

Law of Cosines

Name: _____

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Q Quick Review

The Law of Cosines is the Pythagorean theorem's big sibling – it works for any triangle, not just right ones. For any $\triangle ABC$ with sides a, b, c opposite angles A, B, C :

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

The pattern: the side on the left is across from the angle whose cosine appears on the right. The two sides multiplied are the *two sides that meet at that angle*.

When to use Law of Cosines. It's the right tool for:

SAS (two sides and the included angle) – find the opposite side directly.

SSS (all three sides) – solve for any angle by rearranging the formula: $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$.

Connection to Pythagoras. If $A = 90^\circ$, then $\cos A = 0$ and the formula collapses to $a^2 = b^2 + c^2$. So the Law of Cosines includes the Pythagorean theorem as a special case.

Which angle is which? The largest angle is across from the longest side; the smallest is across from the shortest. Use that as a sanity check after you solve.

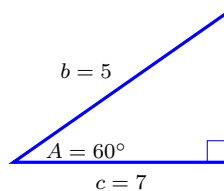
Strategy for SSS. To find all three angles, compute one with the Law of Cosines, then use Law of Sines (or another Law of Cosines) for the second, and subtract from 180° for the third. Computing the *largest* angle first dodges the ambiguous-case issue (the largest angle could be obtuse; the others must be acute).

Common slips. Confusing $-2bc \cos A$ with $+2bc \cos A$ (the minus sign is always there). Forgetting to take the square root at the end – you get a^2 , not a . Plugging the wrong angle's cosine in (it must be the angle across from the side you're solving for).

PRACTICE

Round side lengths to one decimal place and angles to the nearest tenth of a degree unless told otherwise.

- Write the Law of Cosines formula for side a . _____
- In $\triangle ABC$, $b = 5$, $c = 7$, $A = 60^\circ$. Find a . The schematic marks the included angle $A = 60^\circ$ with its two sides $b = 5$ and $c = 7$; the opposite side a is the blank one. _____

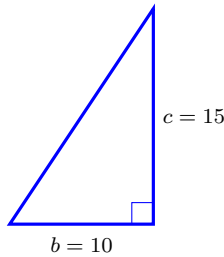


- In $\triangle ABC$, $a = 7$, $b = 8$, $c = 13$. Find $\cos C$. _____
- In $\triangle ABC$, $\cos C = -\frac{1}{2}$. What is angle C ? _____

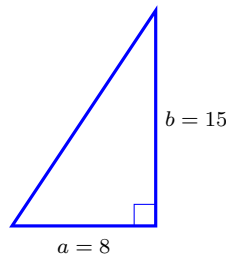


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5. In $\triangle ABC$, $b = 10$, $c = 15$, $A = 90^\circ$. Find a (exact form). Since $A = 90^\circ$, side a is the hypotenuse; legs $b = 10$ and $c = 15$ are labeled below and a is blank. _____



6. A triangle has sides 5, 6, 7. Find the largest angle (nearest degree). _____
7. In $\triangle ABC$, sides 9, 13 have included angle 120° . Find the third side. _____
8. In $\triangle ABC$, $a = 10$, $b = 14$, $c = 16$. Find the angle opposite the side of length 16 (nearest tenth). _____
9. In $\triangle ABC$, $a = 8$, $b = 11$, $c = 7$. Find angle B (nearest tenth). _____
10. Which information triggers the Law of Cosines (not Law of Sines)? _____
11. In $\triangle ABC$, $a = 5$, $b = 12$, $c = 13$. Find angle C . _____
12. In $\triangle ABC$, $b = 6$, $c = 10$, $A = 45^\circ$. Find a (nearest tenth). _____
13. True/false: The Law of Cosines requires the triangle to be acute. _____
14. In $\triangle ABC$ with $a = 4$, $b = 5$, $c = 6$, find $\cos B$. _____
15. In $\triangle ABC$, $a = 8$, $b = 15$, $C = 90^\circ$. Find c . With $C = 90^\circ$ the triangle is right-angled, as shown; legs $a = 8$ and $b = 15$ are labeled and c is blank. _____



16. In $\triangle ABC$, $a = 2$, $b = 3$, $C = 150^\circ$. Find c (exact form). _____
17. Solve for $\cos A$ in terms of sides only. _____
18. In $\triangle ABC$ with $a = 10$, $b = 24$, $c = 26$, identify the right angle. _____
19. In $\triangle ABC$, $a = 11$, $b = 9$, $C = 45^\circ$. Find c (nearest tenth). _____
20. In $\triangle ABC$ with $a = 7$, $b = 24$, $c = 25$, find angle A (nearest tenth). _____

◆ Word Problems

21. Two roads leave a town and form a 60° angle. One car drives 30 miles down the first road; another drives 40 miles down the second. How far apart are the cars? Give an exact answer. _____
22. A triangular plot of land has sides 50, 60, and 70 feet. What is the angle opposite the 70-foot side (the largest angle), to the nearest tenth of a degree? _____
23. A baseball diamond's bases are 90 feet apart. The pitcher's mound sits 60.5 feet from home plate along the line from home to second base. How far is the mound from first base? Round to the nearest tenth of a foot. _____
24. A ship sails 20 km on bearing 30° , then turns and sails 15 km on bearing 100° . How far is it from where it started? Round to the nearest tenth of a km. _____



Additional Practice

25. Find $\sin \theta$ if opposite = 5, hypotenuse = 13. _____

26. Find $\cos \theta$ if adjacent = 12, hypotenuse = 13. _____

27. Find $\tan \theta$ if opposite = 7, adjacent = 4. _____

28. Find $\sin 30^\circ$. _____



Answer Keys

<p>1. $a^2 = b^2 + c^2 - 2bc \cos A$</p> <p>2. $\sqrt{39}$</p> <p>3. $-\frac{1}{2}$</p> <p>4. 120°</p> <p>5. $5\sqrt{13}$</p> <p>6. $\approx 78^\circ$</p> <p>7. $\sqrt{367}$</p> <p>8. $\approx 81.8^\circ$</p> <p>9. $\approx 94.1^\circ$</p> <p>10. SSS or SAS</p> <p>11. 90°</p> <p>12. ≈ 7.2</p>	<p>13. False</p> <p>14. $\frac{9}{16}$</p> <p>15. 17</p> <p>16. $\sqrt{13 + 6\sqrt{3}}$</p> <p>17. $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$</p> <p>18. $C = 90^\circ$</p> <p>19. ≈ 7.9</p> <p>20. $\approx 16.3^\circ$</p> <p>21. $10\sqrt{13}$ miles</p> <p>22. $\approx 78.5^\circ$</p> <p>23. ≈ 63.7 ft</p> <p>24. ≈ 28.8 km</p>
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Additional Practice Answers

25. $\frac{5}{13}$	27. $\frac{7}{4}$
26. $\frac{12}{13}$	28. $\frac{1}{2}$

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. A careful way to see it: Side a is opposite angle A ; the two sides meeting at A are b and c . That gives a quick check on the answer.
2. This is SAS: the angle $A = 60^\circ$ sits between its two sides $b = 5$ and $c = 7$, and a is opposite it. Apply the Law of Cosines: $a^2 = b^2 + c^2 - 2bc \cos A = 25 + 49 - 2(5)(7) \cos 60^\circ = 74 - 70 \cdot \frac{1}{2} = 74 - 35 = 39$. Take the square root (don't forget this final step): $a = \sqrt{39}$.
3. One steady path is: $169 = 49 + 64 - 2(7)(8) \cos C$, so $169 = 113 - 112 \cos C$, giving $112 \cos C = -56$ and $\cos C = -\frac{1}{2}$. That gives a quick check on the answer.
4. Cosine is $-\frac{1}{2}$ at 120° (which is in $[0, 180^\circ]$, the only range a triangle angle can live in).
5. Side a is opposite the right angle, so it's the hypotenuse. Since $\cos 90^\circ = 0$, the $-2bc \cos A$ term vanishes: $a^2 = b^2 + c^2 = 100 + 225 = 325$. Then $a = \sqrt{325} = \sqrt{25 \cdot 13} = 5\sqrt{13}$. With the cosine term gone, the Law of Cosines is just the Pythagorean theorem here.
6. Largest angle is opposite the longest side (7). $49 = 25 + 36 - 60 \cos C \Rightarrow 60 \cos C = 12 \Rightarrow \cos C = \frac{1}{5} = 0.2$. Then $C = \arccos(0.2) \approx 78.46^\circ \approx 78^\circ$.
7. One steady path is: $x^2 = 81 + 169 - 2(9)(13) \cos 120^\circ = 250 - 234(-1/2) = 250 + 117 = 367$. So $x = \sqrt{367}$. That gives a quick check on the answer.
8. Start with the key idea: $256 = 100 + 196 - 280 \cos C \Rightarrow 280 \cos C = 40 \Rightarrow \cos C = \frac{1}{7} \approx 0.1429$. Then $C \approx \arccos(0.1429) \approx 81.79^\circ \approx 81.8^\circ$. That gives a quick check on the answer.
9. A careful way to see it: $121 = 64 + 49 - 2(8)(7) \cos B \Rightarrow 121 = 113 - 112 \cos B \Rightarrow 112 \cos B = -8 \Rightarrow \cos B = -\frac{1}{14} \approx -0.0714$. Then $B \approx \arccos(-0.0714) \approx 94.1^\circ$. That gives a quick check on the answer.
10. Three sides, or two sides with the angle between them. Law of Sines needs a side-angle pair.
11. One steady path is: $169 = 25 + 144 - 120 \cos C \Rightarrow 169 = 169 - 120 \cos C \Rightarrow \cos C = 0 \Rightarrow C = 90^\circ$. (Right triangle - 5-12-13.) That gives a quick check on the answer.
12. Start with the key idea: $a^2 = 36 + 100 - 2(6)(10) \cos 45^\circ = 136 - 120(\sqrt{2}/2) = 136 - 60\sqrt{2} \approx 136 - 84.85 \approx 51.15$. So $a \approx \sqrt{51.15} \approx 7.15$, which rounds to 7.2. That gives a quick check on the answer.
13. A careful way to see it: It works for any triangle - acute, right, or obtuse. This is

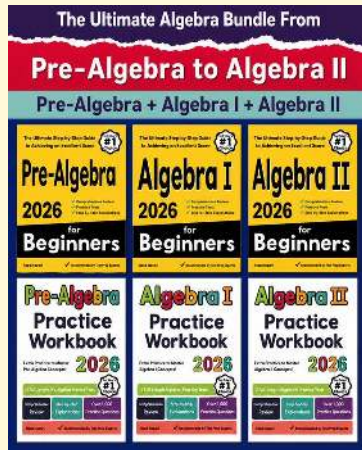
- the part to check before moving on, because it keeps the answer tied to the original question.
14. Keep the rule visible: $25 = 16 + 36 - 48 \cos B \Rightarrow 25 = 52 - 48 \cos B \Rightarrow 48 \cos B = 27 \Rightarrow \cos B = \frac{27}{48} = \frac{9}{16}$. This is the part to check before moving on, because it keeps the answer tied to the original question.
 15. Side c is across from the right angle C , so it's the hypotenuse. Because $\cos 90^\circ = 0$, $c^2 = a^2 + b^2 - 2ab \cos C = 64 + 225 - 0 = 289$, giving $c = \sqrt{289} = 17$. This is the familiar 8-15-17 Pythagorean triple.
 16. Start with the key idea: $\cos 150^\circ = -\frac{\sqrt{3}}{2}$. So $c^2 = 4 + 9 - 2(2)(3)(-\sqrt{3}/2) = 13 + 6\sqrt{3}$ and $c = \sqrt{13 + 6\sqrt{3}}$. That gives a quick check on the answer.
 17. A careful way to see it: Rearrange $a^2 = b^2 + c^2 - 2bc \cos A$: add $2bc \cos A$, then divide by $2bc$. That gives a quick check on the answer.
 18. Keep the rule visible: $\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{100 + 576 - 676}{480} = \frac{0}{480} = 0$, so $C = 90^\circ$. (Pythagorean triple 10-24-26 which scales to 5-12-13.) That gives a quick check on the answer.
 19. One steady path is: $c^2 = 121 + 81 - 2(11)(9)(\sqrt{2}/2) = 202 - 99\sqrt{2} \approx 202 - 140.0 \approx 62.0$. So $c \approx \sqrt{62} \approx 7.87$, which rounds to 7.9. That gives a quick check on the answer.
 20. Start with the key idea: $\cos A = \frac{576 + 625 - 49}{2(24)(25)} = \frac{1152}{1200} = 0.96$, so $A \approx \arccos(0.96) \approx 16.26^\circ \approx 16.3^\circ$. (Right triangle 7-24-25, so $C = 90^\circ$.) That gives a quick check on the answer.
 21. This is SAS: two sides (30 and 40) with included angle 60° . By the Law of Cosines: $d^2 = 30^2 + 40^2 - 2(30)(40) \cos 60^\circ = 900 + 1600 - 2400(1/2) = 2500 - 1200 = 1300$. So $d = \sqrt{1300} = \sqrt{100 \cdot 13} = 10\sqrt{13}$ miles (≈ 36.06 miles).
 22. Largest angle is across from longest side. $70^2 = 50^2 + 60^2 - 2(50)(60) \cos C \Rightarrow 4900 = 2500 + 3600 - 6000 \cos C \Rightarrow 6000 \cos C = 1200 \Rightarrow \cos C = 0.2$. So $C = \arccos(0.2) \approx 78.46^\circ \approx 78.5^\circ$.
 23. The angle at home plate between the line to second base and the line to first base is 45° (the diamond is a square, and the diagonal to second cuts the 90° corner in half). Apply Law of Cosines with sides 60.5 (home to mound) and 90 (home to first) with included angle 45° : $d^2 = 60.5^2 + 90^2 - 2(60.5)(90) \cos 45^\circ = 3660.25 + 8100 - 10890(\sqrt{2}/2) \approx 11760.25 - 7700.0 \approx 4060.25$. So $d \approx \sqrt{4060.25} \approx 63.72$ ft, which rounds to 63.7 ft.



24. At the turn point, the interior angle of the triangle is $180^\circ - (100^\circ - 30^\circ) = 180^\circ - 70^\circ = 110^\circ$ (the supplement of the change in bearing). Apply Law of Cosines with the two leg lengths and the included 110° : $d^2 = 20^2 + 15^2 - 2(20)(15) \cos 110^\circ \approx 400 + 225 - 600(-0.342) \approx 625 + 205.2 \approx 830.2$. So $d \approx \sqrt{830.2} \approx 28.81$ km, which rounds to 28.8 km.



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