

# Infinite Geometric Series

Name: \_\_\_\_\_

Date: \_\_\_\_\_

Score: \_\_\_\_\_ / 30

## Q Quick Review

What happens to  $a_1 + a_1r + a_1r^2 + \dots$  when we never stop adding? Two possibilities, and the answer hinges on  $r$ .

**Convergence condition.** For a nonzero  $a_1$ , the infinite geometric series converges *if and only if*  $|r| < 1$ . The intuition:  $r^n \rightarrow 0$  when  $|r| < 1$ , so the finite formula  $\frac{a_1(1-r^n)}{1-r}$  approaches  $\frac{a_1}{1-r}$  as  $n \rightarrow \infty$ . If  $|r| \geq 1$ , the terms don't shrink to zero – they stay big (or grow) and the sum runs off to infinity (or oscillates).

**Sum formula (only when it converges).**  $S = \frac{a_1}{1-r}$ .

**The  $|r| = 1$  edge.**  $r = 1$  adds  $a_1$  forever (diverges to  $\pm\infty$ ).  $r = -1$  flips back and forth (no settled value). Either way, no convergence.

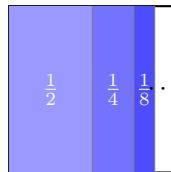
**Repeating decimals.** Any repeating decimal is an infinite geometric series in disguise.  $0.\overline{3} = 0.3 + 0.03 + 0.003 + \dots$  with  $a_1 = 0.3$ ,  $r = 0.1$ . So  $S = \frac{0.3}{1-0.1} = \frac{0.3}{0.9} = \frac{1}{3}$ .  $0.\overline{27}$  has  $a_1 = 0.27$ ,  $r = 0.01$ , giving  $\frac{0.27}{0.99} = \frac{27}{99} = \frac{3}{11}$ .

**Common slips.** Using the formula when  $|r| \geq 1$  – the answer is meaningless (the series diverges). Forgetting the absolute value:  $r = -0.5$  converges (since  $|-0.5| = 0.5 < 1$ ) but  $r = -1.2$  diverges (since  $|-1.2| = 1.2 > 1$ ). Mistaking  $a_1$  for the first power of  $r - a_1$  is the actual first term you see, including any leading constant.

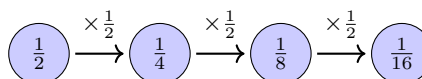
## PRACTICE

Decide whether each infinite geometric series converges. If it does, find the sum using  $S = a_1/(1-r)$ .

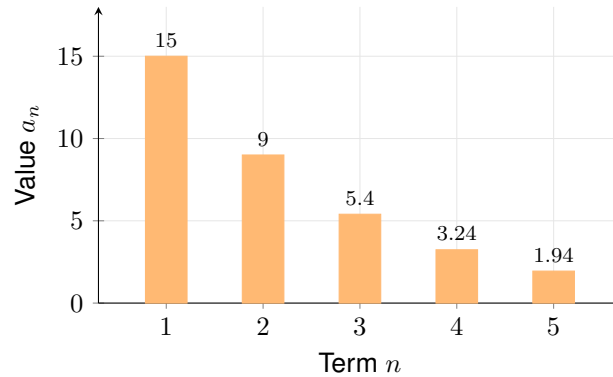
- Does  $\sum_{k=0}^{\infty} 6 \cdot \left(\frac{1}{2}\right)^k$  converge? If yes, find the sum. \_\_\_\_\_
- Does  $\sum_{k=0}^{\infty} 4 \cdot 2^k$  converge? \_\_\_\_\_
- Write  $0.\overline{3}$  as a fraction. \_\_\_\_\_
- An infinite geometric series has  $r = \frac{1}{4}$  and  $S = 20$ . Find  $a_1$ . \_\_\_\_\_
- Find the sum of the shaded area shown. \_\_\_\_\_



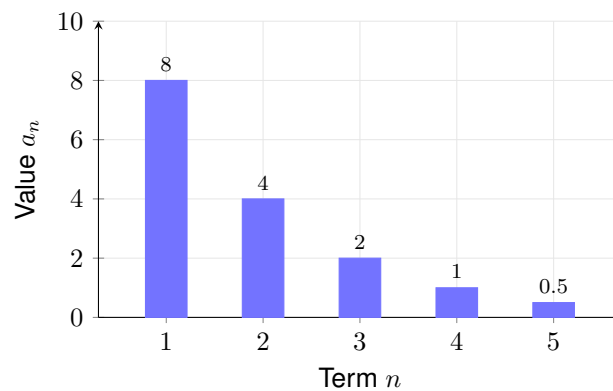
- Find  $\sum_{k=0}^{\infty} 3(0.4)^k$ . \_\_\_\_\_
- Does  $5 + 5(-1.2) + 5(-1.2)^2 + \dots$  converge? \_\_\_\_\_
- Find the sum:  $12 - 6 + 3 - \frac{3}{2} + \dots$  \_\_\_\_\_
- Write  $0.\overline{27}$  as a fraction in lowest terms. \_\_\_\_\_
- Compute  $\sum_{k=1}^{\infty} \frac{1}{2^k}$ . \_\_\_\_\_



11. A ball rebounds to  $\frac{3}{5}$  of its previous height. If the first rebound is 15 feet, find the total of all rebound heights. \_\_\_\_\_



12. True or False:  $r = -1$  makes an infinite geometric series converge. \_\_\_\_\_
13. Find  $\sum_{k=0}^{\infty} \left(-\frac{1}{3}\right)^k$ . \_\_\_\_\_
14. Write  $0.\bar{6}$  as a fraction. \_\_\_\_\_
15. An infinite geometric series converges to 24 with  $a_1 = 18$ . Find  $r$ . \_\_\_\_\_
16. Find the sum:  $9 + 3 + 1 + \frac{1}{3} + \dots$  \_\_\_\_\_
17. Compute  $\sum_{k=0}^{\infty} 5 \left(-\frac{1}{2}\right)^k$ . \_\_\_\_\_
18. True or False: every infinite geometric series with  $a_1 > 0$  and  $r > 0$  converges. \_\_\_\_\_
19. Find the bar-chart series sum:  $a_1 = 8, r = \frac{1}{2}$ . \_\_\_\_\_



20. A pendulum's arc length is 20 cm on the first swing, and each later swing is 80% of the previous arc. Find the total arc length over all swings (assuming it never stops). \_\_\_\_\_



**◆ Word Problems**

21. A super ball is dropped from a height of 9 feet. Each bounce reaches  $\frac{2}{3}$  of the previous bounce's height. \_\_\_\_\_  
What is the total distance (up and down) the ball travels before coming to rest?
22. A factory's output drops by 10% each year due to aging equipment. If it produced 5,000 widgets in year 1, \_\_\_\_\_  
find the total number of widgets produced over all future years.
23. Express  $0.\overline{45}$  as a fraction in lowest terms using an infinite geometric series. \_\_\_\_\_
24. A drug-dosing model: a patient takes 80 mg daily, and the body retains 50% of the previous day's amount \_\_\_\_\_  
each day. The long-run level (after many days) is the sum of the residuals from all prior doses, plus today's  
new dose. Compute this long-run equilibrium dose.

**Additional Practice**

25. Find the next term: 4, 9, 14, 19, ... \_\_\_\_\_
26. Find  $a_{10}$  if  $a_1 = 3$  and  $d = 5$ . \_\_\_\_\_
27. Find the next term: 2, 6, 18, 54, ... \_\_\_\_\_
28. Find  $a_6$  if  $a_1 = 5$  and  $r = 2$ . \_\_\_\_\_
29. Sum  $1 + 2 + 3 + \cdots + 20$ . \_\_\_\_\_
30. Find  $S_5$  for 3, 6, 12, 24, 48. \_\_\_\_\_



Answer Keys

|                                    |                       |
|------------------------------------|-----------------------|
| 1. converges to 12                 | 14. $\frac{2}{3}$     |
| 2. No (diverges)                   | 15. $r = \frac{1}{4}$ |
| 3. $\frac{1}{3}$                   | 16. $\frac{27}{2}$    |
| 4. $a_1 = 15$                      | 17. $\frac{10}{3}$    |
| 5. 1                               | 18. False             |
| 6. 5                               | 19. 16                |
| 7. No (diverges)                   | 20. 100 cm            |
| 8. 8                               | 21. 45 ft             |
| 9. $\frac{3}{11}$                  | 22. 50,000 widgets    |
| 10. 1                              | 23. $\frac{5}{11}$    |
| 11. 37.5 ft                        | 24. 160 mg            |
| 12. False                          |                       |
| 13. $\frac{3}{4}$                  |                       |
| <b>Additional Practice Answers</b> |                       |
| 25. 24                             | 28. 160               |
| 26. 48                             | 29. 210               |
| 27. 162                            | 30. 93                |

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

- A careful way to see it:  $|r| = \frac{1}{2} < 1$ , so it converges.  $a_1 = 6$ , so  $S = \frac{6}{1 - 1/2} = \frac{6}{1/2} = 12$ . That gives a quick check on the answer.
- Keep the rule visible:  $|r| = 2 \geq 1$ . The terms 4, 8, 16, 32, ... blow up, so the sum runs to infinity. That gives a quick check on the answer.
- One steady path is:  $0.\overline{3} = 0.3 + 0.03 + 0.003 + \dots$ . Geometric with  $a_1 = 0.3$ ,  $r = 0.1$ .  $S = \frac{0.3}{0.9} = \frac{1}{3}$ . That gives a quick check on the answer.
- Start with the key idea:  $S = \frac{a_1}{1 - r} = \frac{a_1}{3/4} = \frac{4a_1}{3} = 20$ . So  $a_1 = 20 \cdot \frac{3}{4} = 15$ . That gives a quick check on the answer.
- A careful way to see it:  $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots$  is geometric with  $a_1 = \frac{1}{2}$  and  $r = \frac{1}{2}$ .  $S = \frac{1/2}{1 - 1/2} = \frac{1/2}{1/2} = 1$ . (The picture makes it obvious – the shaded region fills the whole square.) That gives a quick check on the answer.
- Keep the rule visible:  $|0.4| < 1$ , so converges.  $S = \frac{3}{1 - 0.4} = \frac{3}{0.6} = 5$ . This is the part to check before moving on, because it keeps the answer tied to the original question.
- One steady path is:  $|r| = 1.2 \geq 1$ , so the series diverges. (The terms grow in magnitude while alternating sign – no settled value.) That gives a quick check on the answer.
- Ratio:  $-6/12 = -\frac{1}{2}$ .  $|r| = \frac{1}{2} < 1$ , so it converges.  $S = \frac{12}{1 - (-1/2)} = \frac{12}{3/2} = 8$ . (Negative  $r$  is fine – only  $|r| < 1$  matters for convergence.)
- A careful way to see it:  $0.\overline{27} = 0.27 + 0.0027 + \dots$ . Geometric with  $a_1 = 0.27$ ,  $r = 0.01$ .  $S = \frac{0.27}{0.99} = \frac{27}{99} = \frac{3}{11}$  (divide by 9). That gives a quick check on the answer.
- Pull out  $\frac{1}{2}$ :  $\sum_{k=1}^{\infty} \frac{1}{2^k} = \frac{1}{2} + \frac{1}{4} + \dots$  with  $a_1 = \frac{1}{2}$  and  $r = \frac{1}{2}$ .  $S = \frac{1/2}{1/2} = 1$ . (Same as the shaded-square sum.)
- One steady path is:  $a_1 = 15$ ,  $r = \frac{3}{5}$ .  $S = \frac{15}{1 - 3/5} = \frac{15}{2/5} = 15 \cdot \frac{5}{2} = \frac{75}{2} = 37.5$  feet. (The bars show how quickly the rebound heights shrink.) That

- gives a quick check on the answer.
- Start with the key idea:  $|-1| = 1$ , which fails the strict condition  $|r| < 1$ . The partial sums oscillate between two values (e.g.  $a_1, 0, a_1, 0, \dots$  for  $a_1 + (-a_1) + a_1 + \dots$ ), never settling. That gives a quick check on the answer.
  - A careful way to see it:  $a_1 = 1$  (since  $(-\frac{1}{3})^0 = 1$ ),  $r = -\frac{1}{3}$ .  $|r| = \frac{1}{3} < 1$ , so converges.  $S = \frac{1}{1 - (-1/3)} = \frac{1}{4/3} = \frac{3}{4}$ . That gives a quick check on the answer.
  - Keep the rule visible:  $a_1 = 0.6$ ,  $r = 0.1$ .  $S = \frac{0.6}{0.9} = \frac{6}{9} = \frac{2}{3}$ . This is the part to check before moving on, because it keeps the answer tied to the original question.
  - One steady path is:  $24 = \frac{18}{1 - r}$ , so  $24(1 - r) = 18 \Rightarrow 1 - r = \frac{18}{24} = \frac{3}{4} \Rightarrow r = \frac{1}{4}$ . (Quick check:  $|r| = \frac{1}{4} < 1 \checkmark$ .) That gives a quick check on the answer.
  - Start with the key idea:  $r = 3/9 = \frac{1}{3}$ .  $S = \frac{9}{1 - 1/3} = \frac{9}{2/3} = \frac{27}{2}$ . This is the part to check before moving on, because it keeps the answer tied to the original question.
  - A careful way to see it:  $a_1 = 5$ ,  $r = -\frac{1}{2}$ .  $|r| = \frac{1}{2} < 1 \checkmark$ .  $S = \frac{5}{1 - (-1/2)} = \frac{5}{3/2} = \frac{10}{3}$ . That gives a quick check on the answer.
  - Convergence depends on  $|r| < 1$ , not on the signs of  $a_1$  or  $r$ .  $a_1 = 2$ ,  $r = 3$  gives  $2 + 6 + 18 + \dots$  – positive and increasing, but it diverges.
  - One steady path is:  $S = \frac{8}{1 - 1/2} = \frac{8}{1/2} = 16$ . (The bar chart makes the geometric shrinkage visible – after term 5, there's almost nothing left to add.) That gives a quick check on the answer.
  - Start with the key idea:  $a_1 = 20$ ,  $r = 0.8$ .  $|r| < 1 \checkmark$ .  $S = \frac{20}{1 - 0.8} = \frac{20}{0.2} = 100$  cm. (Reality check: positive total, larger than the first swing – both expected.) That gives a quick check on the answer.
  - The first drop is 9 feet (down only). After that, each bounce goes up and then back down by the same height, so the total up-and-down contribution from the bounces is  $2(9 \cdot \frac{2}{3} + 9 \cdot (\frac{2}{3})^2 + \dots)$ . The geometric sum inside is



$$\frac{9 \cdot 2/3}{1 - 2/3} = \frac{6}{1/3} = 18. \text{ Doubling: } 2 \cdot 18 = 36. \text{ Total: } 9 + 36 = 45 \text{ feet.}$$

(Reality check: the ball falls 9 ft, then bounces small amounts forever – the total is finite because the bounces shrink geometrically.)

**22.** Keep the rule visible:  $a_1 = 5,000$ ,  $r = 0.9$  (each year is 90% of the previous).

$$|r| < 1 \checkmark. S = \frac{5,000}{1 - 0.9} = \frac{5,000}{0.1} = 50,000 \text{ widgets. (Reality check: a 10\%}$$

drop is slow decay, so the total is 10 times the first year – this matches the rule  $S = a_1/(1 - r)$  when  $r$  is close to 1.) That gives a quick check on the answer.

**23.** Write  $0.\overline{45} = 0.45 + 0.0045 + 0.000045 + \dots$ . Geometric with  $a_1 = 0.45 =$

$$\frac{45}{100} \text{ and } r = 0.01 = \frac{1}{100}. \text{ Sum: } S = \frac{45/100}{1 - 1/100} = \frac{45/100}{99/100} = \frac{45}{99} = \frac{5}{11}$$

(divide top and bottom by 9).

**24.** At equilibrium, today's level is 80 mg new + 80(0.5) from yesterday + 80(0.5)<sup>2</sup> from the day before +  $\dots$ , an infinite geometric series with  $a_1 = 80$  and  $r = 0.5$ .

$$S = \frac{80}{1 - 0.5} = \frac{80}{0.5} = 160 \text{ mg. (Reality check: equilibrium should be larger than}$$

a single dose – 160 mg, double of 80, makes sense because with 50% retention the long-run level settles at twice the daily intake.)



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