Identity and Zero Matrices

Name: ________________	Date: ____________	Score: _____ / 32

Quick Review

Two special matrices act as the neutral elements of matrix arithmetic. The **identity matrix** I_n is the $n \times n$ matrix with 1s on the main diagonal and 0s everywhere else. It plays the role of 1 for matrix multiplication: $AI = IA = A$ whenever the dimensions line up. The **zero matrix** $\mathbf{0}_{m\times n}$ has every entry equal to 0 and plays the role of 0 for addition: $A + \mathbf{0} = A$.

$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Note that the all-ones matrix is *not* the identity, and a permutation matrix like $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ swaps rows — also not the identity. Only the diagonal-of-1s pattern preserves the matrix it multiplies.

Dimension matching matters. If M is $m \times n$, then $MI_n = M$ on the right (the n in I_n matches the n columns of M) and $I_m M = M$ on the left. Picking the wrong-size identity gives an undefined product.

Three identities to remember. (1) $I_n^k = I_n$ for any positive integer k (the identity raised to any power stays the identity). (2) $\det(I_n) = 1$, so I_n is always invertible. (3) $I_n^{-1} = I_n$ — the identity is its own inverse. The zero matrix, by contrast, has $\det(\mathbf{0}_n) = 0$ for $n \geq 1$, so it's never invertible.

Critical trap. $A + \mathbf{0} = A$ (additive identity) but $A \cdot \mathbf{0} = \mathbf{0}$ (multiplicative *annihilator*) — not A. Zero kills under multiplication; only the identity preserves.

PRACTICE

Apply identity and zero-matrix properties. Compute each product or sum directly.

1. Which of these is the 2×2 identity matrix: $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$, $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$? ________

2. Compute AI_2 where A is laid out below and I_2 is the 2×2 identity: $A = \begin{bmatrix} 4 & 7 \\ -2 & 3 \end{bmatrix}$. ________

	Col 1	Col 2
Row 1	4	7
Row 2	−2	3

3. $A + \mathbf{0}_{2\times 2}$ where $A = \begin{bmatrix} 5 & -1 \\ 0 & 8 \end{bmatrix}$ ________

4. Which is the 3×3 identity: a 3×3 matrix of all 1s, a 3×3 matrix with 1s on the anti-diagonal, a 3×3 matrix with 1s on the main diagonal, or a 3×3 matrix with 3s on the main diagonal? ________

5. For any square A of size n, which always holds: $A\mathbf{0} = A$, $AI_n = \mathbf{0}$, $AI_n = I_nA = A$, or $I_nA = I_n$? ________

6. Which is FALSE: $I_n^k = I_n$ for any positive integer k; $\det(I_n) = 1$; $\mathbf{0}_n \cdot \mathbf{0}_n = I_n$; $I_n^{-1} = I_n$? ________

7. For $A = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix}$, find entry $(2, 2)$ of AI_2. ________

8. True or false: $A \cdot \mathbf{0} = A$ for every compatible square A and zero matrix $\mathbf{0}$. ________

9. Matrix M is 2×3. Which identity gives M when multiplied: MI_3, MI_2, I_2M, or $M\mathbf{0}_{3\times 2}$? ________

10. For $B = \begin{bmatrix} 1 & -2 & 4 \\ 0 & 3 & 5 \end{bmatrix}$, find entry $(1, 3)$ of BI_3. ________

11. $\det(I_3)$ ________

12. $\det(\mathbf{0}_2)$ ________

13. $I_2 \cdot I_2$ ________

14. I_3^5 ________

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15. $\mathbf{0}_2 \cdot A$ where $A = \begin{bmatrix} 3 & 1 \\ -2 & 4 \end{bmatrix}$ ________

16. True or false: the identity matrix I_2 is invertible. ________

17. $5I_2$ ________

18. Compute $A - I_2$ where A is laid out below: $A = \begin{bmatrix} 3 & 0 \\ 0 & 7 \end{bmatrix}$. ________

	Col 1	Col 2
Row 1	3	0
Row 2	0	7

19. Compute $(A + I_2)(A - I_2)$ where A is laid out below: $A = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$. ________

	Col 1	Col 2
Row 1	2	1
Row 2	0	2

20. True or false: for any $n \geq 1$, $\det(\mathbf{0}_n) = 0$. ________

Word Problems

21. In a control system, the state of the system at time $t + 1$ is $\vec{x}_{t+1} = M\vec{x}_t$. If the transition matrix at some step is $M = I_3$, what happens to the state vector $\vec{x}_t$ after that step? ____

22. In an encryption scheme, applying the matrix K then the matrix K^{-1} should return the original plaintext. Show why this works using identity matrices. ____

23. A computer-graphics rendering pipeline applies transformations as matrix products. If a pipeline applies T_1, then $T_2 = I_3$, then T_3, what is the effective transformation applied to a point $\vec{v}$? ____

24. A linear system $A\vec{x} = \vec{b}$ has $A = I_n$. Without doing any computation, what is the solution $\vec{x}$ in terms of $\vec{b}$? ____

Additional Practice

25. State the dimensions of $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$. ________

26. Add $\begin{bmatrix} 1 & 4 \\ 2 & 0 \end{bmatrix} + \begin{bmatrix} 3 & -1 \\ 5 & 6 \end{bmatrix}$. ________

27. Subtract $\begin{bmatrix} 7 & 2 \\ 1 & 5 \end{bmatrix} - \begin{bmatrix} 3 & 4 \\ -2 & 1 \end{bmatrix}$. ________

28. Find $\det \begin{bmatrix} 3 & 2 \\ 5 & 4 \end{bmatrix}$. ________

29. Find entry a_{21} in $\begin{bmatrix} 8 & 9 \\ -3 & 4 \end{bmatrix}$. ________

30. Can a 2×3 matrix multiply a 3×4 matrix? ________

31. Product size: $(2 \times 3)(3 \times 4)$. ________

32. Multiply $\begin{bmatrix} 2 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \end{bmatrix}$. ________

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Answer Keys

1. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
2. $\begin{bmatrix} 4 & 7 \\ -2 & 3 \end{bmatrix}$
3. $\begin{bmatrix} 5 & -1 \\ 0 & 8 \end{bmatrix}$
4. 1s on main diagonal
5. $AI_n = I_nA = A$
6. $\mathbf{0}_n \cdot \mathbf{0}_n = I_n$
7. 4
8. false
9. MI_3 and I_2M both give M
10. 4
11. 1
12. 0
13. I_2
14. I_3
15. $\mathbf{0}_2$
16. true
17. $\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$
18. $\begin{bmatrix} 2 & 0 \\ 0 & 6 \end{bmatrix}$
19. $\begin{bmatrix} 3 & 4 \\ 0 & 3 \end{bmatrix}$
20. true
21. $\vec{x}_{t+1} = \vec{x}_t$ (unchanged)
22. $K^{-1}K\vec{p} = I\vec{p} = \vec{p}$
23. $T_3T_1\vec{v}$ (the I_3 has no effect)
24. $\vec{x} = \vec{b}$

Additional Practice Answers

25. 2×3
26. $\begin{bmatrix} 4 & 3 \\ 7 & 6 \end{bmatrix}$
27. $\begin{bmatrix} 4 & -2 \\ 3 & 4 \end{bmatrix}$
28. 2
29. -3
30. yes
31. 2×4
32. 13

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. Only the matrix with 1s on the main diagonal and 0s elsewhere acts like multiplying by 1. The all-ones matrix sums entries, the swap matrix permutes them, and the zero matrix wipes them out — none of those is the identity.

2. Multiplying by the identity returns the original matrix. Verify $(1,1)$: $(4)(1) + (7)(0) = 4$, matching. (Same fact: $AI = A$ for any A when I has matching size.)

3. The zero matrix is the additive identity: $A + \mathbf{0} = A$. Every entry of A stays put because adding 0 changes nothing.

4. The identity pattern is the same in every size: 1s on the main diagonal, 0s everywhere else. The anti-diagonal matrix reverses the order of entries; the all-1s matrix sums entries; the all-3s diagonal scales by 3. Only the main-diagonal 1s give the identity.

5. The identity preserves any matrix it multiplies on either side (when dimensions match). The other three are distractors that swap zero with identity or confuse multiplication with 0 vs A.

6. Zero times zero is zero, not identity: $\mathbf{0} \cdot \mathbf{0} = \mathbf{0}$. The other three are true: identity to any power stays identity, $\det(I_n) = 1$, and identity is its own inverse.

7. One steady path is: $AI_2 = A$, so entry $(2,2)$ is just A's entry $(2,2) = 4$. The identity doesn't change anything. That gives a quick check on the answer.

8. Start with the key idea: $A \cdot \mathbf{0} = \mathbf{0}$, not A. Zero annihilates under multiplication. The additive identity rule $A + \mathbf{0} = A$ is different — zero is neutral for addition but absorbing for multiplication. That gives a quick check on the answer.

9. A careful way to see it: M is 2×3, so MI_3 matches inner dim $3 = 3$ and gives M (a 2×3). And I_2M matches $2 = 2$, giving M too. Both work — the identity has to match the side it's on. MI_2 fails (inner dim $3 \neq 2$), and the zero matrix annihilates. That gives a quick check on the answer.

10. Keep the rule visible: $BI_3 = B$, so entry $(1,3)$ is just B's entry $(1,3) = 4$. (Same fact as before: the identity preserves columns when applied on the right.) That gives a quick check on the answer.

11. The identity matrix has determinant 1: it's a triangular matrix with 1s on the diagonal, and the product of diagonal entries is 1. (This is why I is always invertible.)

12. The zero matrix has determinant 0 — all entries zero, so $ad - bc = 0 - 0 = 0$. (Consistent with the fact that the zero matrix has no inverse: $\det = 0 \Rightarrow$ singular.)

13. Identity times identity is identity. Check: $(1,1)$ entry is $(1)(1) + (0)(0) = 1$, and the off-diagonals are zero. So $I_2^2 = I_2$. (Hence $I_2^k = I_2$ for any positive k.)

14. Keep the rule visible: I_n raised to any positive integer power is still I_n. So $I_3^5 = I_3$. (You can keep multiplying by the identity forever; it never changes.) That gives a quick check on the answer.

15. Zero times anything (compatible) is zero. Every entry of the product is a sum of products with 0 — always zero. (This is exactly the absorbing behavior of zero under multiplication.)

16. Start with the key idea: $\det(I_2) = 1 \neq 0$, so I_2 is invertible. Its inverse is itself: $I_2 \cdot I_2 = I_2$, so $I_2^{-1} = I_2$. That gives a quick check on the answer.

17. Scalar times the identity scales each 1 on the diagonal by 5; the 0s stay 0. The result is a *scalar matrix* — diagonal with the same entry, behaving like multiplying by 5.

18. Subtract entry by entry: $(3-1, 0-0, 0-0, 7-1) = (2, 0, 0, 6)$. Subtracting the identity from a diagonal matrix shifts each diagonal entry down by 1. (This shows up in eigenvalue work later.)

19. Use $(A + I)(A - I) = A^2 - I$ (since A commutes with I). $A^2 = [(2)(2) + (1)(0), (2)(1) + (1)(2); (0), (0)(1) + (2)(2)] = [4, 4; 0, 4]$. Subtract I_2: $[3, 4; 0, 3]$. (The difference-of-squares factoring works for any A because A and I commute.)

20. Every entry of $\mathbf{0}_n$ is 0, so any expansion of the determinant has a factor of 0 in every term — the determinant is 0. Hence the zero matrix is always singular.

21. A careful way to see it: $I_3\vec{x}_t = \vec{x}_t$. The identity transition leaves the state vector exactly where it was. (Physically: the system has zero change at that step.) That gives a quick check on the answer.

22. By definition of inverse, $K^{-1}K = I$. Multiplying any vector by I leaves it unchanged: $I\vec{p} = \vec{p}$. So applying K then K^{-1} is the same as multiplying by I, which recovers the original $\vec{p}$. (This is exactly why decryption works in practice.)

23. The full pipeline is $T_3T_2T_1\vec{v} = T_3I_3T_1\vec{v} = T_3T_1\vec{v}$. Inserting I_3 in the middle of a product is the same as inserting 1 in a product of numbers — it has no effect. (Identity matrices are useful placeholders in software when a step is skipped.)

24. Start with the key idea: $I_n\vec{x} = \vec{b}$ simplifies immediately to $\vec{x} = \vec{b}$ because the identity doesn't change anything. (This is the trivial case of $\vec{x} = A^{-1}\vec{b}$ with $A = I$ and $A^{-1} = I$.) That gives a quick check on the answer.

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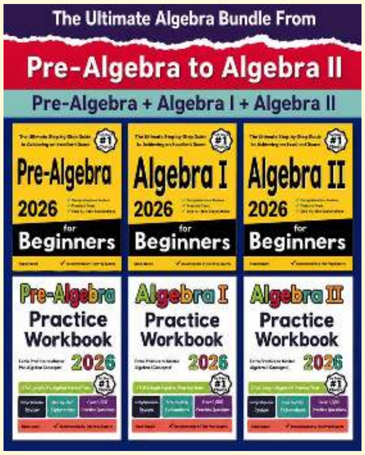

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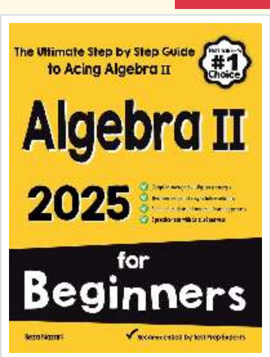

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