

# Independent and Dependent Events

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Score: \_\_\_\_\_ / 35

## Q Quick Review

Two events  $A$  and  $B$  are **independent** when one happening doesn't change the probability of the other. The clean test:  $P(A \cap B) = P(A) \cdot P(B)$ . Equivalently,  $P(A | B) = P(A)$  – knowing  $B$  tells you nothing new about  $A$ .

**Dependent** just means *not* independent. The first event leaves a fingerprint on the second. Drawing two cards *without* replacement is the classic case: pull a red, and the deck now has one fewer red and one fewer card total – the next draw lives in a different world.

**With or without replacement?** That little phrase decides the answer. *With* replacement restores the bag (independent draws). *Without* replacement shrinks the pool (dependent).

**Don't confuse independent with mutually exclusive.** Mutually exclusive means the two events can't happen together ( $P(A \cap B) = 0$ ). If both events have positive probability, mutually exclusive events *can't* be independent (the product  $P(A)P(B) > 0$  would have to match a zero – impossible).

**Multi-step trick.** For a string of independent events – flip, roll, spin – multiply the individual probabilities. If any step is dependent, update the probability for that step using what's left after the earlier step.

**Common slips.** Calling separate experiments dependent just because they happen in sequence. Forgetting to update the denominator after a no-replacement draw. Multiplying probabilities when you should be adding (or vice versa).

## PRACTICE

Decide independence or compute the joint/conditional probability requested.

1. Find  $P(A \cap B)$  if  $A, B$  are independent,  $P(A) = 0.3, P(B) = 0.4$ . \_\_\_\_\_
2. Are rolling a 4 and flipping heads independent? \_\_\_\_\_
3. Draw without replacement: independent or dependent? \_\_\_\_\_
4.  $P(A \cap B)$  if  $P(A) = 0.5, P(B) = 0.6$ , independent \_\_\_\_\_
5.  $P(A | B)$  when  $A, B$  independent and  $P(A) = 0.7$  \_\_\_\_\_
6. A fair 4-section spinner is spun twice; the single-spin probabilities are shown. Find  $P(2 \text{ then } 3)$ . \_\_\_\_\_

outcome	1	2	3	4
$P$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

7. Using the bag below, find  $P(\text{two reds, no replacement})$ . \_\_\_\_\_

Color	Count
Red	6
Blue	4
Total	10

8. Bag: 6 red, 4 blue.  $P(\text{two reds, with replacement})$  \_\_\_\_\_
9.  $P(A) = 0.5, P(B) = 0.4, P(C) = 0.2$  mutually independent. Find  $P(A \cap B \cap C^c)$ . \_\_\_\_\_
10. If  $P(A \cap B) = 0$  and  $P(A), P(B) > 0$ , independent? \_\_\_\_\_
11. Coin flip then die roll.  $P(\text{H and } 5)$  \_\_\_\_\_
12. Draw, replace, draw.  $P(\text{red both times}) = \left(\frac{1}{2}\right)^2$  \_\_\_\_\_
13.  $P(A) = 0.6, P(B) = 0.5, P(A \cap B) = 0.30$ . Independent? \_\_\_\_\_
14.  $P(A) = 0.4, P(B) = 0.5, P(A \cap B) = 0.15$ . Independent? \_\_\_\_\_



15. Roll two dice. Using the per-die tally of even vs. odd faces below, find  $P(\text{both even})$ . \_\_\_\_\_

Face type	Faces
Even	2, 4, 6
Odd	1, 3, 5

16. Three independent events, each  $p = 0.2$ .  $P(\text{all three})$  \_\_\_\_\_

17. Two events with  $P(A | B) = P(A)$ . Independent? \_\_\_\_\_

18. Drawing two cards with replacement: independent? \_\_\_\_\_

19.  $P(\text{both heads in two flips of a fair coin})$  \_\_\_\_\_

20.  $P(A) = \frac{1}{3}, P(B) = \frac{1}{2}$ , independent.  $P(A \cup B)$  \_\_\_\_\_

◆ Word Problems

21. A bag holds 7 red marbles and 3 blue marbles. Drawing two marbles *without* replacement, what's the probability that both are red? \_\_\_\_\_

22. A fair coin is flipped and a fair six-sided die is rolled. What's the probability of getting heads *and* an even number? Treat the two experiments as independent. \_\_\_\_\_

23. A drawer has 5 pairs of socks: 3 black and 2 white. You grab one sock, set it aside, and grab another. What's the probability both socks are black? (Treat the 6 individual black socks as distinct and the 4 individual white socks as distinct.) \_\_\_\_\_

24. A factory has three independent machines  $A, B, C$ . On any given day,  $A$  breaks with probability 0.05,  $B$  with 0.10, and  $C$  with 0.20. What's the probability all three machines work today? \_\_\_\_\_

Additional Practice

25. Probability of rolling an even number on a fair die. \_\_\_\_\_

26. Probability of drawing a heart from a standard deck. \_\_\_\_\_

27. Complement of  $P(A) = 0.37$ . \_\_\_\_\_

28. If events are independent,  $P(A) = 0.4, P(B) = 0.5$ , find  $P(A \cap B)$ . \_\_\_\_\_

29. Find  $P(A \cup B)$  if  $P(A) = 0.6, P(B) = 0.3, P(A \cap B) = 0.1$ . \_\_\_\_\_

30. Choose 3 from 8. \_\_\_\_\_

31. Arrange 4 distinct books. \_\_\_\_\_

32. Find  ${}^7P_2$ . \_\_\_\_\_

33. Find  ${}^7C_2$ . \_\_\_\_\_

34. Probability of two heads in two coin flips. \_\_\_\_\_

35. Expected wins in 80 trials with  $p = 0.25$ . \_\_\_\_\_



## Answer Keys

1. 0.12  
 2. Yes  
 3. Dependent  
 4. 0.30  
 5. 0.7  
 6.  $\frac{1}{16}$   
 7.  $\frac{1}{3}$   
 8.  $\frac{9}{25}$   
 9. 0.16  
 10. No  
 11.  $\frac{1}{12}$   
 12.  $\frac{1}{4}$   
 13. Yes  
 14. No  
 15.  $\frac{1}{4}$   
 16. 0.008  
 17. Yes  
 18. Yes  
 19.  $\frac{1}{4}$   
 20.  $\frac{2}{3}$   
 21.  $\frac{7}{15}$   
 22.  $\frac{1}{4}$   
 23.  $\frac{1}{3}$   
 24. 0.684

## Additional Practice Answers

25.  $\frac{1}{2}$   
 26.  $\frac{1}{4}$   
 27. 0.63  
 28. 0.20  
 29. 0.8  
 30. 56  
 31. 24  
 32. 42  
 33. 21  
 34.  $\frac{1}{4}$   
 35. 20

**Additional Practice:** Answers for all numbered items, including the added practice, are shown in the grid above.

## Step-by-Step Explanations

1. Independent events multiply:  $0.3 \times 0.4 = 0.12$ . Notice the product comes out smaller than either factor – typical when both probabilities are below 1.
2. A die and a coin are physically separate – one can't possibly affect the other. The events are independent.
3. Pulling a card and leaving it out changes the deck for the next draw. The first outcome echoes into the second. Without replacement  $\Rightarrow$  dependent.
4. For independent events the joint probability is the product, so multiply (don't add):  $P(A \cap B) = 0.5 \times 0.6 = 0.30$ . Adding here would give 1.1, an impossible probability – a quick check that multiplying is the right move.
5. Independence means  $B$  gives no info about  $A$ , so  $P(A | B) = P(A) = 0.7$  – same as the unconditional probability.
6. Each spin is independent with probability  $\frac{1}{4}$  for any single number. So  $\frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$ .
7. First red:  $\frac{6}{10}$ . Second red, given one red already gone:  $\frac{5}{9}$ . Multiply:  $\frac{6}{10} \cdot \frac{5}{9} = \frac{30}{90} = \frac{1}{3}$ .
8. With replacement the bag resets, so each draw is  $\frac{6}{10} = \frac{3}{5}$ . Multiply:  $\frac{3}{5} \cdot \frac{3}{5} = \frac{9}{25}$ .
9. A careful way to see it:  $P(C^c) = 1 - 0.2 = 0.8$ . Then  $0.5 \times 0.4 \times 0.8 = 0.16$ . The complement only changes that one factor. That gives a quick check on the answer.
10. Independence would force  $P(A \cap B) = P(A)P(B) > 0$ . But the joint is 0, so the events are mutually exclusive and therefore dependent.
11. A coin and a die can't affect each other, so the events are independent and we multiply.  $P(H) = \frac{1}{2}$  and  $P(5) = \frac{1}{6}$ , so  $\frac{1}{2} \cdot \frac{1}{6} = \frac{1}{12}$ .
12. Replacement makes the two draws independent, each with probability  $\frac{1}{2}$ . Multiply:  $\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$ .
13. Check  $P(A)P(B) = 0.6 \times 0.5 = 0.30$ , which equals  $P(A \cap B)$ . The product matches the joint, so the events are independent.
14. Keep the rule visible:  $P(A)P(B) = 0.4 \times 0.5 = 0.20$ , but  $P(A \cap B) = 0.15$ . The product doesn't match, so the events are dependent. That gives a quick check on the answer.
15. Each die has 3 even faces out of 6, so  $P(\text{even}) = \frac{1}{2}$  per die. The rolls are independent, so multiply:  $\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$ .
16. For independent events we multiply, and here all three have the same probability, so  $0.2 \times 0.2 \times 0.2 = 0.2^3 = 0.008$ . The chance of all three happening together is much smaller than any single event – multiplying small numbers shrinks fast.
17. That's the definition. If conditioning on  $B$  leaves  $P(A)$  unchanged, the events are independent.
18. Putting the first card back resets the deck. The second draw happens from the same full deck of 52 – independence restored.
19. Each flip is independent with  $P(H) = \frac{1}{2}$ , so multiply:  $\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$ . You can see it directly too – HH is one of the four equally-likely outcomes HH, HT, TH, TT.
20. Start with the key idea:  $P(A \cap B) = \frac{1}{3} \cdot \frac{1}{2} = \frac{1}{6}$ . Then  $P(A \cup B) = \frac{1}{3} + \frac{1}{2} - \frac{1}{6} = \frac{2}{6} + \frac{3}{6} - \frac{1}{6} = \frac{4}{6} = \frac{2}{3}$ . That gives a quick check on the answer.
21. First red:  $\frac{7}{10}$ . After pulling one red, 6 reds remain in a bag of 9, so the second red has probability  $\frac{6}{9} = \frac{2}{3}$ . Multiply:  $\frac{7}{10} \cdot \frac{2}{3} = \frac{14}{30} = \frac{7}{15}$ . The classic



without-replacement update: shrink both numerator and denominator by one for the color that just left.

22. Keep the rule visible:  $P(\text{heads}) = \frac{1}{2}$ . The die's even outcomes are  $\{2, 4, 6\}$

out of 6, so  $P(\text{even}) = \frac{3}{6} = \frac{1}{2}$ . The coin and the die share no causal link, so

multiply:  $\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$ . (Sanity check: of the 12 equally-likely coin-die outcomes,

exactly 3 are H2, H4, H6 – that's  $\frac{3}{12} = \frac{1}{4}$ , matches.) That gives a quick check on the answer.

23. There are 6 black socks among 10 total. First black:  $\frac{6}{10}$ . After one black is out, 5 blacks remain in a drawer of 9:  $\frac{5}{9}$ . Multiply:  $\frac{6}{10} \cdot \frac{5}{9} = \frac{30}{90} = \frac{1}{3}$ . The setup is dependent because we removed a sock and didn't replace it – the second draw lives in a smaller drawer.

24. "All three work" means none of them breaks. The non-break probabilities are 0.95, 0.90, and 0.80. The machines are independent (the problem says so), so multiply:  $0.95 \times 0.90 \times 0.80 = 0.684$ . So a typical day has about a 68.4% chance of trouble-free operation. (Cross-check: roughly  $1 - 0.05 - 0.10 - 0.20 = 0.65$  is the wrong shortcut for independent events – that approximation overcounts the rare "two break at once" day. The exact product is the way.)



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