Increasing and Decreasing Functions

Name: ____________ Date: ____________ Score: ______ / 34

Quick Review

A function is **increasing** on an interval when its outputs grow as the inputs grow: for any $a < b$ in the interval, $f(a) < f(b)$. It's **decreasing** when outputs shrink: $f(a) > f(b)$. Walk left-to-right along the graph — climbing is increasing, sliding down is decreasing.

Positive vs. negative (about the sign of the output, not the slope): f is positive on an interval if the graph is *above* the x-axis there, negative if *below. Sign and monotonicity are independent.* $f(x) = x$ on $[-2, -1]$ is negative but increasing. Don't confuse the two.

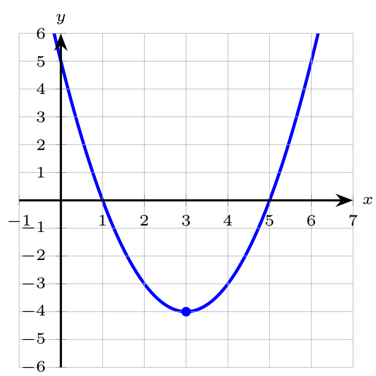
Turning points. Local maxima and minima are where increase changes to decrease (or vice versa). For a vertex-form quadratic $f(x) = a(x-h)^2 + k$ with $a > 0$, the function decreases on $x \le h$ and increases on $x \ge h$ (the parabola opens up, vertex at the bottom). With $a < 0$ the direction flips: increase on $x \le h$, decrease on $x \ge h$.

Sign of a factored polynomial. For $f(x) = (x - r_1)(x - r_2)$, test each interval between zeros. Quick check: $g(x) = -x(x-6)$ has zeros at 0 and 6, opens downward, so $g(x) > 0$ on $(0, 6)$ and $g(x) < 0$ outside. The vertex sits halfway between the zeros, at $x = 3$.

PRACTICE

Identify positive/negative intervals and increasing/decreasing intervals.

1. Define "f is positive on an interval I." ________
2. Define "f is increasing on I." ________
3. $f(x) = (x-2)(x+3)$. On what interval is $f < 0$? ________
4. $f(x) = (x-1)^2$. On what interval is f increasing? ________
5. $f(x) = x^2 - 4$. Where is $f > 0$ AND increasing? ________
6. Vertex of $f(x) = x^2 - 6x + 5$. ________
7. The graph of $f(x) = x^2 - 6x + 5$ is shown. On what interval is f decreasing? ________



8. $f(x) = -(x-1)^2 + 9$. Where is $f > 0$ AND decreasing? ________
9. $g(x) = -x(x-6)$. Positive interval? ________
10. Same g. Where is g decreasing? ________
11. $h(x) = 2x + 3$. Where is it increasing? ________
12. $h(x) = -3x + 1$. Where is it increasing? ________
13. f has a local max at $(-1, 4)$ and local min at $(2, -3)$, increasing on $x \le -1$, decreasing on $-1 \le x \le 2$, increasing on $x \ge 2$. Increasing interval after the min? ________
14. Even or odd? "$f(x) = x$ is decreasing on $[-2, -1]$." ________
15. $f(x) = x^3$. Where is f increasing? ________

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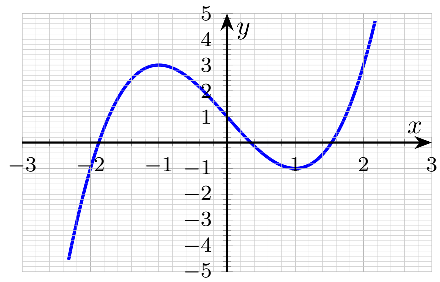
16. $f(x) = x^3 - 3x + 1$ (graph below). Decreasing interval? ________



17. $y = (x+1)(x-2)(x-4)$. Where is $y > 0$? ________

18. f is positive on $(0, 5)$ and decreasing on $(2, 5)$. Is f positive AND decreasing on $(2, 5)$? ________

19. $f(x) = |x - 2|$. Where is f increasing? ________

20. $f(x) = |x - 2|$. Where is f positive? ________

♦ Word Problems

21. A ball's height in feet is $h(t) = -16t^2 + 64t + 5$ for $t \geq 0$. State the intervals where the ball is rising and where it is falling, and find the highest point. ____

22. Daily profit (in thousands of dollars) for x items sold: $P(x) = -x^2 + 10x - 9$. On what interval is the company making money ($P > 0$), and on what subset of that interval is profit still climbing? ____

23. A car's distance from home in miles is $d(t) = t^3 - 9t^2 + 15t$ for $t \geq 0$ (hours). At what times is the car closest to home and farthest? Discuss when it's moving toward home vs. away. ____

24. A baker's monthly profit (in thousands of dollars) is $P(x) = (x-2)(8-x)$ for $0 \leq x \leq 10$, where x is price per pastry in dollars. State the price range that is profitable, and the price that maximizes profit. ____

Additional Practice

25. If $f(x) = 2x - 5$, find $f(4)$. ________

26. If $g(x) = x^2 + 1$, find $g(-3)$. ________

27. For $f(x) = 3x + 2$, solve $f(x) = 14$. ________

28. Find $(f+g)(x)$ if $f = x + 1$, $g = 2x - 5$. ________

29. Find $(fg)(x)$ if $f = x - 2$, $g = x + 3$. ________

30. Find $f(g(x))$ if $f(x) = 2x$, $g(x) = x + 7$. ________

31. Find the inverse of $f(x) = x - 9$. ________

32. Find the inverse of $f(x) = 3x + 1$. ________

33. Domain of $f(x) = \sqrt{x - 4}$. ________

34. Domain of $f(x) = \dfrac{1}{x+6}$. ________

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Answer Keys

1. $f(x) > 0$ for every $x \in I$
2. $a < b \Rightarrow f(a) < f(b)$ for $a, b \in I$
3. $(-3, 2)$
4. $[1, \infty)$
5. $x > 2$
6. $(3, -4)$
7. $(-\infty, 3]$
8. $(1, 4)$
9. $(0, 6)$
10. $[3, \infty)$
11. all of $\mathbb{R}$
12. nowhere (it is decreasing)
13. $x \geq 2$
14. false (it is increasing)
15. all of $\mathbb{R}$
16. $(-1, 1)$
17. $(-1, 2) \cup (4, \infty)$
18. yes
19. $[2, \infty)$
20. $\{x : x \neq 2\}$
21. rising on $[0, 2]$, falling on $[2, 4.08]$; peak 69 ft at $t = 2$
22. $P > 0$ on $(1, 9)$; climbing on $(1, 5]$
23. toward home on $[1, 5]$; away on $[0, 1] \cup [5, \infty)$
24. profitable on $(2, 8)$; max at $x = \$5$

Additional Practice Answers

25. 3
26. 10
27. $x = 4$
28. $3x - 4$
29. $x^2 + x - 6$
30. $2x + 14$
31. $f^{-1}(x) = x + 9$
32. $f^{-1}(x) = \frac{x-1}{3}$
33. $x \geq 4$
34. $x \neq -6$

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. A careful way to see it: All outputs on I are above the x-axis. This is the part to check before moving on, because it keeps the answer tied to the original question.
2. Keep the rule visible: Bigger input $\Rightarrow$ bigger output, throughout the interval. This is the part to check before moving on, because it keeps the answer tied to the original question.
3. The zeros are where each factor is zero: $x = 2$ and $x = -3$. Since the parabola opens upward (positive leading coefficient), it dips below the x-axis between its zeros. So $f < 0$ on $(-3, 2)$.
4. The vertex is at $x = 1$ (from $x - 1$), and the parabola opens upward. Left of the vertex it falls; right of it, it climbs. So f is increasing on $[1, \infty)$.
5. Two conditions to intersect. First, $f > 0$ means $x^2 > 4$, so $|x| > 2$, i.e. $x < -2$ or $x > 2$. Second, this upward parabola increases for $x \geq 0$. The overlap of both is $x > 2$.
6. Use $x = -\frac{b}{2a} = -\frac{-6}{2(1)} = 3$ for the vertex's x-coordinate, then evaluate: $f(3) = 3^2 - 6(3) + 5 = 9 - 18 + 5 = -4$. Vertex $(3, -4)$.
7. Walking left to right, the curve slides downward until it reaches the lowest point (the vertex at $x = 3$), then climbs. So it decreases on $(-\infty, 3]$.
8. Find the zeros: $-(x-1)^2 + 9 = 0$ gives $(x-1)^2 = 9$, so $x - 1 = \pm 3$ and $x = -2$ or 4. This downward parabola is positive between its zeros, $(-2, 4)$, and (opening down with vertex at $x = 1$) decreases for $x \geq 1$. The overlap is $(1, 4)$.
9. The zeros are $x = 0$ and $x = 6$. The leading term is $-x^2$, so the parabola opens downward and rises above the axis between its zeros. So $g > 0$ on $(0, 6)$.
10. Expand: $g(x) = -x^2 + 6x$. Vertex at $x = -\frac{b}{2a} = -\frac{6}{2(-1)} = 3$. Opening downward, the parabola climbs up to the vertex and falls after, so it decreases on $[3, \infty)$.
11. One steady path is: Linear with positive slope: increasing everywhere. This is the part to check before moving on, because it keeps the answer tied to the original question.
12. Start with the key idea: Linear with negative slope: decreasing everywhere. This is the part to check before moving on, because it keeps the answer tied to the original question.
13. A careful way to see it: After the local min at $x = 2$, the function increases again. That gives a quick check on the answer.
14. Keep the rule visible: $f(x) = x$ has positive slope everywhere; even though outputs are negative on $[-2, -1]$, the function is increasing. That gives a quick check on the answer.
15. One steady path is: Cube is strictly increasing. This is the part to check before moving on, because it keeps the answer tied to the original question.
16. Critical points where $f'(x) = 3x^2 - 3 = 0$: $x = \pm 1$. Function decreases between them.
17. The zeros are $-1, 2, 4$, splitting the line into four intervals. Test a point in each: at $x = -2$ all three factors are negative (product negative); at $x = 0$ two negatives one positive (positive); at $x = 3$ one negative (negative); at $x = 5$ all positive (positive). So $y > 0$ on $(-1, 2) \cup (4, \infty)$.
18. Keep the rule visible: Both conditions hold throughout $(2, 5)$, so the intersection is $(2, 5)$. That gives a quick check on the answer.
19. One steady path is: V-vertex at $x = 2$; opens up. Increase on the right. This is the part to check before moving on, because it keeps the answer tied to the original question.
20. Start with the key idea: Absolute value is nonnegative and zero only at the vertex. That gives a quick check on the answer.
21. Vertex at $t = -b/(2a) = -64/(-32) = 2$ s. Height at vertex: $h(2) = -64 + 128 + 5 = 69$ ft. Opens downward (coefficient -16), so rising before the vertex and falling after. Lands when $h(t) = 0$: $t \approx 4.08$ s (or roughly 4 s for a clean reading).
22. Factor: $P(x) = -(x-1)(x-9)$. Zeros $1, 9$; opens downward, so positive between the zeros: $(1, 9)$. Vertex at $x = 5$, so P rises on $[1, 5]$ and falls on $[5, 9]$. Profit climbs on $(1, 5]$ within the profitable range. Best to sell exactly 5 items per day in this model.
23. Set $d'(t) = 3t^2 - 18t + 15 = 3(t-1)(t-5) = 0$: critical points $t = 1, t = 5$. Sign chart of d': positive on $[0, 1)$, negative on $(1, 5)$, positive on $(5, \infty)$. So distance is rising (moving *away*) on $[0, 1]$, dropping (heading *toward home*) on $[1, 5]$, rising again on $[5, \infty)$. Local max distance at $t = 1$, local min at $t = 5$.
24. Zeros at $x = 2$ and $x = 8$. Expanding: $P(x) = -x^2 + 10x - 16$. Opens downward; max at $x = -b/(2a) = 10/2 = 5$. Profit is positive between $x = 2$ and $x = 8$. So pricing under \$2 or above \$8 loses money, and \$5 per pastry is the sweet spot. (Profit at \$5: $P(5) = (3)(3) = 9$, i.e. \$9,000.)

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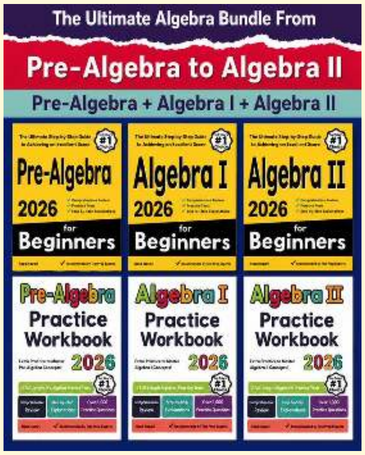

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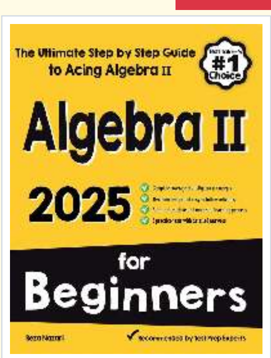

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