Inverse Variation

Name: _____ Date: _____ Score: _____ / 37

Quick Review

Inverse variation means two quantities have a constant product: $xy = k$, or equivalently $y = \frac{k}{x}$ for some nonzero constant k (the *constant of variation*). As x doubles, y halves — they move in opposite directions, multiplicatively. This is *not* a linear relationship, despite the name; the graph is a hyperbola with the coordinate axes as asymptotes.

Finding the constant. Use one (x, y) pair: $k = xy$. Then the model is $y = \frac{k}{x}$ for any other input. Quick check: $y = 6$ when $x = 4$ gives $k = 24$, so $y = \frac{24}{x}$. When $x = 5$, $y = \frac{24}{5} = 4.8$.

Inverse-square variation (like gravity or light intensity) uses $y = \frac{k}{x^2}$. Same logic: find k from one data point, then evaluate. Light intensity at $d = 2$ m equals 100 lux means $k = 100 \cdot 4 = 400$, and $I(d) = \frac{400}{d^2}$. At $d = 5$: $I = 16$ lux.

Joint inverse variation: $z = \frac{k}{xy}$ has z varying inversely with the *product* xy. **Common traps**: confusing direct ($y = kx$) and inverse ($y = k/x$); writing $y = \frac{x}{k}$ by accident (which is direct, with constant $\frac{1}{k}$). And: $y = \frac{k}{x}$ is undefined at $x = 0$ — the graph never touches either axis.

PRACTICE

Set up the model, find k, and answer. Watch direct vs. inverse.

1. y varies inversely with x. General form? _____
2. Inverse variation: $y = 6$ when $x = 4$. Find k. _____
3. The table shows an inverse variation. Find y when $x = 4$. _____

x	2	3	5	6
y	30	20	12	10

4. Pressure P varies inversely with volume V. $P = 80$ at $V = 6$. Find P at $V = 10$. _____
5. What feature of the table below confirms it is an inverse variation? _____

x	1	2	3	4
y	12	6	4	3

6. z varies inversely with both x and y. General form? _____
7. Inverse-square: $I = k/d^2$, $I = 100$ at $d = 2$. Find k. _____
8. Does the graph of $y = k/x$ pass through the origin? _____
9. Inverse variation, $y = 8$ at $x = 3$. Equation? _____
10. $F = k/r^2$, $F = 18$ at $r = 4$. Find F at $r = 6$. _____
11. Inverse variation, $y = 12$ at $x = 3$. Find y at $x = 9$. _____
12. Direct variation: general form? _____
13. For $y = k/x$ with $k > 0$ and $x > 0$: as x rises, y does what? _____
14. Inverse variation, $y = 20$ at $x = 2$. Find y at $x = 8$. _____
15. $y = \frac{k}{x}$ at $x = 0$ is defined. True or false? _____

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16. Does the table show inverse variation? _____

x	1	2	3	4
y	3	6	9	12

17. Travel time T vs. speed S for a fixed distance: what kind of variation? _____

18. $y = \frac{36}{x}$; find y at $x = -3$. _____

19. $xy = 24$; if $x = 6$, find y _____

20. y varies inversely with x. $y = 10$ at $x = 2$. Find y at $x = 5$. _____

Word Problems

21. At constant temperature, the pressure P (in psi) of a sealed gas varies inversely with the volume V (in liters). When $V = 8$ liters, $P = 60$ psi. Find P when $V = 12$ liters. ____

22. The intensity of a sound, I, varies inversely with the square of the distance d from the source. At $d = 3$ meters, the intensity is 50 units. Find the intensity at $d = 10$ meters. ____

23. It takes 6 workers 12 days to paint a building. Assuming the number of days varies inversely with the number of workers (everyone works at the same steady rate), how many days will 9 workers need? ____

24. The gravitational force between two objects varies inversely with the square of the distance between them. If the force is 200 newtons when they are 4 meters apart, what is the force when they are 8 meters apart? ____

Additional Practice

25. If y varies directly with x and $y = 18$ when $x = 6$, find k. _____

26. Direct variation with $k = 5$: write the equation. _____

27. If y varies inversely with x and $y = 4$ when $x = 3$, find k. _____

28. Inverse variation with $k = 20$: write the equation. _____

29. If $y = 7x$, find y when $x = 9$. _____

30. If $y = \frac{30}{x}$, find y when $x = 5$. _____

31. Joint variation: $z = kxy$, $z = 24$, $x = 3$, $y = 2$. Find k. _____

32. Combined variation: $y = \frac{kx}{z}$, $y = 10$, $x = 5$, $z = 2$. Find k. _____

33. Does $y = 2x + 1$ show direct variation? _____

34. Does $xy = 16$ show inverse variation? _____

35. If y varies directly and x doubles, what happens to y? _____

36. If y varies inversely and x doubles, what happens to y? _____

37. Find x if $y = \frac{18}{x}$ and $y = 6$. _____

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Answer Keys

1. $y = \frac{k}{x},\ k \neq 0$
2. 24
3. 15
4. 48
5. products xy all equal
6. $z = \frac{k}{xy}$
7. 400
8. no
9. $y = \frac{24}{x}$
10. 8
11. 4
12. $y = kx$
13. falls
14. 5
15. false
16. no (direct, $k = 3$)
17. inverse: $T = D/S$
18. -12
19. 4
20. 4
21. $P = 40$ psi
22. $I = 4.5$ units
23. 8 days
24. 50 newtons

Additional Practice Answers

25. $k = 3$
26. $y = 5x$
27. $k = 12$
28. $y = \frac{20}{x}$
29. 63
30. 6
31. 4
32. 4
33. no
34. yes
35. doubles
36. halves
37. 3

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. Inverse variation has the form $y = \frac{k}{x}$ — reciprocal of the input, scaled by k.

2. In inverse variation the product xy stays constant and equals k. Plug in the given pair: $k = xy = 4 \cdot 6 = 24$.

3. Every product xy is the same: $2 \cdot 30 = 60$, $5 \cdot 12 = 60$, and so on, so $k = 60$ and $y = \frac{60}{x}$. At $x = 4$: $y = \frac{60}{4} = 15$.

4. Inverse variation means the product PV is constant (Boyle's law). Find k from the given pair: $k = 80 \cdot 6 = 480$. Then at $V = 10$, $P = \frac{k}{V} = \frac{480}{10} = 48$ psi.

5. Multiply each pair: $1 \cdot 12$, $2 \cdot 6$, $3 \cdot 4$, $4 \cdot 3$ — all equal 12. A constant product $xy = k$ is the signature of inverse variation.

6. Keep the rule visible: Joint inverse variation: both variables in the denominator. That gives a quick check on the answer.

7. For inverse-square variation $I = \frac{k}{d^2}$, multiply both sides by d^2 to isolate k: $k = I\,d^2$. With $I = 100$ and $d = 2$: $k = 100 \cdot 2^2 = 100 \cdot 4 = 400$.

8. Start with the key idea: $y = \frac{k}{x}$ is undefined at $x = 0$. Vertical asymptote there. This is the part to check before moving on, because it keeps the answer tied to the original question.

9. Find the constant from the given pair: $k = xy = 3 \cdot 8 = 24$. The model is then $y = \frac{k}{x} = \frac{24}{x}$.

10. First get k: from $F = \frac{k}{r^2}$, $k = Fr^2 = 18 \cdot 4^2 = 18 \cdot 16 = 288$. Then at $r = 6$: $F = \frac{288}{6^2} = \frac{288}{36} = 8$.

11. Constant product: $k = xy = 3 \cdot 12 = 36$. Then at $x = 9$, $y = \frac{k}{x} = \frac{36}{9} = 4$. Notice tripling the input divided the output by 3.

12. Start with the key idea: y scales linearly with x. Compare to inverse: $y = \frac{k}{x}$. That gives a quick check on the answer.

13. A careful way to see it: Inverse means opposite direction. Doubling x halves y. That gives a quick check on the answer.

14. Constant product: $k = xy = 2 \cdot 20 = 40$. At $x = 8$: $y = \frac{40}{8} = 5$. Quadrupling the input quartered the output — the inverse relationship at work.

15. One steady path is: Denominator zero. Vertical asymptote at $x = 0$. This is the part to check before moving on, because it keeps the answer tied to the original question.

16. The products xy give $3, 12, 27, 48$ — not constant, so it is not inverse variation. But $y/x = 3$ throughout, so it is direct variation with $k = 3$.

17. A careful way to see it: Fixed distance D gives $T \cdot S = D$. Going twice as fast takes half the time. That gives a quick check on the answer.

18. Keep the rule visible: $y = \frac{36}{-3} = -12$. (Inverse variation works for negative inputs too, in the same branch.) That gives a quick check on the answer.

19. One steady path is: $y = \frac{24}{6} = 4$. Constant product means easy reuse of k. This is the part to check before moving on, because it keeps the answer tied to the original question.

20. Start with the key idea: Constant product gives $k = xy = 2 \cdot 10 = 20$. At $x = 5$: $y = \frac{20}{5} = 4$. That gives a quick check on the answer.

21. Use Boyle's law: $PV = k$. From the given pair, $k = 8 \cdot 60 = 480$. At $V = 12$: $P = \frac{480}{12} = 40$ psi. (Increasing the volume by $1.5\times$ drops the pressure by the same factor — $60 \div 1.5 = 40$. That's the inverse relationship in action.)

22. Model: $I = \frac{k}{d^2}$. From the given pair, $k = I \cdot d^2 = 50 \cdot 9 = 450$. At $d = 10$: $I = \frac{450}{100} = 4.5$ units. The inverse-square law explains why sound (and light) drop off quickly with distance — tripling the distance cuts the intensity to about $\frac{1}{9}$.

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23. One steady path is: $N \cdot D = k$. From the given pair: $6 \cdot 12 = 72$ worker-days for the whole job. With 9 workers: $D = \frac{72}{9} = 8$ days. (Whole-day answer makes sense for a planning problem.) That gives a quick check on the answer.

24. Start with the key idea: $F = \frac{k}{d^2}$. From the given pair, $k = 200 \cdot 16 = 3200$. At $d = 8$: $F = \frac{3200}{64} = 50$ newtons. (Doubling distance quarters the force — the $1/d^2$ hallmark.) That gives a quick check on the answer.

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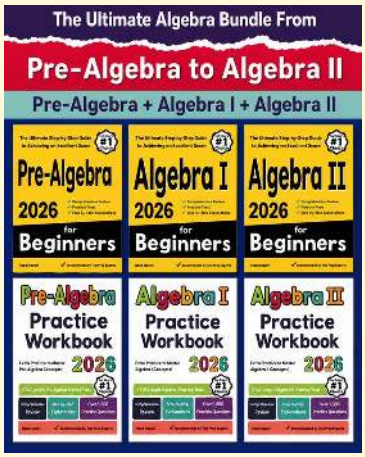

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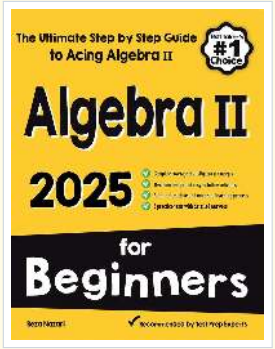

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