. ________
4. In $\triangle ABC$, $A = 42°$, $B = 68°$, $a = 9$. Which expression gives b? ________
5. In $\triangle ABC$, $A = 30°$, $C = 45°$, $a = 10$. Find c. (Round to one decimal place.) The schematic marks angle $A = 30°$ with its opposite side $a = 10$; pair c with $\sin C$. ________



6. Which case is the "ambiguous case" of the Law of Sines? ________
7. In $\triangle ABC$, $A = 35°$, $a = 10$, $b = 14$. Find the smaller possible value of $\sin B$ (to four decimals). ________
8. In $\triangle ABC$, $A = 35°$, $a = 10$, $b = 14$. Find both possible values of B (nearest tenth). ________
9. In $\triangle ABC$, $A = 120°$, $a = 10$, $b = 15$. Find $\sin B$. ________

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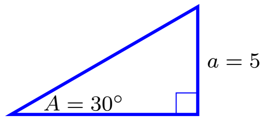
10. True/false: The Law of Sines applies only to right triangles.

11. In $\triangle ABC$, $A = 60°$, $B = 80°$, $a = 12$. Find c (nearest tenth).

12. In $\triangle ABC$, $C = 90°$, $A = 30°$, $a = 5$. Find c (the hypotenuse). The right triangle below shows angle A with its opposite side $a = 5$; the hypotenuse c is unlabeled.



13. In $\triangle ABC$, $A = 20°$, $B = 130°$, $a = 5$. Find b (nearest tenth).

14. What is $\dfrac{a}{\sin A}$ called geometrically (one short phrase)?

15. In $\triangle ABC$, $A = 45°$, $a = 8\sqrt{2}$, $B = 60°$. Find b exactly.

16. For SSA triangle with $A = 30°$, $a = 5$, $b = 10$. Find $\sin B$.

17. In $\triangle ABC$, $a = 7$, $A = 25°$, $B = 110°$. Find C.

18. In $\triangle ABC$, $A = 40°$, $a = 12$, $b = 15$. Decide if any triangle exists.

19. In $\triangle ABC$, $A = 70°$, $a = 10$, $b = 8$. Find B (nearest tenth).

20. For $\triangle ABC$ with $A = 50°$, $a = 12$, find b in terms of B only.

♦ Word Problems

21. A surveyor places stakes at endpoints A and B of a 100-foot baseline. From A, the angle to a marker M is $30°$; from B, the angle to the marker is $45°$. How far is the marker from endpoint A? Round to the nearest foot.

22. In $\triangle ABC$, $A = 50°$, $B = 50°$, and the included side $c = 10$. Find side a, rounded to the nearest tenth.

23. In $\triangle PQR$, $P = 50°$, $Q = 70°$, and $p = 12$. Find q (nearest tenth).

24. A boat's two sonar pings hit a sunken wreck. From position A, the wreck is at angle $55°$ from the line AB; from position B, the wreck is at angle $75°$ from the same line AB. If $AB = 200$ meters, how far is the wreck from position A? Round to the nearest meter.

Additional Practice

25. Find $\sin\theta$ if opposite $= 5$, hypotenuse $= 13$.

26. Find $\cos\theta$ if adjacent $= 12$, hypotenuse $= 13$.

27. Find $\tan\theta$ if opposite $= 7$, adjacent $= 4$.

28. Find $\sin 30°$.

29. Find $\cos 60°$.

30. Find $\tan 45°$.

31. Convert $180°$ to radians.

32. Convert $\frac{\pi}{3}$ radians to degrees.

33. Find a coterminal angle with $70°$.

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Answer Keys

1. $\boxed{10\sqrt{3}}$
2. $\boxed{60^\circ}$
3. $\boxed{\approx 0.96}$
4. $\boxed{\dfrac{9\sin 68^\circ}{\sin 42^\circ}}$
5. $\boxed{\approx 14.1}$
6. SSA
7. $\boxed{\approx 0.8030}$
8. $B \approx 53.4^\circ$ or 126.6°
9. $\boxed{\dfrac{15\sin 120^\circ}{10} \approx 1.30}$
10. False
11. $\boxed{\approx 8.9}$
12. $\boxed{10}$
13. $\boxed{\approx 11.2}$
14. Diameter of the circumscribed circle
15. $\boxed{8\sqrt{3}}$
16. $\boxed{1}$
17. $\boxed{45^\circ}$
18. Two triangles
19. $\boxed{\approx 48.7^\circ}$
20. $\boxed{b = \dfrac{12\sin B}{\sin 50^\circ}}$
21. ≈ 73 feet
22. $\boxed{\approx 7.8}$
23. $\boxed{\approx 14.7}$
24. ≈ 252 m

Additional Practice Answers

25. $\boxed{\frac{5}{13}}$
26. $\boxed{\frac{12}{13}}$
27. $\boxed{\frac{7}{4}}$
28. $\boxed{\frac{1}{2}}$
29. $\boxed{\frac{1}{2}}$
30. $\boxed{1}$
31. $\boxed{\pi}$
32. $\boxed{60^\circ}$
33. $\boxed{430^\circ}$

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. Pair each side with the angle across from it: $\frac{b}{\sin B} = \frac{a}{\sin A}$, so $\frac{b}{\sin 60^\circ} = \frac{10}{\sin 30^\circ}$. Solve for b and plug in exact values: $b = \frac{10\sin 60^\circ}{\sin 30^\circ} = \frac{10(\sqrt{3}/2)}{1/2} = 10\sqrt{3}$. The bigger side faces the bigger angle, which checks out.

2. Keep the rule visible: All three angles sum to 180°: $C = 180-50-70 = 60^\circ$. This is the part to check before moving on, because it keeps the answer tied to the original question.

3. One steady path is: $\frac{\sin B}{12} = \frac{\sin 40^\circ}{8} \Rightarrow \sin B = \frac{12\sin 40^\circ}{8} \approx \frac{12(0.6428)}{8} \approx 0.96$. This is the part to check before moving on, because it keeps the answer tied to the original question.

4. Start with the key idea: $\frac{a}{\sin A} = \frac{b}{\sin B}$. Plug in and solve for b: $b = \frac{9\sin 68^\circ}{\sin 42^\circ}$. This is the part to check before moving on, because it keeps the answer tied to the original question.

5. Pair c with $\sin C$ and a with $\sin A$: $\frac{c}{\sin 45^\circ} = \frac{10}{\sin 30^\circ}$. Solving, $c = \frac{10\sin 45^\circ}{\sin 30^\circ} = \frac{10(\sqrt{2}/2)}{1/2} = 10\sqrt{2} \approx 14.14$, which rounds to 14.1. (You don't even need angle B here – the A-a pair is enough to start.)

6. SSA – two sides and a non-included angle – can lead to zero, one, or two triangles.

7. One steady path is: $\frac{\sin B}{14} = \frac{\sin 35^\circ}{10} \Rightarrow \sin B = \frac{14\sin 35^\circ}{10} = 1.4\sin 35^\circ \approx 1.4(0.5736) \approx 0.8030$. This is the part to check before moving on, because it keeps the answer tied to the original question.

8. From $\sin B \approx 0.8030$, the acute solution is $\arcsin(0.8030) \approx 53.4^\circ$; the supplement is $180 - 53.4 = 126.6^\circ$. Check both: $35 + 53.4 = 88.4 < 180$ ✓and $35 + 126.6 = 161.6 < 180$ ✓. Both are valid triangles.

9. A careful way to see it: $\sin B = \frac{15\sin 120^\circ}{10} = \frac{15(\sqrt{3}/2)}{10} \approx 1.299$. Since this exceeds 1, no triangle exists with these measurements. That gives a quick check on the answer.

10. It applies to any triangle. (Right triangles are a special case, but the law is general.)

11. First the third angle: $C = 180 - 60 - 80 = 40^\circ$. Then by Law of Sines, $\frac{c}{\sin 40^\circ} = \frac{12}{\sin 60^\circ} \Rightarrow c = \frac{12\sin 40^\circ}{\sin 60^\circ} \approx \frac{12(0.6428)}{0.8660} \approx 8.91$, which rounds to 8.9.

12. Pair the hypotenuse c with its opposite angle $C = 90^\circ$, and a with A: $\frac{c}{\sin 90^\circ} = \frac{5}{\sin 30^\circ}$. Then $c = \frac{5\sin 90^\circ}{\sin 30^\circ} = \frac{5(1)}{1/2} = 10$. Cross-check with plain right-triangle trig: $\sin 30^\circ = \frac{\text{opp}}{\text{hyp}} = \frac{5}{c} = \frac{1}{2}$, so $c = 10$ again.

13. A careful way to see it: $b = \frac{a\sin B}{\sin A} = \frac{5\sin 130^\circ}{\sin 20^\circ} \approx \frac{5(0.7660)}{0.3420} \approx 11.20$. Rounds to 11.2. That gives a quick check on the answer.

14. Each side over the sine of its opposite angle equals $2R$, where R is the circumradius. The diameter, not just the radius.

15. One steady path is: $b = \frac{8\sqrt{2}\sin 60^\circ}{\sin 45^\circ} = \frac{8\sqrt{2}\cdot(\sqrt{3}/2)}{\sqrt{2}/2} = \frac{8\sqrt{2}\cdot\sqrt{3}}{\sqrt{2}} = 8\sqrt{3}$ (the $\sqrt{2}$'s cancel). That gives a quick check on the answer.

16. Start with the key idea: $\sin B = \frac{10\sin 30^\circ}{5} = \frac{10(1/2)}{5} = 1$. That means $B = 90^\circ$ – exactly one triangle (a right triangle). That gives a quick check on the answer.

17. A careful way to see it: $C = 180 - 25 - 110 = 45^\circ$. Straight from the angle-sum. This is the part to check before moving on, because it keeps the answer tied to the original question.

18. Keep the rule visible: $\sin B = \frac{15\sin 40^\circ}{12} \approx 0.804 < 1$, so a triangle exists. Acute: $B \approx 53.5^\circ$, $A + B \approx 93.5 < 180$ ✓. Obtuse: $B \approx 126.5^\circ$, $A + B \approx 166.5 < 180$ ✓. Both valid – ambiguous case. That gives a quick check on the answer.

19. Pair side b with angle B: $\frac{\sin B}{8} = \frac{\sin 70^\circ}{10}$, so $\sin B = \frac{8\sin 70^\circ}{10} \approx 0.7518$. Take the arcsine: $B \approx 48.74^\circ$, which rounds to 48.7°. Always check the supplement too: 131.3° would give $A + B > 180^\circ$, so reject it – only one triangle

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exists here.

20. Start with the key idea: From $\frac{a}{\sin A} = \frac{b}{\sin B}$: $b = \frac{a \sin B}{\sin A} = \frac{12 \sin B}{\sin 50^\circ}$. This is the part to check before moving on, because it keeps the answer tied to the original question.

21. The third angle of $\triangle ABM$ is $180 - 30 - 45 = 105^\circ$. The side AM is opposite the 45° angle, and the baseline $AB = 100$ is opposite the 105° angle. So $AM = \frac{100 \sin 45^\circ}{\sin 105^\circ} \approx \frac{100(0.7071)}{0.9659} \approx 73.2$ ft, which rounds to 73 ft.

22. The third angle is $C = 180 - 50 - 50 = 80^\circ$. Then by Law of Sines, $\frac{a}{\sin 50^\circ} = \frac{10}{\sin 80^\circ}$, so $a = \frac{10 \sin 50^\circ}{\sin 80^\circ} \approx \frac{10(0.7660)}{0.9848} \approx 7.78$, which rounds to 7.8. (Sanity check: the triangle is isoceles in the two 50° angles, so $a = b$ – a known shape.)

23. One steady path is: $\frac{p}{\sin P} = \frac{q}{\sin Q} \Rightarrow q = \frac{12 \sin 70^\circ}{\sin 50^\circ} \approx \frac{12(0.9397)}{0.7660} \approx 14.72$. Rounds to 14.7. That gives a quick check on the answer.

24. The third angle of the triangle (at the wreck) is $180 - 55 - 75 = 50^\circ$. The side from A to the wreck is opposite the angle at B (75°), and the known side $AB = 200$ is opposite the angle at the wreck (50°). By Law of Sines: $AW = \frac{200 \sin 75^\circ}{\sin 50^\circ} \approx \frac{200(0.9659)}{0.7660} \approx 252.2$ m, which rounds to 252 m.

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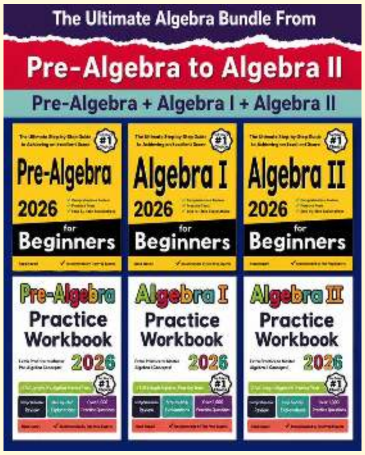

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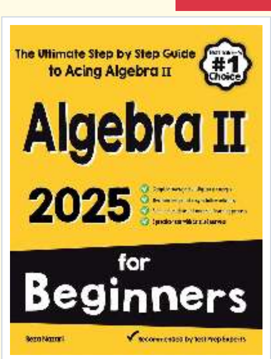

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