# Independent and Dependent Events

| Name: ____________ | Date: ____________ | Score: ______ / 35 |
|---|---|---|

## Quick Review

Two events $A$ and $B$ are **independent** when one happening doesn't change the probability of the other. The clean test: $P(A \cap B) = P(A) \cdot P(B)$. Equivalently, $P(A \mid B) = P(A)$ – knowing $B$ tells you nothing new about $A$.

**Dependent** just means *not* independent. The first event leaves a fingerprint on the second. Drawing two cards *without* replacement is the classic case: pull a red, and the deck now has one fewer red and one fewer card total – the next draw lives in a different world.

**With or without replacement?** That little phrase decides the answer. *With* replacement restores the bag (independent draws). *Without* replacement shrinks the pool (dependent).

**Don't confuse independent with mutually exclusive.** Mutually exclusive means the two events can't happen together ($P(A \cap B) = 0$). If both events have positive probability, mutually exclusive events *can't* be independent (the product $P(A)P(B) > 0$ would have to match a zero – impossible).

**Multi-step trick.** For a string of independent events – flip, roll, spin – multiply the individual probabilities. If any step is dependent, update the probability for that step using what's left after the earlier step.

**Common slips.** Calling separate experiments dependent just because they happen in sequence. Forgetting to update the denominator after a no-replacement draw. Multiplying probabilities when you should be adding (or vice versa).

## PRACTICE

*Decide independence or compute the joint/conditional probability requested.*

1. Find $P(A \cap B)$ if $A, B$ are independent, $P(A) = 0.3$, $P(B) = 0.4$. ________
2. Are rolling a $4$ and flipping heads independent? ________
3. Draw without replacement: independent or dependent? ________
4. $P(A \cap B)$ if $P(A) = 0.5, P(B) = 0.6$, independent ________
5. $P(A \mid B)$ when $A, B$ independent and $P(A) = 0.7$ ________
6. A fair $4$-section spinner is spun twice; the single-spin probabilities are shown. Find $P(2 \text{ then } 3)$. ________

| *outcome* | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $P$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ |

7. Using the bag below, find $P(\text{two reds, no replacement})$. ________

| Color | Count |
|---|---|
| Red | 6 |
| Blue | 4 |
| Total | 10 |

8. Bag: $6$ red, $4$ blue. $P(\text{two reds, with replacement})$ ________
9. $P(A) = 0.5, P(B) = 0.4, P(C) = 0.2$ mutually independent. Find $P(A \cap B \cap C^c)$. ________
10. If $P(A \cap B) = 0$ and $P(A), P(B) > 0$, independent? ________
11. Coin flip then die roll. $P(\text{H and } 5)$ ________
12. Draw, replace, draw. $P(\text{red both times}) = \left(\frac{1}{2}\right)^2$ ________
13. $P(A) = 0.6, P(B) = 0.5, P(A \cap B) = 0.30$. Independent? ________
14. $P(A) = 0.4, P(B) = 0.5, P(A \cap B) = 0.15$. Independent? ________

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**15.** Roll two dice. Using the per-die tally of even vs. odd faces below, find $P(\text{both even})$. ________

| Face type | Faces |
|---|---|
| Even | $2, 4, 6$ |
| Odd | $1, 3, 5$ |

**16.** Three independent events, each $p = 0.2$. $P(\text{all three})$ ________

**17.** Two events with $P(A \mid B) = P(A)$. Independent? ________

**18.** Drawing two cards with replacement: independent? ________

**19.** $P(\text{both heads in two flips of a fair coin})$ ________

**20.** $P(A) = \frac{1}{3}, P(B) = \frac{1}{2}$, independent. $P(A \cup B)$ ________

## Word Problems

**21.** A bag holds $7$ red marbles and $3$ blue marbles. Drawing two marbles *without* replacement, what's the probability that both are red? ____

**22.** A fair coin is flipped and a fair six-sided die is rolled. What's the probability of getting heads *and* an even number? Treat the two experiments as independent. ____

**23.** A drawer has $5$ pairs of socks: $3$ black and $2$ white. You grab one sock, set it aside, and grab another. What's the probability both socks are black? (Treat the $6$ individual black socks as distinct and the $4$ individual white socks as distinct.) ____

**24.** A factory has three independent machines $A$, $B$, $C$. On any given day, $A$ breaks with probability $0.05$, $B$ with $0.10$, and $C$ with $0.20$. What's the probability all three machines work today? ____

## Additional Practice

**25.** Probability of rolling an even number on a fair die. ________

**26.** Probability of drawing a heart from a standard deck. ________

**27.** Complement of $P(A) = 0.37$. ________

**28.** If events are independent, $P(A) = 0.4$, $P(B) = 0.5$, find $P(A \cap B)$. ________

**29.** Find $P(A \cup B)$ if $P(A) = 0.6$, $P(B) = 0.3$, $P(A \cap B) = 0.1$. ________

**30.** Choose $3$ from $8$. ________

**31.** Arrange $4$ distinct books. ________

**32.** Find $7P2$. ________

**33.** Find $7C2$. ________

**34.** Probability of two heads in two coin flips. ________

**35.** Expected wins in $80$ trials with $p = 0.25$. ________

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## Answer Keys

1. 0.12
2. Yes
3. Dependent
4. 0.30
5. 0.7
6. $\frac{1}{16}$
7. $\frac{1}{3}$
8. $\frac{9}{25}$
9. 0.16
10. No
11. $\frac{1}{12}$
12. $\frac{1}{4}$
13. Yes
14. No
15. $\frac{1}{4}$
16. 0.008
17. Yes
18. Yes
19. $\frac{1}{4}$
20. $\frac{2}{3}$
21. $\frac{7}{15}$
22. $\frac{1}{4}$
23. $\frac{1}{3}$
24. 0.684

### Additional Practice Answers

25. $\frac{1}{2}$
26. $\frac{1}{4}$
27. 0.63
28. 0.20
29. 0.8
30. 56
31. 24
32. 42
33. 21
34. $\frac{1}{4}$
35. 20

**Additional Practice:** Answers for all numbered items, including the added practice, are shown in the grid above.

## Step-by-Step Explanations

**1.** Independent events multiply: $0.3 \times 0.4 = 0.12$. Notice the product comes out smaller than either factor – typical when both probabilities are below $1$.

**2.** A die and a coin are physically separate – one can't possibly affect the other. The events are independent.

**3.** Pulling a card and leaving it out changes the deck for the next draw. The first outcome echoes into the second. Without replacement $\Rightarrow$ dependent.

**4.** For independent events the joint probability is the product, so multiply (don't add): $P(A \cap B) = 0.5 \times 0.6 = 0.30$. Adding here would give $1.1$, an impossible probability – a quick check that multiplying is the right move.

**5.** Independence means $B$ gives no info about $A$, so $P(A \mid B) = P(A) = 0.7$ – same as the unconditional probability.

**6.** Each spin is independent with probability $\frac{1}{4}$ for any single number. So $\frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$.

**7.** First red: $\frac{6}{10}$. Second red, given one red already gone: $\frac{5}{9}$. Multiply: $\frac{6}{10} \cdot \frac{5}{9} = \frac{30}{90} = \frac{1}{3}$.

**8.** With replacement the bag resets, so each draw is $\frac{6}{10} = \frac{3}{5}$. Multiply: $\frac{3}{5} \cdot \frac{3}{5} = \frac{9}{25}$.

**9.** A careful way to see it: $P(C^c) = 1 - 0.2 = 0.8$. Then $0.5 \times 0.4 \times 0.8 = 0.16$. The complement only changes that one factor. That gives a quick check on the answer.

**10.** Independence would force $P(A \cap B) = P(A)P(B) > 0$. But the joint is $0$, so the events are mutually exclusive and therefore dependent.

**11.** A coin and a die can't affect each other, so the events are independent and we multiply. $P(\text{H}) = \frac{1}{2}$ and $P(5) = \frac{1}{6}$, so $\frac{1}{2} \cdot \frac{1}{6} = \frac{1}{12}$.

**12.** Replacement makes the two draws independent, each with probability $\frac{1}{2}$. Multiply: $\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$.

**13.** Check $P(A)P(B) = 0.6 \times 0.5 = 0.30$, which equals $P(A \cap B)$. The product matches the joint, so the events are independent.

**14.** Keep the rule visible: $P(A)P(B) = 0.4 \times 0.5 = 0.20$, but $P(A \cap B) = 0.15$. The product doesn't match, so the events are dependent. That gives a quick check on the answer.

**15.** Each die has $3$ even faces out of $6$, so $P(\text{even}) = \frac{1}{2}$ per die. The rolls are independent, so multiply: $\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$.

**16.** For independent events we multiply, and here all three have the same probability, so $0.2 \times 0.2 \times 0.2 = 0.2^3 = 0.008$. The chance of all three happening together is much smaller than any single event – multiplying small numbers shrinks fast.

**17.** That's the definition. If conditioning on $B$ leaves $P(A)$ unchanged, the events are independent.

**18.** Putting the first card back resets the deck. The second draw happens from the same full deck of $52$ – independence restored.

**19.** Each flip is independent with $P(\text{H}) = \frac{1}{2}$, so multiply: $\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$. You can see it directly too – HH is one of the four equally-likely outcomes HH, HT, TH, TT.

**20.** Start with the key idea: $P(A \cap B) = \frac{1}{3} \cdot \frac{1}{2} = \frac{1}{6}$. Then $P(A \cup B) = \frac{1}{3} + \frac{1}{2} - \frac{1}{6} = \frac{2}{6} + \frac{3}{6} - \frac{1}{6} = \frac{4}{6} = \frac{2}{3}$. That gives a quick check on the answer.

**21.** First red: $\frac{7}{10}$. After pulling one red, $6$ reds remain in a bag of $9$, so the second red has probability $\frac{6}{9} = \frac{2}{3}$. Multiply: $\frac{7}{10} \cdot \frac{2}{3} = \frac{14}{30} = \frac{7}{15}$. The classic

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without-replacement update: shrink both numerator and denominator by one for the color that just left.

**22.** Keep the rule visible: $P(\text{heads}) = \frac{1}{2}$. The die's even outcomes are $\{2, 4, 6\}$ out of $6$, so $P(\text{even}) = \frac{3}{6} = \frac{1}{2}$. The coin and the die share no causal link, so multiply: $\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$. (Sanity check: of the $12$ equally-likely coin-die outcomes, exactly $3$ are H2, H4, H6 – that's $\frac{3}{12} = \frac{1}{4}$, matches.) That gives a quick check on the answer.

**23.** There are $6$ black socks among $10$ total. First black: $\frac{6}{10}$. After one black is out, $5$ blacks remain in a drawer of $9$: $\frac{5}{9}$. Multiply: $\frac{6}{10} \cdot \frac{5}{9} = \frac{30}{90} = \frac{1}{3}$. The setup is dependent because we removed a sock and didn't replace it – the second draw lives in a smaller drawer.

**24.** "All three work" means none of them breaks. The non-break probabilities are $0.95$, $0.90$, and $0.80$. The machines are independent (the problem says so), so multiply: $0.95 \times 0.90 \times 0.80 = 0.684$. So a typical day has about a $68.4\%$ chance of trouble-free operation. (Cross-check: roughly $1 - 0.05 - 0.10 - 0.20 = 0.65$ is the wrong shortcut for independent events – that approximation overcounts the rare "two break at once" day. The exact product is the way.)