Graphing the Tangent Function

Name: ____________________	Date: ____________	Score: ______ / 32

Quick Review

Tangent is the misfit of the trig family. It isn't a smooth wave – it's a stack of repeating branches, each shooting from $-\infty$ up to $+\infty$ between vertical asymptotes.

Why the asymptotes show up. $\tan x = \dfrac{\sin x}{\cos x}$, so wherever cosine is zero, tangent is undefined – it blows up. Cosine is zero at $x = \dfrac{\pi}{2} + n\pi$, and those are exactly the tangent asymptotes.

Key facts.

Domain: all x except $\dfrac{\pi}{2} + n\pi$.

Range: $\mathbb{R}$ – between asymptotes, tangent sweeps every real value.

Period: π – much shorter than sine's 2π. (Reason: $\tan(x+\pi) = \tan x$, because both sine and cosine flip sign and the ratio is unchanged.)

Zeros: $x = n\pi$ – wherever sine is zero.

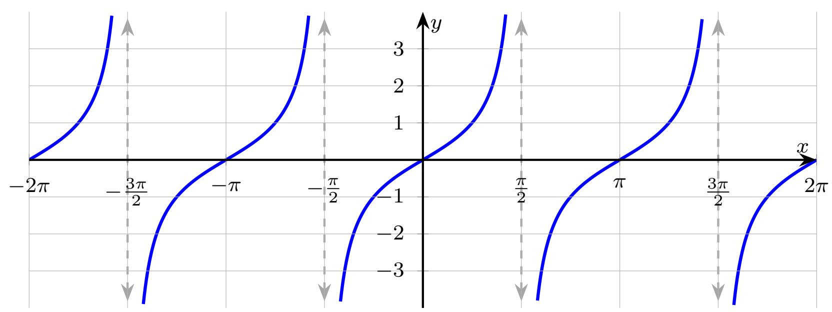
Symmetry: about the origin – tangent is odd, $\tan(-x) = -\tan x$.

Amplitude doesn't apply. Tangent has no top or bottom, so "amplitude" isn't a thing. People sometimes call the leading coefficient a "stretch factor" instead.

Transformations. For $y = a\tan(bx)$, period $= \dfrac{\pi}{|b|}$ (note the π, not 2π). The coefficient a stretches the curve vertically, but the asymptote locations only depend on b.

Common slips. Using $\dfrac{2\pi}{|b|}$ for the period – it's $\dfrac{\pi}{|b|}$ for tangent. Putting asymptotes at the zeros of sine (those are tangent's *zeros*, not asymptotes). Drawing tangent as a wave – it goes up forever, not up-and-down.

The parent tangent graph, with dashed vertical asymptotes at $\pm\dfrac{\pi}{2}$ and $\pm\dfrac{3\pi}{2}$:



PRACTICE

Find period, asymptotes, and zeros; sketch transformations of tangent.

1. State the period of $y = \tan x$. ________

2. Where are the vertical asymptotes of $y = \tan x$? ________

3. State the range of $y = \tan x$. ________

4. Where does $y = \tan x$ cross the x-axis? ________

5. Evaluate $\tan\left(\dfrac{\pi}{4}\right)$. ________

6. What is the period of $y = \tan(2x)$? ________

7. What is the period of $y = \tan\left(\dfrac{x}{3}\right)$? ________

8. True or false: tangent has an amplitude. ________

9. $y = \tan x$ is symmetric about which feature? ________

10. At what x-value (in $[0, \pi]$) is $\tan x$ undefined? ________

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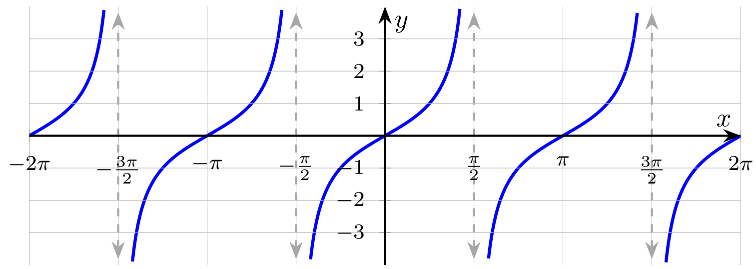
11. Identify the period and a vertical asymptote of the graph below. ________



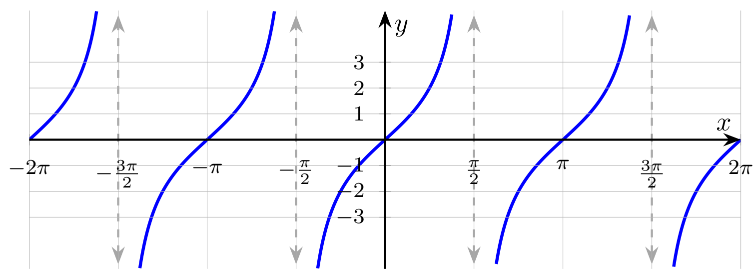
12. Evaluate $\tan(\pi)$. ________

13. What is the value of $\tan\left(\frac{\pi}{3}\right)$? ________

14. How many vertical asymptotes does $y = \tan x$ have on $[0, 2\pi]$? ________

15. How many vertical asymptotes does $y = \tan(2x)$ have on $[0, 2\pi]$? ________

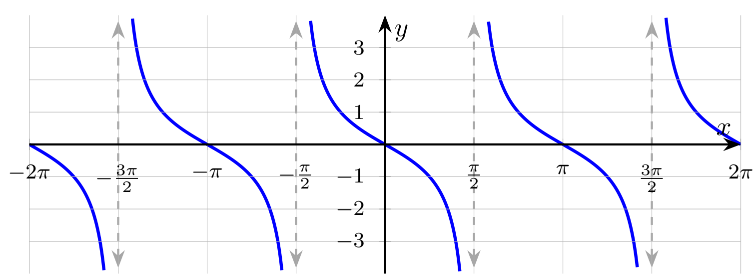
16. From the graph below (a transformation of $\tan x$), identify the period and the stretch factor. ________



17. True or false: $\tan x > 0$ in Q2. ________

18. At what x-values does $\tan x = 1$ on $[0, 2\pi)$? ________

19. How does the graph of $y = -\tan x$ (shown below) compare to $y = \tan x$? ________



20. State the period of $y = \tan\left(\frac{\pi}{2}x\right)$. ________

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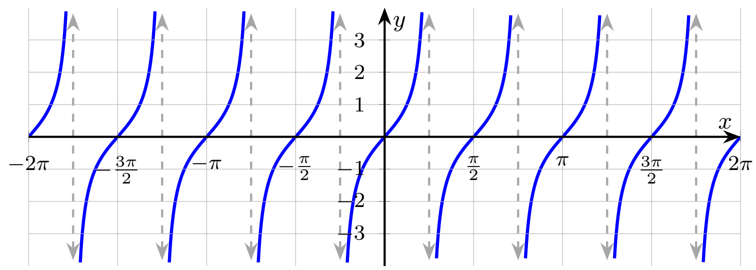
Word Problems

21. A laser at a fixed point shines on a wall 10 meters away and sweeps through an angle θ from straight ahead. The horizontal distance from the point of straight-ahead contact to the lit spot is $d(\theta) = 10\tan\theta$ meters. Where does the laser "go to infinity," and what does that mean physically? ____

22. The graph below shows $y = \tan(2x)$. Find the period and the locations of two consecutive vertical asymptotes. ________



23. A surveyor measures the angle of elevation θ from a point on the ground to the top of a 50-ft pole. The horizontal distance from the point to the pole's base is $h(\theta) = \dfrac{50}{\tan\theta}$ ft. For which angles in $(0, \pi/2)$ is this model undefined or nonsensical? ____

24. A rotating spotlight on a stage tracks an actor walking parallel to the stage. The angle θ of the spotlight from straight ahead and the actor's distance x from the spotlight's straight-ahead position satisfy $\tan\theta = \dfrac{x}{20}$. Solve for θ as a function of x (using arctangent) and explain why θ stays inside $(-\pi/2, \pi/2)$. ____

Additional Practice

25. Amplitude of $y = 4\sin x$. ______

26. Period of $y = \sin(2x)$. ______

27. Amplitude of $y = -3\cos x$. ______

28. Period of $y = \tan(5x)$. ______

29. Midline of $y = 2\sin x - 7$. ______

30. Phase shift of $y = \sin(x - \pi/3)$. ______

31. Range of $y = 5\cos x$. ______

32. Range of $y = 2\sin x + 1$. ______

Answer Keys

1. π
2. $x = \frac{\pi}{2} + n\pi,\ n \in \mathbb{Z}$
3. $\mathbb{R}$
4. $x = n\pi$
5. 1
6. $\frac{\pi}{2}$
7. 3π
8. False
9. the origin
10. $x = \frac{\pi}{2}$
11. period π; $x = \frac{\pi}{2}$
12. 0
13. $\sqrt{3}$
14. 2
15. 4
16. period π; vertical stretch by 2
17. False
18. $x = \frac{\pi}{4}, \frac{5\pi}{4}$
19. Reflected across the x-axis
20. 2
21. $\theta = \pm\frac{\pi}{2}$; laser parallel to wall, never hits it
22. period $\frac{\pi}{2}$; $x = \frac{\pi}{4}, \frac{3\pi}{4}$
23. $\theta = 0$ (and at $\theta = \pi/2,\ h = 0$)
24. $\theta = \arctan\left(\frac{x}{20}\right)$

Additional Practice Answers

25. 4
26. π
27. 3
28. $\frac{\pi}{5}$
29. $y = -7$
30. $\frac{\pi}{3}$ right
31. $[-5, 5]$
32. $[-1, 3]$

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. Tangent repeats every π – half as long as sine and cosine – because both $\sin$ and $\cos$ flip sign over π and the ratio stays the same.
2. Keep the rule visible: Tangent dies wherever cosine is zero. Cosine is zero at $\frac{\pi}{2} + n\pi$. That gives a quick check on the answer.
3. Between any two consecutive asymptotes, tangent climbs from $-\infty$ to $+\infty$, hitting every real value.
4. Start with the key idea: Tangent is zero wherever sine is zero – at every integer multiple of π. That gives a quick check on the answer.
5. A careful way to see it: At $\frac{\pi}{4}$, sine and cosine are equal ($\frac{\sqrt{2}}{2}$ each), so their ratio is 1. That gives a quick check on the answer.
6. For tangent the period is $\frac{\pi}{|b|}$ – note it's π on top, not 2π. With $b = 2$, period $= \frac{\pi}{2}$, so the curve compresses horizontally.
7. Here $b = \frac{1}{3}$. For tangent, period $= \frac{\pi}{|b|} = \frac{\pi}{1/3} = 3\pi$. A small b stretches the branches wider apart.
8. Start with the key idea: Tangent shoots to $\pm\infty$, so there's no max or min to define an amplitude. That gives a quick check on the answer.
9. A careful way to see it: Tangent is odd: $\tan(-x) = -\tan x$. That gives $180°$ rotational symmetry. That gives a quick check on the answer.
10. Keep the rule visible: $\cos\left(\frac{\pi}{2}\right) = 0$, so tangent has an asymptote there. This is the part to check before moving on, because it keeps the answer tied to the original question.
11. Between any two dashed lines, the curve completes one branch. The lines are π apart, so the period is π. The asymptote nearest $x = 0$ is at $x = \frac{\pi}{2}$.
12. Start with the key idea: $\sin\pi = 0$, $\cos\pi = -1$, so $\tan\pi = \frac{0}{-1} = 0$. This is the part to check before moving on, because it keeps the answer tied to the original question.
13. A careful way to see it: $\tan\left(\frac{\pi}{3}\right) = \frac{\sin(\pi/3)}{\cos(\pi/3)} = \frac{\sqrt{3}/2}{1/2} = \sqrt{3}$. This is the part to check before moving on, because it keeps the answer tied to the original question.
14. Keep the rule visible: On $[0, 2\pi]$ the asymptotes are $\frac{\pi}{2}$ and $\frac{3\pi}{2}$ – exactly two. This is the part to check before moving on, because it keeps the answer tied to the original question.
15. For $\tan(2x)$, asymptotes occur where $\cos(2x) = 0$, i.e. $2x = \frac{\pi}{2} + n\pi$, so $x = \frac{\pi}{4} + \frac{n\pi}{2}$. On $[0, 2\pi]$ that gives $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$ – four of them.
16. Same asymptotes as the parent (π apart, at $\frac{\pi}{2} + n\pi$), so period is π. The branches rise twice as steeply – a vertical stretch of 2. Equation: $y = 2\tan x$.
17. A careful way to see it: In Q2, sine is positive and cosine is negative, so tangent is negative. That gives a quick check on the answer.
18. Keep the rule visible: $\tan x = 1$ means sine and cosine are equal, which has reference angle $\frac{\pi}{4}$. Tangent is positive in Q1 and Q3, giving $x = \frac{\pi}{4}$ and $x = \frac{\pi}{4} + \pi = \frac{5\pi}{4}$ on $[0, 2\pi)$. That gives a quick check on the answer.
19. The negative sign flips outputs to their opposite. Branches now descend from $+\infty$ to $-\infty$ instead of climbing.
20. Here $b = \frac{\pi}{2}$. For tangent, period $= \frac{\pi}{|b|} = \frac{\pi}{\pi/2} = 2$. The π's cancel neatly, leaving a period of 2.
21. A careful way to see it: $\tan\theta$ blows up at $\theta = \pm\frac{\pi}{2}$. At those angles the laser is pointing straight along the wall and never touches it – so the model says the lit spot is infinitely far away (or doesn't exist). That gives a quick check on the answer.
22. For $\tan(2x)$, period $= \frac{\pi}{2}$. Asymptotes at $2x = \frac{\pi}{2} + n\pi$, i.e. $x = \frac{\pi}{4} + \frac{n\pi}{2}$ – so two in a row are $\frac{\pi}{4}$ and $\frac{3\pi}{4}$, $\frac{\pi}{2}$ apart (one period).
23. At $\theta = 0$, $\tan 0 = 0$ and we divide by zero – physically, looking horizontally means the pole is infinitely far away. At $\theta = \pi/2$, we'd be looking straight up and $h = 0$ (we're at the pole). The $1/\tan$ form shows tangent's reciprocal (cotangent) producing the asymptote at $\theta = 0$.
24. Since the spotlight can swivel left or right but never look behind itself, $\theta \in (-\pi/2, \pi/2)$ – exactly the principal range of arctangent. Solving $\tan\theta = \frac{x}{20}$ in that range gives $\theta = \arctan\left(\frac{x}{20}\right)$ uniquely.

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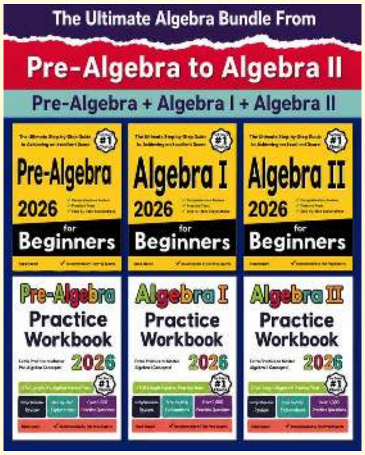

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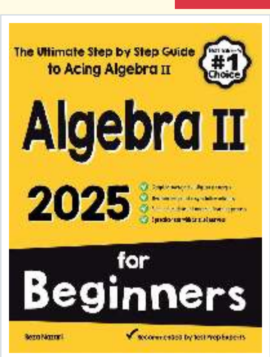

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