Inverse Relationship of Exponential and Logarithmic Functions

Name: ____________	Date: ____________	Score: ______ / 37

Quick Review

For any valid base $b > 0, b \neq 1$, the functions $f(x) = b^x$ and $g(x) = \log_b(x)$ are **inverses**. They undo each other:

- $b^{\log_b(x)} = x$ for $x > 0$, and
- $\log_b(b^x) = x$ for every real x.

Swap of domain and range. The exponential $y = b^x$ has domain all real numbers and range $y > 0$. The logarithm $y = \log_b(x)$ has domain $x > 0$ and range all real numbers. Inverses always swap the two.

Reflection across $y = x$. The graph of $y = \log_b(x)$ is the mirror image of $y = b^x$ across the line $y = x$. Every point (a, b^a) on the exponential corresponds to the point (b^a, a) on the logarithm — inputs and outputs trade places.

Two ways to write the same statement. $\log_b(y) = x \iff b^x = y$. This is the master conversion. Every log equation has an exponential twin, and vice versa. Solving $\log_4(64) = x$ and solving $4^x = 64$ give the same answer ($x = 3$).

Solving exponential equations using logs. When the unknown is in an exponent, take a logarithm with the same base. To solve $4^{x-1} = 28$: apply $\log_4$ to both sides, giving $x - 1 = \log_4(28)$, so $x = 1 + \log_4(28) \approx 3.40$.

Common slips. Confusing $\log_b(b^x) = x$ with $\log_b(x^b)$ (those are different). Reading the swapped point as (b, a) instead of (b^a, a). Forgetting that $b^x > 0$ for every real x — the exponential never hits zero.

PRACTICE

Switch between log and exponential forms; use the inverse identities; reflect graphs across $y = x$.

1. The inverse of $f(x) = b^x$ $(b > 0,\ b \neq 1)$ is what? ________
2. Rewrite $\log_4(64) = 3$ as an exponential equation. ________
3. Simplify $5^{\log_5(12)}$. ________
4. Simplify $\log_3(3^8)$. ________
5. How is the graph of $y = \log_2(x)$ related to $y = 2^x$? ________
6. Solve $e^{2\ln(x)} = 49$ for $x > 0$. ________
7. $f(x) = 5^x$ and $g(x) = \log_5(x)$. Compute $f(g(25))$. ________
8. Which inverse facts are TRUE for $b > 0,\ b \neq 1$? $b^{\log_b(x)} = x$ for $x > 0$; $\log_b(b^x) = x$ for real x; graphs reflect across $y = x$; same domain. ________
9. Point $(3, 8)$ lies on $y = 2^x$. What point lies on $y = \log_2(x)$? ________
10. Solve $4^{x-1} = 28$ to two decimals. ________

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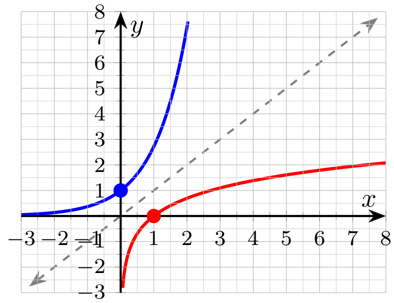
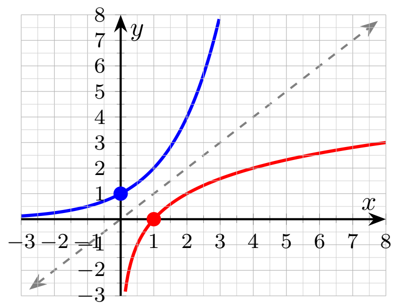
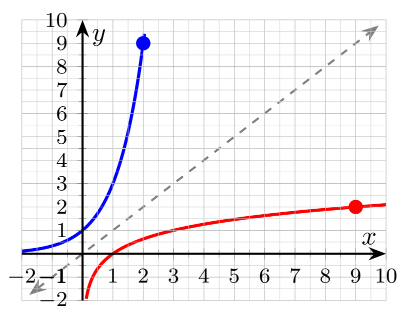
11. The graphs of $y = e^x$ and $y = \ln(x)$ meet which line? State the inverse-form rewrite of $e^x = 7$. ________



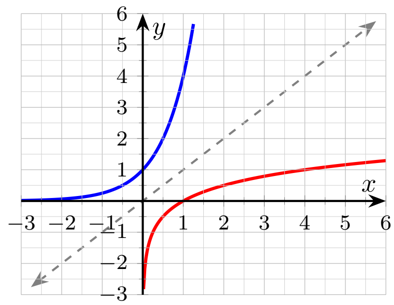
12. Which equation is equivalent to $2^x = 5$? ________



13. If $(2, 9)$ is on $y = 3^x$, which point is on $y = \log_3(x)$? ________



14. For the inverse pair $y = 4^x$ and $y = \log_4(x)$, state the domain and range of each. ________



15. Solve $\log_2(x + 1) = 3$ by switching to exponential form. ________

16. Solve $3^{2x} = 27$ by switching to logarithmic form. ________

17. Simplify $\ln(e^{x+3})$. ________

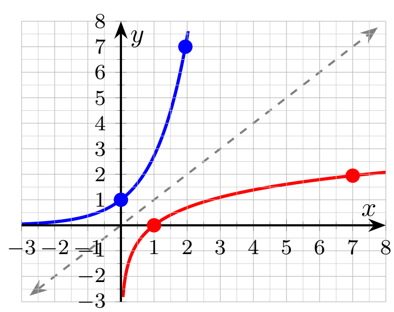
18. Simplify $e^{\ln(x^2+1)}$ (the argument is positive for every real x). ________

19. Convert $\log_2(32) = 5$ to exponential form, then verify. ________

20. If $f(x) = \log_3(x)$, what is $f^{-1}(2)$? ________

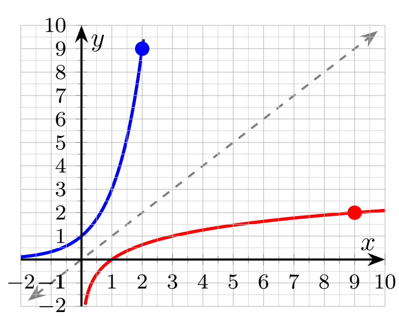
♦ Word Problems

21. The graph below shows $y = e^x$ (blue) and $y = \ln(x)$ (red), with the dashed line $y = x$ shown as the mirror line. Use the inverse relationship to rewrite $e^x = 7$ as a logarithmic equation, then solve to four decimals. ____



22. For $f(x) = 5^x$ and $g(x) = \log_5(x)$, verify that f and g are inverses by computing both $f(g(25))$ and $g(f(2))$. ____

23. The point $(2, 9)$ lies on the graph of $y = 3^x$. State the corresponding point on $y = \log_3(x)$ and explain how the dashed reflection line $y = x$ makes the swap visible. The two graphs are sketched below. ____



24. Solve $7^{x+2} = 50$ using the inverse relationship. Round to two decimal places. Verify your answer by back-substitution. ____

Additional Practice

25. Evaluate $3 \cdot 2^4$. ______

26. Find a in $y = a \cdot 3^x$ if $y(0) = 7$. ______

27. Growth or decay: $y = 12(0.8)^x$. ______

28. Growth or decay: $y = 5(1.12)^t$. ______

29. Find y when $x = 3$ for $y = 2^x + 1$. ______

30. Solve $2^x = 32$. ______

31. Solve $5^x = 125$. ______

32. Initial value of $P = 400(1.05)^t$. ______

33. Rate in $P = 900(1.08)^t$. ______

34. Rate in $A = 1200(0.92)^t$. ______

35. Double 60 three times. ______

36. Half 256 four times. ______

37. Horizontal asymptote of $y = 2^x + 5$. ______

Answer Keys

1. $f^{-1}(x) = \log_b(x)$
2. $4^3 = 64$
3. 12
4. 8
5. reflection across $y = x$
6. $x = 7$
7. 25
8. first three
9. $(8, 3)$
10. $x = 1 + \log_4(28) \approx 3.40$
11. $y = x;\ x = \ln(7)$
12. $x = \log_2(5)$
13. $(9, 2)$
14. 4^x: $D=\mathbb{R}$, $R=(0,\infty)$; $\log_4(x)$: $D=(0,\infty)$, $R=\mathbb{R}$
15. $x = 7$
16. $x = \dfrac{3}{2}$
17. $x + 3$
18. $x^2 + 1$
19. $2^5 = 32$
20. 9
21. $x = \ln(7) \approx 1.9459$
22. $f(g(25)) = 25;\ g(f(2)) = 2$
23. $(9, 2)$
24. $x \approx 0.01$

Additional Practice Answers

25. 48
26. $a = 7$
27. decay
28. growth
29. 9
30. $x = 5$
31. $x = 3$
32. 400
33. 8%
34. 8% decay
35. 480
36. 16
37. $y = 5$

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. To find an inverse you swap input and output, and the equation $y = b^x$ rewrites as $x = \log_b(y)$. So the inverse is $f^{-1}(x) = \log_b(x)$. Because the swap trades x and y, the domain and range trade places too.

2. Use the master conversion $\log_b(y) = x \iff b^x = y$. The base 4 stays the base; the value 3 becomes the exponent and 64 stays the result: $4^3 = 64$.

3. The exponent $\log_5(12)$ is exactly the power you must put on 5 to get 12, so raising 5 to it lands on 12. That is the inverse identity $b^{\log_b(x)} = x$ for $x > 0$ — and the bases match here, so it applies.

4. Ask: $3^? = 3^8$. The exponents must match, so the answer is 8. The log undoes the exponential of the same base: $\log_b(b^x) = x$.

5. They are inverses, and inverse graphs are mirror images across the line $y = x$. Every point $(a, 2^a)$ on the exponential has a twin $(2^a, a)$ on the log — the coordinates simply swap.

6. Keep the rule visible: Power rule: $2\ln x = \ln(x^2)$. Inverse: $e^{\ln(x^2)} = x^2 = 49$. Since $x > 0$, $x = 7$. That gives a quick check on the answer.

7. One steady path is: $g(25) = \log_5(25) = 2$, then $f(2) = 5^2 = 25$. The composition is the identity. That gives a quick check on the answer.

8. The last is wrong: domains differ. Exponential accepts all real x; log requires $x > 0$.

9. The functions $y = 2^x$ and $y = \log_2(x)$ are inverses, so their points swap coordinates: (a, b) on f becomes (b, a) on f^{-1}. So $(3, 8)$ becomes $(8, 3)$. Check: $2^3 = 8$ rewrites as $\log_2(8) = 3$.

10. Keep the rule visible: $x - 1 = \log_4(28) = \dfrac{\ln 28}{\ln 4} \approx \dfrac{3.332}{1.386} \approx 2.40$, so $x \approx 3.40$. This is the part to check before moving on, because it keeps the answer tied to the original question.

11. Inverse graphs reflect across $y = x$. To undo $e^x = 7$, apply $\ln$ to both sides: $\ln(e^x) = \ln(7)$, so $x = \ln(7) \approx 1.946$.

12. Start with the key idea: Apply $\log_2$ to both sides: $\log_2(2^x) = \log_2(5)$, so $x = \log_2(5) \approx 2.32$. That gives a quick check on the answer.

13. Inverse statements swap inputs and outputs: $3^2 = 9$ means $\log_3(9) = 2$. So $(9, 2)$ is on the logarithm graph.

14. Inverses swap domain and range. Exponential: all real x in, positive outputs only. Logarithm: positive inputs only, all real outputs.

15. One steady path is: $x+1 = 2^3 = 8$, so $x = 7$. Domain check: $x+1 = 8 > 0$, valid. This is the part to check before moving on, because it keeps the answer tied to the original question.

16. Switch to log form by applying $\log_3$ to both sides: $2x = \log_3(27)$. Since $27 = 3^3$, $\log_3(27) = 3$, so $2x = 3$ and $x = \frac{3}{2}$. Verify: $3^{2(3/2)} = 3^3 = 27$ ✓.

17. The natural log undoes e^u for any real exponent: $\ln(e^u) = u$. Here the exponent is $u = x + 3$, so the expression collapses to $x + 3$.

18. Keep the rule visible: $e^{\ln(u)} = u$ for $u > 0$. Since $x^2 + 1 \geq 1 > 0$ for all real x, the simplification is legal. That gives a quick check on the answer.

19. One steady path is: Base 2, result 32, exponent 5. Check: $2^5 = 2\cdot2\cdot2\cdot2\cdot2 = 32$ ✓. That gives a quick check on the answer.

20. Start with the key idea: $f^{-1}(x) = 3^x$, so $f^{-1}(2) = 3^2 = 9$. (Or by definition: $f^{-1}(2)$ is the x with $f(x) = 2$, i.e., $\log_3(x) = 2$, so $x = 3^2 = 9$.) That gives a quick check on the answer.

21. To undo e^x, apply its inverse $\ln$ to both sides. $\ln(e^x) = \ln(7)$ simplifies to $x = \ln(7)$ (since $\ln$ and e^x undo each other for any real x). Calculator: $\ln(7) \approx 1.9459$. The marked points $(1.9459, 7)$ on $y = e^x$ and $(7, 1.9459)$ on $y = \ln(x)$ show the swap exactly — inputs and outputs trade places across $y = x$.

22. Inverse functions undo each other — one check goes both directions. $g(25) = \log_5(25) = 2$ because $5^2 = 25$. Then $f(2) = 5^2 = 25$. So $f(g(25)) = 25$. The other direction: $f(2) = 5^2 = 25$, then $g(25) = \log_5(25) = 2$. So $g(f(2)) = 2$. Both compositions return the original input on their appropriate domains, which is the formal definition of inverses. (General identities: $f(g(x)) = x$ for $x > 0$, and $g(f(x)) = x$ for all real x.)

23. Inverse functions swap inputs and outputs, and that swap shows up geometrically as a reflection across $y = x$. If (a, b) is on the exponential, then (b, a) is on the logarithm. So $(2, 9)$ on $y = 3^x$ corresponds to $(9, 2)$ on $y = \log_3(x)$. Algebraically: $3^2 = 9$ rewrites as $\log_3(9) = 2$. The dashed line acts as the literal mirror — each colored point is the reflection of its partner across that line.

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24. Apply $\log_7$ to both sides (the inverse of base-7 exponential): $x + 2 = \log_7(50) = \frac{\ln 50}{\ln 7} \approx \frac{3.912}{1.946} \approx 2.0106$. So $x \approx 2.0106 - 2 = 0.0106$, or about 0.01. Verify: $7^{0.01+2} = 7^{2.0106} = 7^2 \cdot 7^{0.0106} \approx 49 \cdot 1.0185 \approx 49.9 \approx 50$ ✓. (The fact that the answer is so close to zero makes sense: $7^2 = 49$ is already a hair under 50, so just a tiny bump above the exponent 2 gets us to 50.)

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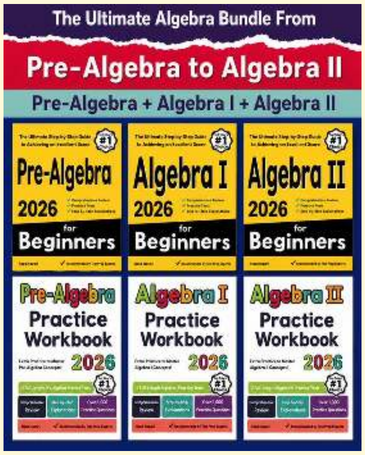

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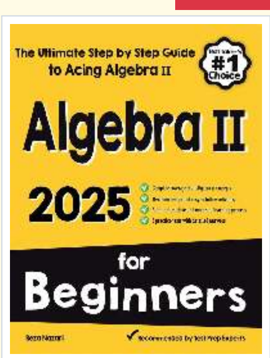

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