Irrational Functions

Name: ____________________ Date: ______________ Score: _____ / 36

Quick Review

An **irrational function** packs a variable inside a radical: $f(x) = \sqrt{\text{stuff}(x)}$ or a cube root, fourth root, and so on. The two questions you ask first are always domain and range.

Domain. For an even-index radical, the radicand must be ≥ 0. For a cube root or any odd-index root, every real input is fine. If the function also has a denominator, add that restriction too.

Range. The principal square root is always ≥ 0, so $\sqrt{\text{stuff}} \geq 0$ – vertical shifts and reflections move the floor (or turn it into a ceiling). Quick check: $-2\sqrt{x+5}+1 \leq 1$ for every x in the domain.

Evaluating. Plug in carefully – $\sqrt{9} = 3$ (not ± 3). The radical symbol always returns the principal (non-negative) root.

Solving irrational equations. Isolate the radical, raise both sides to the matching power (square for $\sqrt{\ }$, cube for $\sqrt[3]{\ }$), solve, and **check** – squaring can introduce extraneous roots.

Common slips. Treating cube roots like square roots (they accept any real radicand). Forgetting that the *right* side of $\sqrt{\text{stuff}} = \text{expression}$ must also be non-negative (radical's output is). Dropping the check after squaring.

PRACTICE

For each function or equation, identify the domain and range when asked, evaluate, or solve and check.

1. Which is an irrational function: $f(x) = \sqrt{x+1}$, $3x+2$, x^2+5, $\dfrac{1}{x}$? ________
2. Find the domain of $f(x) = \sqrt{x-3}$. ________
3. Evaluate $f(9)$ for $f(x) = 2\sqrt{x}+5$. The table lists a few smaller inputs to help you see the pattern. ________

x	0	1	4
$2\sqrt{x}+5$	5	7	9

4. Find the range of $g(x) = \sqrt{x} - 4$. ________
5. Solve $\sqrt{x+4} = 6$. ________
6. Find the domain of $h(x) = \dfrac{\sqrt{x}}{x-4}$. ________
7. Find the domain of $f(x) = \sqrt{x+2} - 3$. ________
8. Find the range of $f(x) = \sqrt{x+2} - 3$. The table samples a few inputs inside the domain. ________

x	-1	2	7
$\sqrt{x+2}-3$	-2	-1	0

9. Mark TRUE or FALSE: The range of $y = \sqrt{x}$ includes negative numbers. ________
10. Find the domain of $p(x) = \dfrac{\sqrt{2x-5}}{x-3}$. ________
11. Solve $\sqrt{x+5} = x - 1$. Identify any extraneous roots. ________
12. Mark TRUE or FALSE: The cube-root function $f(x) = \sqrt[3]{x}$ is defined for all real x. ________

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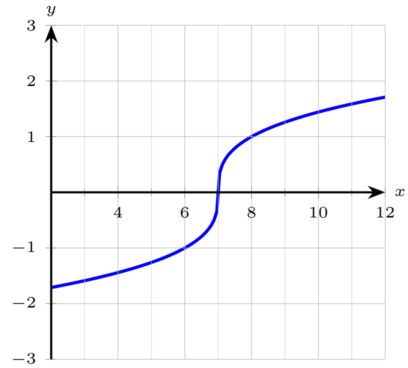
13. Find the domain of $f(x) = \sqrt[3]{x-7}$. The graph shows the cube-root curve; note where it lives along the x-axis. ________



14. Solve $\sqrt{2x+3} = 5$. ________

15. Find the range of $f(x) = -\sqrt{x} + 2$. ________

16. Find the domain of $f(x) = \sqrt{16 - x^2}$. ________

17. Evaluate $f(8)$ for $f(x) = \sqrt[3]{x} + 1$. ________

18. Solve $\sqrt{x-1} + 2 = 5$. ________

19. Find the domain of $f(x) = \sqrt{x^2 - 9}$. ________

20. Mark TRUE or FALSE: Squaring both sides of a radical equation can introduce extraneous solutions. ________

Word Problems

21. The time (seconds) for a free fall from height h (meters) is $t(h) = \sqrt{h/4.9}$. Find the time for a fall from 19.6 m, and state the domain of the model. ____

22. A car's stopping distance on a dry road is $d = \frac{v^2}{20}$ feet, where v is the speed in mph. Solve for v in terms of d to get an irrational function. Find v when $d = 125$ ft. ____

23. Solve $\sqrt{3x+4} = x$ and identify any extraneous solutions, showing the check step explicitly. ____

24. A pendulum's period is $T = 2\pi\sqrt{L/g}$, with L in meters and $g = 9.8$ m/s^2. Solve for L in terms of T, and find L for $T = 1$ second. ____

Additional Practice

25. Simplify $\sqrt{72}$. ________

26. Simplify $\sqrt{45}$. ________

27. Simplify $\sqrt[3]{64}$. ________

28. Solve $\sqrt{x+5} = 9$. ________

29. Solve $\sqrt{x} - 3 = 4$. ________

30. Domain of $y = \sqrt{x-6}$. ________

31. Add $3\sqrt{5} + 2\sqrt{5}$. ________

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32. Multiply $\sqrt{3} \cdot \sqrt{12}$. ________

33. Rationalize $\frac{4}{\sqrt{2}}$. ________

34. Write $x^{3/2}$ using radicals. ________

35. Simplify $(\sqrt{7})^2$. ________

36. Solve $\sqrt{x+1} < 4$. ________

Answer Keys

1. $\boxed{\sqrt{x+1}}$
2. $\boxed{x \geq 3}$
3. $\boxed{11}$
4. $\boxed{y \geq -4}$
5. $\boxed{x = 32}$
6. $\boxed{x \geq 0,\ x \neq 4}$
7. $\boxed{x \geq -2}$
8. $\boxed{y \geq -3}$
9. FALSE
10. $\boxed{x \geq \frac{5}{2},\ x \neq 3}$
11. $\boxed{x = 4}$
12. TRUE
13. all real numbers
14. $\boxed{x = 11}$
15. $\boxed{y \leq 2}$
16. $\boxed{-4 \leq x \leq 4}$
17. $\boxed{3}$
18. $\boxed{x = 10}$
19. $x \leq -3$ or $x \geq 3$
20. TRUE
21. $t = 2$ s; domain $h \geq 0$
22. $v = \sqrt{20d}$; $v = 50$ mph at $d = 125$ ft
23. $\boxed{x = 4}$
24. $L = \frac{gT^2}{4\pi^2}$; $L \approx 0.248$ m

Additional Practice Answers

25. $\boxed{6\sqrt{2}}$
26. $\boxed{3\sqrt{5}}$
27. $\boxed{4}$
28. $\boxed{x = 76}$
29. $\boxed{x = 49}$
30. $\boxed{x \geq 6}$
31. $\boxed{5\sqrt{5}}$
32. $\boxed{6}$
33. $\boxed{2\sqrt{2}}$
34. $\boxed{\sqrt{x^3}}$
35. $\boxed{7}$
36. $\boxed{-1 \leq x < 15}$

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. A careful way to see it: Only the first has a variable inside a radical. This is the part to check before moving on, because it keeps the answer tied to the original question.

2. A square root needs a non-negative radicand, so require $x - 3 \geq 0$. Solving gives $x \geq 3$, i.e. $[3, \infty)$. (For an odd root like a cube root you wouldn't need this – only even-index radicals restrict the domain.)

3. One steady path is: $\sqrt{9} = 3$, so $f(9) = 2(3) + 5 = 11$. (Extend the table one step: at $x = 9$, $\sqrt{9} = 3$ and $f = 2(3) + 5 = 11$.) That gives a quick check on the answer.

4. On its domain $\sqrt{x} \geq 0$, with its smallest value 0 at $x = 0$. Subtracting 4 shifts every output down by 4, so $\sqrt{x} - 4 \geq -4$. The lowest point is $g(0) = -4$, and the range is $[-4, \infty)$.

5. A careful way to see it: Square: $x + 4 = 36$, so $x = 32$. **Check:** $\sqrt{36} = 6$ ✓. This is the part to check before moving on, because it keeps the answer tied to the original question.

6. Two conditions stack here. The square root on top needs $x \geq 0$, and the denominator can't be zero, so $x \neq 4$. Combine both: $x \geq 0$ with $x = 4$ punched out.

7. The radicand must be non-negative: $x + 2 \geq 0$, so $x \geq -2$. The -3 sits outside the radical, so it shifts the graph down but never affects which inputs are allowed.

8. Start with the key idea: $\sqrt{x+2} \geq 0$, so $\sqrt{x+2} - 3 \geq -3$. Lowest value is at $x = -2$: $f(-2) = -3$. (The table values climb as x grows, but they never dip below -3.) That gives a quick check on the answer.

9. A careful way to see it: The principal square root is always ≥ 0. Range: $[0, \infty)$ only. That gives a quick check on the answer.

10. Keep the rule visible: Radical: $2x - 5 \geq 0$, so $x \geq \frac{5}{2}$. Denominator: $x \neq 3$. Both apply. That gives a quick check on the answer.

11. Right side must be ≥ 0: $x \geq 1$. Square: $x + 5 = (x-1)^2 = x^2 - 2x + 1$, so $x^2 - 3x - 4 = 0$ and $(x-4)(x+1) = 0$. Candidates: $x = 4, -1$. Reject $x = -1$ (fails $x \geq 1$). **Check** $x = 4$: $\sqrt{9} = 3$, and $4 - 1 = 3$ ✓.

12. Start with the key idea: Cube roots accept any real radicand – positive, zero, or negative. That gives a quick check on the answer.

13. Cube root accepts any real input – the curve continues without a break in either direction, so there is no value of x it skips.

14. Keep the rule visible: Square: $2x + 3 = 25$, so $x = 11$. **Check:** $\sqrt{25} = 5$ ✓. This is the part to check before moving on, because it keeps the answer tied to the original question.

15. One steady path is: $\sqrt{x} \geq 0$, so $-\sqrt{x} \leq 0$ and $-\sqrt{x} + 2 \leq 2$. Range $(-\infty, 2]$. This is the part to check before moving on, because it keeps the answer tied to the original question.

16. Start with the key idea: $16 - x^2 \geq 0$ means $x^2 \leq 16$, so $|x| \leq 4$. This is the part to check before moving on, because it keeps the answer tied to the original question.

17. A careful way to see it: Since $2^3 = 8$, the cube root $\sqrt[3]{8} = 2$. Then $f(8) = \sqrt[3]{8} + 1 = 2 + 1 = 3$. That gives a quick check on the answer.

18. Keep the rule visible: Isolate: $\sqrt{x-1} = 3$. Square: $x - 1 = 9$, so $x = 10$. **Check:** $\sqrt{9} + 2 = 5$ ✓. That gives a quick check on the answer.

19. One steady path is: $x^2 - 9 \geq 0$ means $x^2 \geq 9$, so $|x| \geq 3$. Two-piece domain $(-\infty, -3] \cup [3, \infty)$. That gives a quick check on the answer.

20. Squaring is not reversible – it can create new solutions that don't satisfy the original. That's why the check is mandatory.

21. At $h = 19.6$: $t = \sqrt{19.6/4.9} = \sqrt{4} = 2$ seconds. Domain: $h/4.9 \geq 0$ requires $h \geq 0$ (also physically; negative height is meaningless). The radicand controls the domain; the output is automatically ≥ 0.

22. From $d = \frac{v^2}{20}$, solve for v: $v^2 = 20d$, so $v = \sqrt{20d}$ (positive root because speed is positive). At $d = 125$: $v = \sqrt{2500} = 50$ mph. **Check:** $d = \frac{50^2}{20} = \frac{2500}{20} = 125$ ft ✓. (This is the inverse of the stopping-distance formula; forensic crash analysts use it to estimate speed from skid marks.)

23. Right side ≥ 0 requires $x \geq 0$. Square both sides: $3x + 4 = x^2$, so $x^2 - 3x - 4 = 0$. Factor: $(x-4)(x+1) = 0$, giving $x = 4$ or $x = -1$. Reject $x = -1$ (violates $x \geq 0$). **Check** $x = 4$: $\sqrt{3(4) + 4} = \sqrt{16} = 4$, matching the right side 4 ✓. Final answer: $x = 4$. (The extraneous root $x = -1$ does satisfy the squared equation, but plugging in: $\sqrt{1} = 1 \neq -1$. Squaring created the phantom root – the check removes it.)

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24. From $T = 2\pi\sqrt{L/g}$, divide and square: $\frac{T}{2\pi} = \sqrt{L/g} \Rightarrow \frac{T^2}{4\pi^2} = \frac{L}{g} \Rightarrow L = \frac{gT^2}{4\pi^2}$. At $T = 1$: $L = \frac{9.8}{4\pi^2} \approx \frac{9.8}{39.48} \approx 0.248$ m, or about 25 cm. (A one-second-period pendulum is about a quarter meter long; that matches intuition for an old wall-clock pendulum.)

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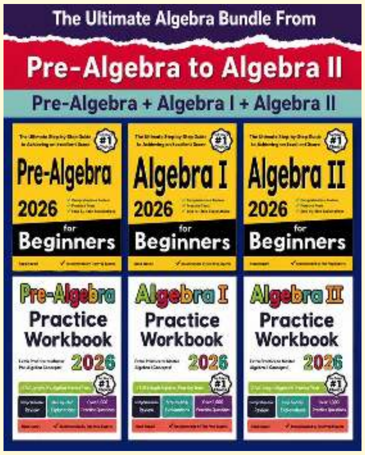

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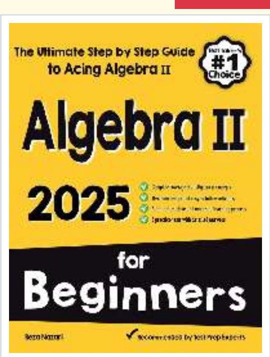

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