Hyperbola in Standard Form

Name: ________________ Date: ____________ Score: ______ / 24

Quick Review

A **hyperbola** is the set of points whose distances to two fixed points (the **foci**) *differ* by a constant. It has two disconnected branches that open away from each other and follow a pair of straight-line asymptotes at large distances.

Standard form, horizontal transverse axis. $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$. Center (h, k). *Vertices* (the closest points on each branch) at $(h \pm a, k)$. *Foci* at $(h \pm c, k)$. *Asymptotes*: $y - k = \pm\frac{b}{a}(x - h)$.

Standard form, vertical transverse axis. $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$. Vertices at $(h, k \pm a)$. Foci at $(h, k \pm c)$. Asymptotes: $y - k = \pm\frac{a}{b}(x - h)$. (Note the slope flips compared to the horizontal case – the a for the transverse axis goes on top.)

The hyperbola focal formula. $c^2 = a^2 + b^2$ (addition, not subtraction). This is the key sign change compared to ellipses. So $c > a$ always, i.e., the foci sit *outside* the vertices.

Identifying the transverse axis. Look at which squared term is *positive*. If x^2 is positive, the transverse axis is horizontal; if y^2 is positive, vertical. The denominator of the positive term is always a^2.

Asymptotes – the rectangle trick. Sketch a rectangle with corners at $(h \pm a, k \pm b)$ centered on (h, k). The asymptotes are the lines through the center and the rectangle's opposite corners. The branches of the hyperbola get arbitrarily close to these asymptotes but never touch.

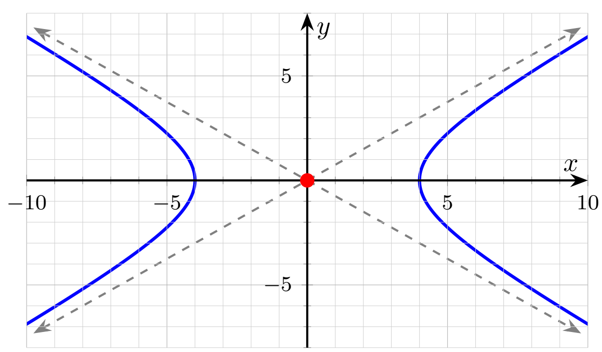
Common slips. Using the ellipse formula $c^2 = a^2 - b^2$ on a hyperbola (wrong sign). Mixing up the transverse axis direction – always check which squared term is positive. Flipping the asymptote slope ($\frac{a}{b}$ vs $\frac{b}{a}$) when the transverse axis is vertical.

PRACTICE

For each hyperbola, identify center, vertices, foci, and asymptotes as the problem requests. Watch for the transverse axis direction.

1. What is the center of $\frac{(x-4)^2}{9} - \frac{(y+2)^2}{16} = 1$? ________

2. For $\frac{x^2}{16} - \frac{y^2}{9} = 1$, find the vertices. ________



3. For $\frac{y^2}{25} - \frac{x^2}{144} = 1$, identify the transverse axis and vertices. ________

4. For $\frac{x^2}{16} - \frac{y^2}{9} = 1$, find the foci. ________

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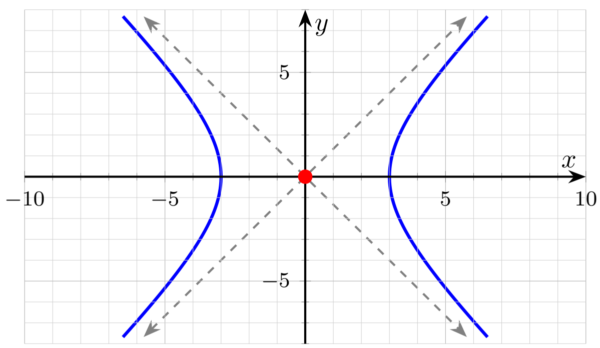
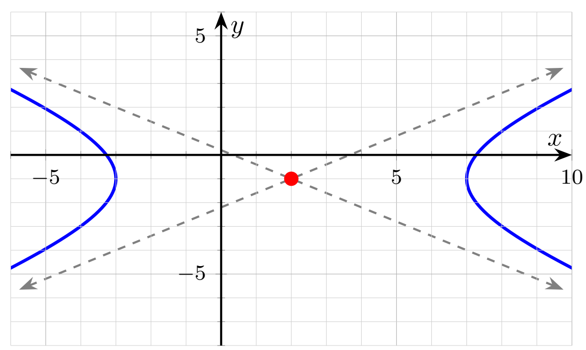
5. For $\frac{x^2}{9} - \frac{y^2}{16} = 1$, find the asymptotes. ________



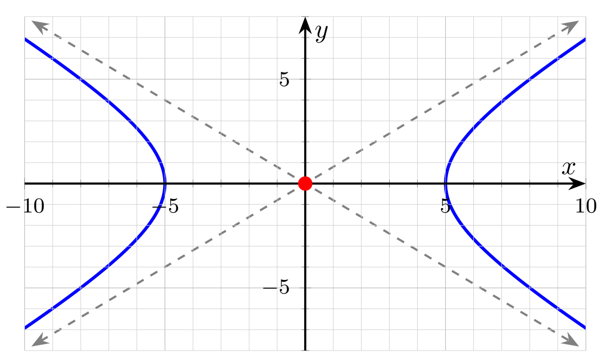
6. For $\frac{x^2}{36} - \frac{y^2}{64} = 1$, write the asymptote equations. ________

7. For $\frac{(x-2)^2}{25} - \frac{(y+1)^2}{9} = 1$, find the center and vertices. ________



8. A hyperbola has center $(0, 0)$, vertices $(0, \pm 4)$, and foci $(0, \pm 7)$. Find its equation. ________

9. For $\frac{x^2}{25} - \frac{y^2}{16} = 1$, find the asymptotes. ________



10. For $\frac{y^2}{25} - \frac{x^2}{144} = 1$, find the foci. ________

11. Mark TRUE or FALSE: For a hyperbola, the foci always sit between the vertices. ________

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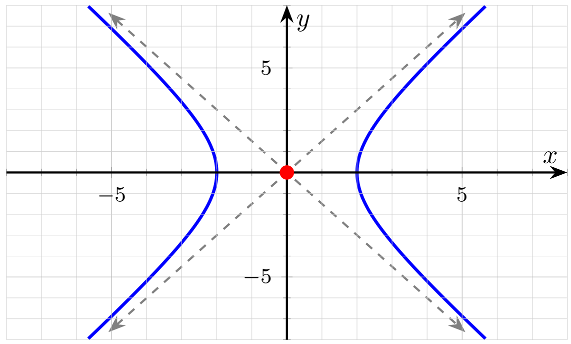
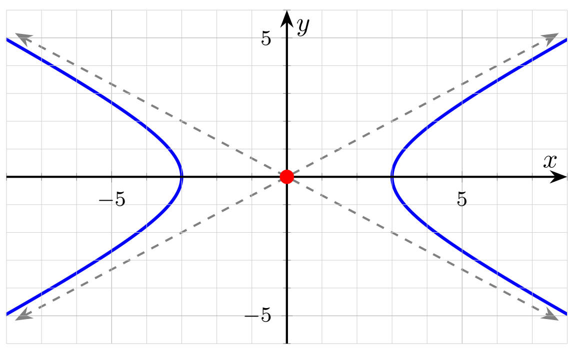
12. For $\frac{x^2}{4} - \frac{y^2}{9} = 1$, find the asymptotes. ________



13. For $\frac{y^2}{36} - \frac{x^2}{9} = 1$, find the asymptotes. ________

14. Find the asymptotes of $\frac{(x-1)^2}{4} - \frac{(y-2)^2}{9} = 1$. ________

15. For $\frac{x^2}{9} - \frac{y^2}{4} = 1$, find the vertices. ________



16. Find the equation of the hyperbola with center at the origin, a horizontal transverse axis with vertices $(\pm 6, 0)$, and asymptote slopes $\pm\frac{1}{2}$. ________

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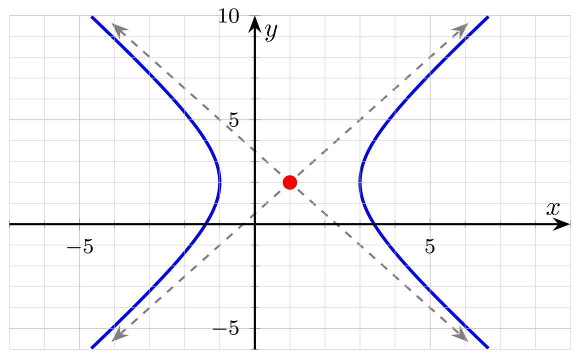
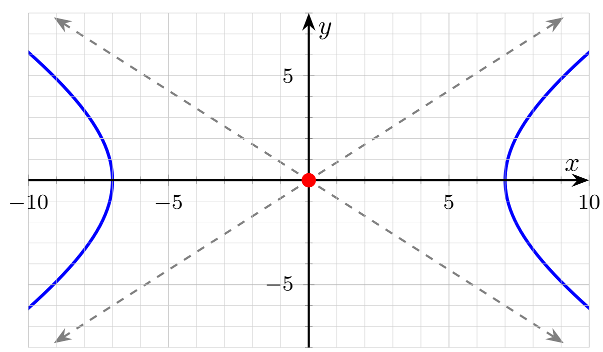
17. Convert $9x^2 - 4y^2 - 18x + 16y - 43 = 0$ to standard form. ________



18. Mark TRUE or FALSE: A hyperbola has two disconnected branches and a pair of asymptotes that the branches approach but never cross. ________

19. Find the foci of $\dfrac{(x-2)^2}{16} - \dfrac{(y+1)^2}{9} = 1$. ________

20. For $\dfrac{x^2}{49} - \dfrac{y^2}{36} = 1$, find the asymptotes. ________

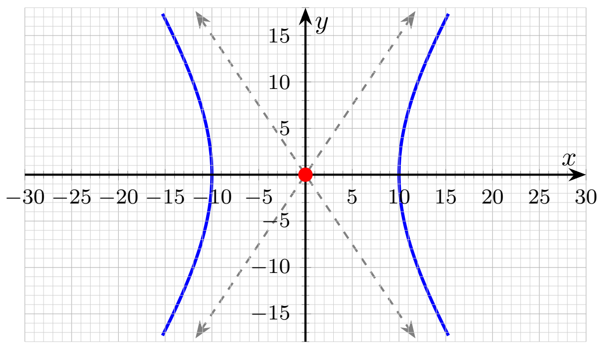


♦ Word Problems

21. A ship uses the LORAN navigation system, which determines the ship's location by measuring the *difference* in time signals from two stations – placing the ship somewhere on a hyperbola with the two stations as foci. Two stations sit at $(\pm 100, 0)$ km, and the time difference indicates the ship is 120 km farther from station B at $(100, 0)$ than from station A at $(-100, 0)$. Find the equation of the hyperbola that contains the ship's possible positions. ____

22. A comet's trajectory through the solar system is hyperbolic (one branch). The Sun sits at one focus of the hyperbola. Suppose the trajectory's equation, in millions of kilometers and centered so the Sun is at $(c, 0)$, is $\dfrac{x^2}{900} - \dfrac{y^2}{1600} = 1$, and only the right branch matters (where the comet actually travels). Find the focal distance c, and find the comet's closest approach to the Sun. ____

23. A cooling tower at a power plant has a hyperbolic cross-section – it's narrowest at the middle and widens both at the top and bottom. Suppose the cross-section is described by $\frac{x^2}{100} - \frac{y^2}{225} = 1$ (measurements in meters). The tower is 30 m tall, with y ranging from -15 m (bottom) to $+15$ m (top). How wide is the tower at the top? _____



24. A student claims that the asymptotes of $\frac{y^2}{16} - \frac{x^2}{9} = 1$ have slopes $\pm\frac{3}{4}$ because $b/a = 3/4$. Find the correct asymptote equations and explain the student's mistake. _____

Answer Keys

1. $(4, -2)$
2. $(\pm 4, 0)$
3. vertical; $(0, \pm 5)$
4. $(\pm 5, 0)$
5. $y = \pm\frac{4}{3}x$
6. $y = \pm\frac{4}{3}x$
7. center $(2, -1)$; vertices $(-3, -1), (7, -1)$
8. $\frac{y^2}{16} - \frac{x^2}{33} = 1$
9. $y = \pm\frac{4}{5}x$
10. $(0, \pm 13)$
11. FALSE
12. $y = \pm\frac{3}{2}x$
13. $y = \pm 2x$
14. $y - 2 = \pm\frac{3}{2}(x - 1)$
15. $(\pm 3, 0)$
16. $\frac{x^2}{36} - \frac{y^2}{9} = 1$
17. $\frac{(x-1)^2}{4} - \frac{(y-2)^2}{9} = 1$
18. TRUE
19. $(2+5, -1),\ (2-5, -1) = (7, -1), (-3, -1)$
20. $y = \pm\frac{6}{7}x$
21. $\frac{x^2}{3600} - \frac{y^2}{6400} = 1$ (left branch)
22. $c = 50$; closest approach: 20 million km
23. width at top: $20\sqrt{2} \approx 28.28$ m
24. $y = \pm\frac{4}{3}x$; student used wrong slope formula

Step-by-Step Explanations

1. Match to $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$. The $(x - 4)$ gives $h = 4$, and $(y + 2) = (y - (-2))$ gives $k = -2$. The sign flips, just as with circles and ellipses. Center $(4, -2)$.

2. Keep the rule visible: x^2 is the positive term, so the transverse axis is horizontal. $a^2 = 16$, so $a = 4$. Vertices $(\pm 4, 0)$. The dashed asymptotes have slope $\pm\frac{b}{a} = \pm\frac{3}{4}$. That gives a quick check on the answer.

3. The positive squared term is y^2, so the transverse axis is vertical. For a vertical hyperbola the denominator under the positive term is a^2, so $a^2 = 25$ and $a = 5$. The vertices sit on the transverse axis, a units from the center: $(0, \pm 5)$.

4. The positive x^2 term means a horizontal transverse axis with $a^2 = 16, b^2 = 9$. The hyperbola focal formula *adds*: $c^2 = a^2 + b^2 = 16 + 9 = 25$, so $c = 5$. Foci sit on the transverse axis: $(\pm 5, 0)$. (Using $a^2 - b^2$ here is the ellipse mistake.)

5. Horizontal hyperbola: slope $\pm\frac{b}{a}$. $a = 3$, $b = 4$, so slopes $\pm\frac{4}{3}$. Asymptotes pass through the center (origin): $y = \pm\frac{4}{3}x$.

6. Positive x^2 term: horizontal transverse axis, so the asymptote slopes are $\pm\frac{b}{a}$. Here $a^2 = 36$ ($a = 6$) and $b^2 = 64$ ($b = 8$), giving slopes $\pm\frac{8}{6} = \pm\frac{4}{3}$. Through the center (origin): $y = \pm\frac{4}{3}x$.

7. One steady path is: $h = 2$, $k = -1$. Horizontal transverse axis (positive x-term), $a = 5$. Vertices: $(2 \pm 5, -1)$, i.e., $(-3, -1)$ and $(7, -1)$. That gives a quick check on the answer.

8. Vertices and foci are on the y-axis, so the transverse axis is vertical with $a = 4$ ($a^2 = 16$) and $c = 7$. Rearrange the hyperbola formula $c^2 = a^2 + b^2$ to $b^2 = c^2 - a^2 = 49 - 16 = 33$. For a vertical hyperbola the y-term is positive: $\frac{y^2}{16} - \frac{x^2}{33} = 1$.

9. A careful way to see it: Horizontal hyperbola: slopes $\pm\frac{b}{a} = \pm\frac{4}{5}$. This is the part to check before moving on, because it keeps the answer tied to the original question.

10. Positive y^2 term: vertical transverse axis, so $a^2 = 25$ (under y^2) and $b^2 = 144$. Add for a hyperbola: $c^2 = a^2 + b^2 = 25 + 144 = 169$, so $c = 13$. Foci on the y-axis: $(0, \pm 13)$. (A 5-12-13 triple.)

11. One steady path is: $c > a$ for any hyperbola (since $c^2 = a^2 + b^2 > a^2$). That means the foci sit *outside* the vertices, farther from the center – the opposite of ellipses, where the foci are between the vertices. That gives a quick check on the answer.

12. Start with the key idea: $a = 2$, $b = 3$: slopes $\pm\frac{3}{2}$. This is the part to check before moving on, because it keeps the answer tied to the original question.

13. The positive term is y^2: vertical transverse axis, so slopes are $\pm\frac{a}{b}$ (the a from the positive term goes on top). Here $a^2 = 36$ ($a = 6$) and $b^2 = 9$ ($b = 3$), so slopes $\pm\frac{6}{3} = \pm 2$, giving $y = \pm 2x$.

14. The center is $(1, 2)$ and the positive x^2 term gives a horizontal transverse axis, so slopes are $\pm\frac{b}{a}$ with $a^2 = 4$ ($a = 2$), $b^2 = 9$ ($b = 3$): $\pm\frac{3}{2}$. The asymptotes pass through the center: $y - 2 = \pm\frac{3}{2}(x - 1)$.

15. One steady path is: Positive x^2 term – horizontal transverse axis. $a = 3$, so vertices $(\pm 3, 0)$. That gives a quick check on the answer.

16. Vertices $(\pm 6, 0)$ give $a = 6$ ($a^2 = 36$) on a horizontal transverse axis. The asymptote slopes are $\pm\frac{b}{a} = \pm\frac{1}{2}$, so $\frac{b}{6} = \frac{1}{2}$ gives $b = 3$ ($b^2 = 9$). Horizontal form: $\frac{x^2}{36} - \frac{y^2}{9} = 1$.

17. Group: $9(x^2 - 2x) - 4(y^2 - 4y) = 43$. Complete: $9[(x-1)^2 - 1] - 4[(y-2)^2 - 4] = 43 \Rightarrow 9(x-1)^2 - 9 - 4(y-2)^2 + 16 = 43 \Rightarrow 9(x-1)^2 - 4(y-2)^2 = 36$. Divide by 36: $\frac{(x-1)^2}{4} - \frac{(y-2)^2}{9} = 1$. Center $(1, 2)$, vertices $(1 \pm 2, 2) = (-1, 2)$ and $(3, 2)$.

18. That's the defining shape. The asymptotes form an "X" through the center; each branch lives in one of the four wedges defined by the X and stretches toward the asymptotes at infinity.

19. Center $(2, -1)$, positive x^2 term, so horizontal transverse axis with $a^2 = 16$, $b^2 = 9$. Add for a hyperbola: $c^2 = a^2 + b^2 = 16 + 9 = 25$, $c = 5$. Foci sit c units left and right of the center: $(2 \pm 5, -1)$, i.e. $(7, -1)$ and $(-3, -1)$.

20. Start with the key idea: $a = 7$, $b = 6$: slopes $\pm\frac{6}{7}$. This is the part to check before moving on, because it keeps the answer tied to the original question.

21. The difference of focal distances is $2a$, so $2a = 120$ and $a = 60$, $a^2 = 3600$. The focal distance is $c = 100$ (half the distance between the foci), so $c^2 = 10000$. Hyperbola: $b^2 = c^2 - a^2 = 10000 - 3600 = 6400$. Equation: $\frac{x^2}{3600} - \frac{y^2}{6400} = 1$.
Branch selection: the ship is farther from B (right focus) than from A (left focus), so it sits on the side closer to A – the *left* branch (negative x). A second pair of stations is needed to pin down the exact position; the intersection of two hyperbolas gives a unique location.

22. Keep the rule visible: $a^2 = 900$, $b^2 = 1600$, so $c^2 = a^2 + b^2 = 2500$, $c = 50$. The Sun sits at the right focus, $(50, 0)$. The right vertex (closest point on the right branch to the Sun) is at $(30, 0)$ (since $a = 30$). Closest approach: $|50 - 30| = 20$ million km. (Real comets that travel hyperbolic paths never return

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– they pass the Sun once and head out into interstellar space.) That gives a quick check on the answer.

23. Plug $y = 15$ into the equation: $\frac{x^2}{100} - \frac{225}{225} = 1 \Rightarrow \frac{x^2}{100} = 2 \Rightarrow x^2 = 200 \Rightarrow x = \pm 10\sqrt{2}$. The tower spans from $-10\sqrt{2}$ to $+10\sqrt{2}$ meters at the top, giving a width of $20\sqrt{2} \approx 28.28$ m. (At the middle $y = 0$, the vertices give a narrower throat of just $2a = 20$ m. Cooling towers actually use this exact hyperbolic profile to maximize airflow up the chimney.)

24. Identify the transverse axis first. The positive term is y^2, so the transverse axis is *vertical*. For a vertical hyperbola $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$, the asymptote slope is $\pm\frac{a}{b}$ (NOT $\pm\frac{b}{a}$). Here $a^2 = 16$ sits under y^2 (so $a = 4$), and $b^2 = 9$ sits under x^2 (so $b = 3$). Slopes: $\pm\frac{a}{b} = \pm\frac{4}{3}$. Asymptote equations: $y = \pm\frac{4}{3}x$. **The student's slip:** they treated the denominators as if the hyperbola were horizontal, with the 9 as a^2 and the 16 as b^2. For a vertical hyperbola, the variable with the *positive* squared term gives a^2 (the transverse semi-axis), regardless of the numerical sizes. Always identify the transverse axis direction first.

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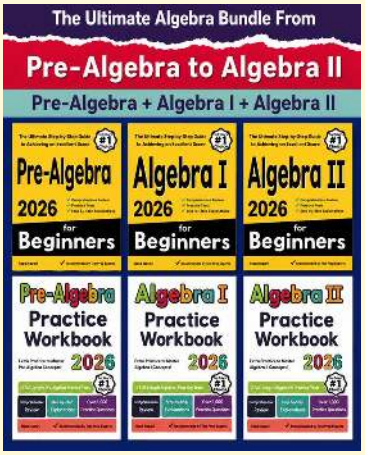

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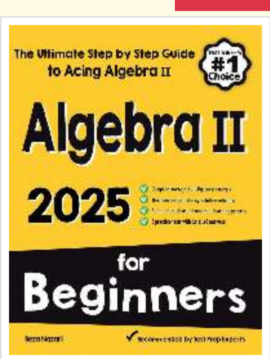

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