Independence and Conditional Probability

Name: _____	Date: _____	Score: _____ / 36

Quick Review

Independence reframed. The cleanest definition runs through conditional probability: events A and B are **independent** exactly when $P(A \mid B) = P(A)$. Knowing B happened gives no info about A. The equivalent product form: $P(A \cap B) = P(A)P(B)$. The two definitions are the same fact written two ways.

Testing for independence. Given numerical probabilities, compute $P(A)P(B)$ and compare with $P(A \cap B)$. Equal $\Rightarrow$ independent. Unequal $\Rightarrow$ dependent. From a two-way table, compute $P(A \mid B)$ and $P(A)$; if they agree, the variables are independent in that sample.

What independence is NOT. Not the same as mutually exclusive (events that can't happen together). Not the same as having equal probabilities. Not the same as $P(A) + P(B) = 1$.

Conditional probability formula. $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$. The condition shrinks the sample space to outcomes where B holds; $P(A \cap B)$ measures the part of A that survives.

Multiplication rule (general). $P(A \cap B) = P(A) \cdot P(B \mid A) = P(B) \cdot P(A \mid B)$. When you know one conditional and one marginal, this rule fills in the joint.

Common slips. Confusing independence with mutual exclusivity (they're opposites for nonzero probabilities!). Treating "without replacement" draws as independent. Plugging $P(A \cap B)$ into the denominator instead of $P(B)$.

PRACTICE

Test independence or compute the requested conditional probability.

1. $P(A) = 0.3, P(B) = 0.5, P(A \cap B) = 0.15$. Independent? _____
2. $P(A) = 0.4, P(B) = 0.6, P(A \cap B) = 0.2$. $P(A \mid B)$ _____
3. $P(A) = 0.4, P(B) = 0.6, P(A \cap B) = 0.2$. Independent? _____
4. $P(A) = 0.4$, A, B independent. $P(A \mid B)$ _____
5. Two draws are made *with* replacement from the bag below. Find $P(\text{two reds})$. _____

Red	Blue	Total
5	5	10

6. Bag: 5R, 5B; without replacement. $P(\text{two reds})$ _____
7. $P(A \mid B) = 0.8, P(B) = 0.4$. $P(A \cap B)$ _____
8. $P(A \cap B) = 0.32, P(A) = 0.5$. $P(B \mid A)$ _____
9. Using the probabilities listed below, find $P(\text{athlete}|\text{tall})$. _____

Event	Probability
Tall	0.6
Athlete	0.4
Tall and athlete	0.3

10. Mutually exclusive with positive probabilities: independent? _____
11. $P(A) = P(B) = 0.5, P(A \cap B) = 0.25$. Independent? _____
12. Two independent flips. $P(\text{first H}|\text{second H})$ _____

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13. From the two-way survey table below, find $P(\text{car}|\text{male})$. _____

	Car	No car	Total
Male	30	30	60
Female	20	20	40
Total	50	50	100

14. Same survey: 20 female car-owners of 40 females. $P(\text{car}|\text{female})$ _____

15. In that 100-person survey, are car ownership and gender independent? _____

16. $P(A) = P(B) = 0.5$. Forces independence? _____

17. $P(A) = 0.6, P(B) = 0.5, P(A \cup B) = 0.8$. $P(A \cap B)$ _____

18. Continued: independent? _____

19. $P(A \mid B) \neq P(A) \Rightarrow A, B$ are ... _____

20. Draw with replacement: independent or dependent? _____

♦ Word Problems

21. A class has $P(\text{tall}) = 0.6$ and $P(\text{athlete}) = 0.4$, with $P(\text{tall and athlete}) = 0.3$. What is the probability that a tall student is an athlete? Are "tall" and "athlete" independent in this class? _____

22. Events X and Y satisfy $P(X) = 0.5$, $P(Y) = 0.4$, and $P(X \mid Y) = 0.8$. Compute $P(X \cap Y)$ and $P(Y \mid X)$. Are X and Y independent? _____

23. A bag has 5 red marbles and 5 blue marbles. Two marbles are drawn *with* replacement. Find the probability that both are red, and explain why the two draws are independent. _____

24. A survey of 100 people on vehicle ownership shows: among 60 males, 30 own a car; among 40 females, 20 own a car. Are gender and car ownership independent in this sample? _____

Additional Practice

25. Probability of rolling an even number on a fair die. _____

26. Probability of drawing a heart from a standard deck. _____

27. Complement of $P(A) = 0.37$. _____

28. If events are independent, $P(A) = 0.4$, $P(B) = 0.5$, find $P(A \cap B)$. _____

29. Find $P(A \cup B)$ if $P(A) = 0.6$, $P(B) = 0.3$, $P(A \cap B) = 0.1$. _____

30. Choose 3 from 8. _____

31. Arrange 4 distinct books. _____

32. Find $7P2$. _____

33. Find $7C2$. _____

34. Probability of two heads in two coin flips. _____

35. Expected wins in 80 trials with $p = 0.25$. _____

36. Is drawing without replacement independent? _____

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Answer Keys

1. Yes
2. $\frac{1}{3}$
3. No
4. 0.4
5. $\frac{1}{4}$
6. $\frac{2}{9}$
7. 0.32
8. 0.64
9. 0.5
10. No
11. Yes
12. $\frac{1}{2}$
13. $\frac{1}{2}$
14. $\frac{1}{2}$
15. Yes
16. No
17. 0.3
18. Yes
19. dependent
20. Independent
21. $P(\text{ath} \mid \text{tall}) = 0.5$; not independent
22. 0.32; 0.64; not independent
23. $\frac{1}{4}$
24. Yes

Additional Practice Answers

25. $\frac{1}{2}$
26. $\frac{1}{4}$
27. 0.63
28. 0.20
29. 0.8
30. 56
31. 24
32. 42
33. 21
34. $\frac{1}{4}$
35. 20
36. no

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. A careful way to see it: $P(A)P(B) = 0.3 \times 0.5 = 0.15 = P(A \cap B)$. Product matches joint, so independent. That gives a quick check on the answer.

2. Conditional formula: $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{0.2}{0.6} = \dfrac{1}{3}$. Divide the joint by $P(B)$, the event you're conditioning on.

3. One steady path is: $P(A)P(B) = 0.4 \times 0.6 = 0.24$, but $P(A \cap B) = 0.2$. Mismatch, so dependent. That gives a quick check on the answer.

4. The definition of independence is $P(A \mid B) = P(A)$ – learning B tells you nothing about A. So $P(A \mid B) = 0.4$, no computation needed.

5. A careful way to see it: With replacement, draws are independent. $\dfrac{5}{10} \cdot \dfrac{5}{10} = \dfrac{1}{4}$. That gives a quick check on the answer.

6. Keep the rule visible: Without replacement: $\dfrac{5}{10} \cdot \dfrac{4}{9} = \dfrac{20}{90} = \dfrac{2}{9}$. Dependent draws. That gives a quick check on the answer.

7. General multiplication rule: $P(A \cap B) = P(A \mid B) \cdot P(B) = 0.8 \times 0.4 = 0.32$. This works whether or not the events are independent, since the conditional already adjusts for B.

8. Conditional formula, conditioning on A this time: $P(B \mid A) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{0.32}{0.5} = 0.64$. Notice $P(B \mid A) \neq P(A \mid B)$ in general – the denominator changes.

9. A careful way to see it: $\dfrac{0.3}{0.6} = 0.5$. Half of the tall students are athletes. This is the part to check before moving on, because it keeps the answer tied to the original question.

10. Keep the rule visible: $P(A \cap B) = 0$ but $P(A)P(B) > 0$. The product can't equal zero, so mutually exclusive events with positive probabilities cannot be independent. That gives a quick check on the answer.

11. One steady path is: $P(A)P(B) = 0.25 = P(A \cap B)$. Product matches the joint. This is the part to check before moving on, because it keeps the answer tied to the original question.

12. Independent means conditioning doesn't matter: $P(\text{first H} \mid \text{second H}) = P(\text{first H}) = \dfrac{1}{2}$.

13. "Given male" sets the denominator to the male row total (60). Car owners among them: 30. So $\dfrac{30}{60} = \dfrac{1}{2}$.

14. Keep the rule visible: $\dfrac{20}{40} = 0.5$. Females and males have the same car-ownership rate – independence sign. That gives a quick check on the answer.

15. One steady path is: $P(\text{car} \mid \text{male}) = P(\text{car} \mid \text{female}) = 0.5 = P(\text{car})$. Conditioning doesn't move the probability, so independent. That gives a quick check on the answer.

16. Equal probabilities alone say nothing about independence. You'd need to compare $P(A \cap B)$ with $P(A)P(B) = 0.25$. Could be anything from 0 (mutually exclusive) to 0.5 (one contains the other).

17. Rearrange inclusion-exclusion $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ to isolate the overlap: $P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.6 + 0.5 - 0.8 = 0.3$.

18. Keep the rule visible: $P(A)P(B) = 0.6 \times 0.5 = 0.3 = P(A \cap B)$. Product matches. This is the part to check before moving on, because it keeps the answer tied to the original question.

19. One steady path is: That's the negation of independence – conditioning changes the probability. That gives a quick check on the answer.

20. Replacement restores the population, so the second draw is unaffected by the first.

21. Conditional: $P(\text{ath} \mid \text{tall}) = \dfrac{P(\text{tall \& ath})}{P(\text{tall})} = \dfrac{0.3}{0.6} = 0.5$. For independence, check whether $P(\text{tall})P(\text{ath}) = P(\text{tall \& ath})$: $0.6 \times 0.4 = 0.24 \neq 0.3$. Product and joint disagree, so the events are dependent. (Tall students are more likely to be athletes than the overall rate of 40% – being tall raises the chance.)

22. Multiplication rule: $P(X \cap Y) = P(X \mid Y)P(Y) = 0.8 \times 0.4 = 0.32$. Flip the conditional: $P(Y \mid X) = \dfrac{P(X \cap Y)}{P(X)} = \dfrac{0.32}{0.5} = 0.64$. Independence check: $P(X)P(Y) = 0.5 \times 0.4 = 0.2$, which doesn't match the joint of 0.32. So

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X and Y are dependent. Notice $P(X \mid Y) = 0.8 > P(X) = 0.5$ – knowing Y pushes the probability of X up.

23. With replacement, the first marble is returned before the second draw, so the bag is identical for both draws. That means $P(\text{red on draw 1}) = \frac{5}{10} = \frac{1}{2}$, and the same for draw 2 no matter what happened on draw 1. Independence holds. Multiply: $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$. (Without replacement, the second draw would have $\frac{4}{9}$, giving $\frac{5}{10} \times \frac{4}{9} = \frac{2}{9}$ instead – different answer, different setup.)

24. Compute conditionals and the marginal: $P(\text{car} \mid \text{male}) = \frac{30}{60} = 0.5$, $P(\text{car} \mid \text{female}) = \frac{20}{40} = 0.5$, $P(\text{car}) = \frac{30+20}{100} = \frac{50}{100} = 0.5$. All three probabilities are the same. Knowing the gender doesn't change the chance of car ownership, so gender and car ownership are independent in this sample. (Raw counts 30 vs. 20 differ, but that's because the groups differ in size – the *proportions* are equal, which is what matters.)

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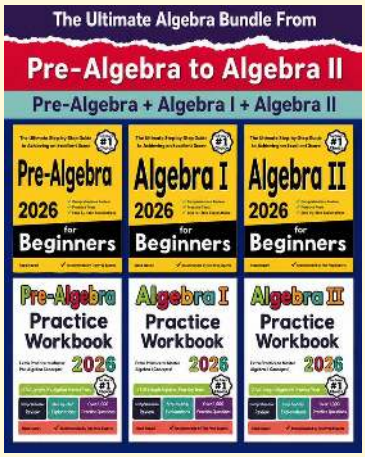

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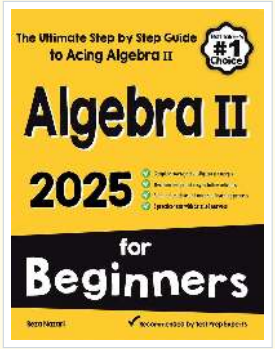

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