Graphing the Secant Function

Name: _____	Date: _____	Score: _____ / 35

Quick Review

Secant is the reciprocal of cosine: $\sec x = \dfrac{1}{\cos x}$. Cosine is small near its zeros, so secant is huge there. Cosine zeros become secant asymptotes.

The U-and-upside-down-U shape (again). Sketch cosine first. Between consecutive cosine zeros (which are π apart, at $\dfrac{\pi}{2} + n\pi$), the secant graph is a U opening up where cosine is positive, or upside-down where cosine is negative.

Key facts.

Domain: all x except $\dfrac{\pi}{2} + n\pi$.

Range: $(-\infty, -1] \cup [1, \infty)$.

Period: 2π, same as cosine.

Vertical asymptotes: $x = \dfrac{\pi}{2} + n\pi$.

Symmetry: about the y-axis – secant is even, $\sec(-x) = \sec x$.

y-intercept: $\sec(0) = \dfrac{1}{\cos 0} = 1$.

Secant vs. cosecant.

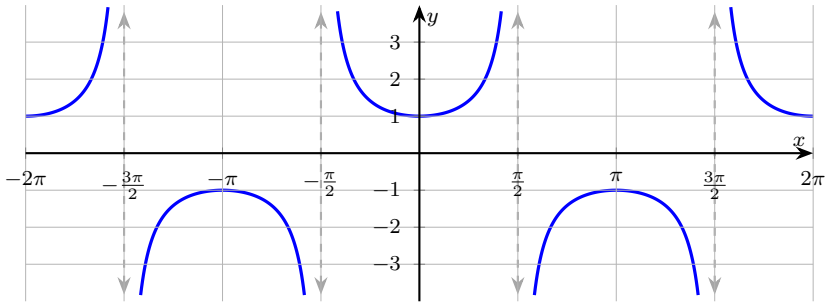
Secant pairs with cosine (asymptotes at $\dfrac{\pi}{2} + n\pi$, even function).

Cosecant pairs with sine (asymptotes at $n\pi$, odd function).

Transformations. For $y = a\sec(bx)$, period $= \dfrac{2\pi}{|b|}$, asymptotes wherever $\cos(bx) = 0$, i.e. $bx = \dfrac{\pi}{2} + n\pi$.

Common slips. Confusing $\sec x$ with $\sin^{-1} x$ or $\dfrac{1}{\sin x}$. The reciprocal of $\sin x$ is *cosecant*; $\sin^{-1} x$ is the inverse, not a reciprocal. Drawing secant inside $(-1, 1)$ – like cosecant, it never enters that band.

Parent secant graph, with cosine asymptotes dashed:



PRACTICE

Find period, asymptotes, range, and intercepts for secant graphs.

1. State the period of $y = \sec x$. _____
2. Where are the vertical asymptotes of $y = \sec x$? _____
3. State the range of $y = \sec x$. _____
4. Evaluate $\sec(0)$. _____
5. Evaluate $\sec\left(\dfrac{\pi}{3}\right)$. _____
6. Is $y = \sec x$ symmetric about the y-axis or the origin? _____
7. What is the y-intercept of $y = \sec x$? _____
8. What is the period of $y = \sec(3x)$? _____

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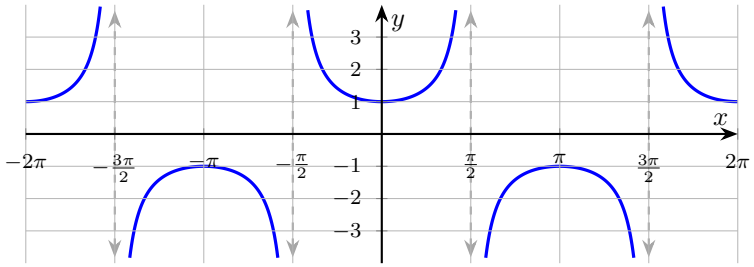
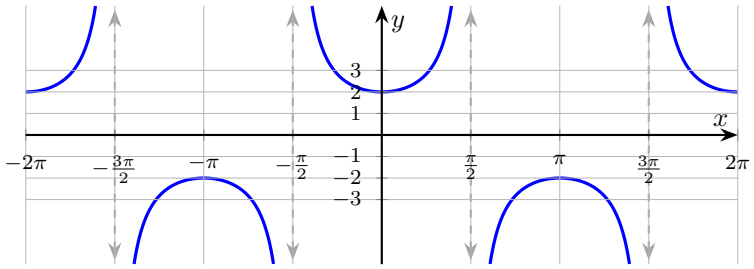
9. Where are the vertical asymptotes of $y = \sec(2x)$ on $[0, 2\pi)$? _____

10. Identify the period from the graph below. _____



11. Does $y = \sec x$ have any x-intercepts? _____

12. State the range of $y = 2\sec x$ shown below. _____

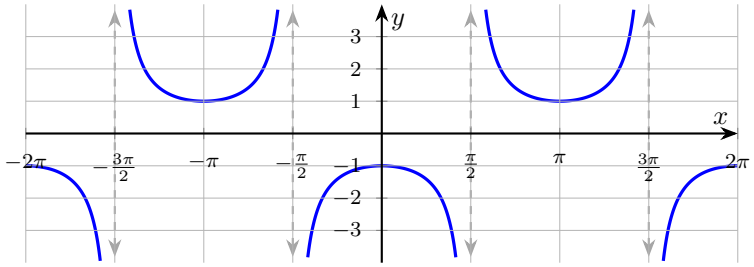


13. Evaluate $\sec(\pi)$. _____

14. True or false: $\sec x$ has period π. _____

15. For what $\cos x$-values is $\sec x$ positive? _____

16. How does $y = -\sec x$ (shown below) compare to $y = \sec x$? _____



17. List the asymptotes of $y = \sec x$ on $[0, 2\pi)$. _____

18. Evaluate $\sec\left(-\frac{\pi}{3}\right)$. _____

19. What is the period of $y = \sec\left(\frac{x}{2}\right)$? _____

20. State the smallest positive $\sec x$ value for $x \in (0, \pi/2)$. _____

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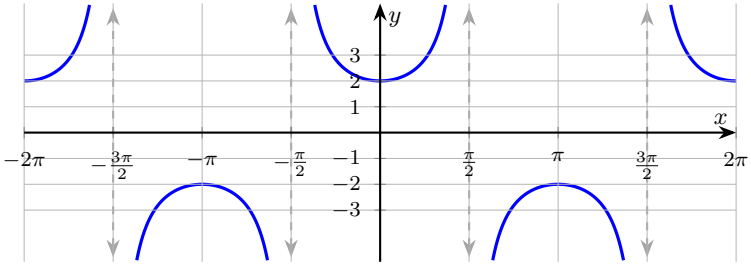
Word Problems

21. A guy-wire stretches from the top of a 10-ft pole to a point on the ground. If the wire makes angle θ with the pole (measured from vertical), the wire's length is $L(\theta) = 10\sec\theta$ ft. Find L when $\theta = \dfrac{\pi}{3}$ and explain why the model fails as $\theta \to \dfrac{\pi}{2}$. ____

22. In a refraction problem, $n(\theta) = \sec\theta$ relates the index of refraction to the incidence angle in a simple limit. State the value of n at $\theta = 0$ and the smallest angle $\theta > 0$ where $n = 2$. ____

23. The graph below shows $y = 2\sec(x)$. State the range and the first two positive asymptotes. _____



24. A surveyor measures the slant distance s across a horizontal ditch of width $W = 6$ ft using $s(\theta) = 6\sec\theta$, where θ is the angle of depression of the line of sight from the horizontal. Find s at $\theta = \dfrac{\pi}{4}$, and explain why $\theta \geq \dfrac{\pi}{2}$ violates the model. ____

Additional Practice

25. Amplitude of $y = 4\sin x$. _____

26. Period of $y = \sin(2x)$. _____

27. Amplitude of $y = -3\cos x$. _____

28. Period of $y = \tan(5x)$. _____

29. Midline of $y = 2\sin x - 7$. _____

30. Phase shift of $y = \sin(x - \pi/3)$. _____

31. Range of $y = 5\cos x$. _____

32. Range of $y = 2\sin x + 1$. _____

33. Asymptotes of $y = \tan x$ in one period. _____

34. Domain of $y = \sec x$ excludes what? _____

35. Range of $y = \csc x$. _____

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Answer Keys

1. 2π
2. $x = \frac{\pi}{2} + n\pi,\ n \in \mathbb{Z}$
3. $(-\infty, -1] \cup [1, \infty)$
4. 1
5. 2
6. y-axis
7. 1
8. $\frac{2\pi}{3}$
9. $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$
10. 2π
11. No
12. $(-\infty, -2] \cup [2, \infty)$
13. -1
14. False
15. $\cos x > 0$
16. Reflected across the x-axis
17. $x = \frac{\pi}{2}, \frac{3\pi}{2}$
18. 2
19. 4π
20. 1 (at $x = 0$)
21. $L = 20$ ft; $\theta = \pi/2$ gives infinite length
22. $n(0) = 1;\ \theta = \frac{\pi}{3}$
23. $(-\infty, -2] \cup [2, \infty);\ x = \frac{\pi}{2}, \frac{3\pi}{2}$
24. $s = 6\sqrt{2}$ ft; $\theta \geq \pi/2$ is past vertical

Additional Practice Answers

25. 4
26. π
27. 3
28. $\frac{\pi}{5}$
29. $y = -7$
30. $\frac{\pi}{3}$ right
31. $[-5, 5]$
32. $[-1, 3]$
33. $x = \pm\frac{\pi}{2}$
34. $\cos x = 0$
35. $(-\infty, -1] \cup [1, \infty)$

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. Secant is the reciprocal of cosine, so its pattern repeats whenever cosine repeats. Cosine's period is 2π, and taking a reciprocal does not change that horizontal repeat.

2. Since $\sec x = \frac{1}{\cos x}$, secant is undefined where the denominator is zero. Cosine is zero at $x = \frac{\pi}{2} + n\pi$, so those become the vertical asymptotes.

3. Cosine values stay between -1 and 1. Reciprocals of positive cosine values are at least 1, and reciprocals of negative cosine values are at most -1, so the middle band is skipped.

4. Use the reciprocal definition first: $\sec 0 = \frac{1}{\cos 0}$. Since $\cos 0 = 1$, the value is $\frac{1}{1} = 1$.

5. Secant is reciprocal cosine. Because $\cos(\pi/3) = \frac{1}{2}$, the secant value is $\frac{1}{1/2} = 2$.

6. Cosine is even, so $\cos(-x) = \cos x$. Taking reciprocals preserves that equality: $\sec(-x) = \sec x$. That is y-axis symmetry.

7. At $x = 0$: $\sec 0 = 1$. (The parent secant graph crosses through $(0, 1)$ – the bottom of its central U.)

8. Start with the key idea: Secant keeps cosine's period rule, $\frac{2\pi}{|b|}$. With $b = 3$, period $= \frac{2\pi}{3}$. That gives a quick check on the answer.

9. Secant blows up where its cosine denominator is zero. Set $\cos(2x) = 0$: $2x = \frac{\pi}{2} + n\pi$, so $x = \frac{\pi}{4} + \frac{n\pi}{2}$. On $[0, 2\pi)$ that gives $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$.

10. Keep the rule visible: The pattern (U-up, U-down) repeats every 2π. The asymptotes are at $\frac{\pi}{2} + n\pi$. That gives a quick check on the answer.

11. An x-intercept would require $\sec x = 0$. But $\sec x = \frac{1}{\cos x}$ has numerator 1, so it can never equal zero.

12. Start with the key idea: Multiply by 2: $|y| \geq 2$. The graph never enters the band $-2 < y < 2$. That gives a quick check on the answer.

13. A careful way to see it: Start with cosine: $\cos \pi = -1$. Secant is the reciprocal, so $\sec \pi = \frac{1}{-1} = -1$. That gives a quick check on the answer.

14. Secant follows cosine's repeat length because it is $1/\cos x$. Cosine repeats after 2π, not after π, so the statement is false.

15. Secant is $\frac{1}{\cos x}$. A reciprocal has the same sign as the denominator, so secant is positive exactly when $\cos x$ is positive.

16. Start with the key idea: The minus sign flips each U. Asymptotes don't move – still at $\frac{\pi}{2} + n\pi$. That gives a quick check on the answer.

17. A careful way to see it: Secant has asymptotes where $\cos x = 0$. On $[0, 2\pi)$, cosine is zero at $\frac{\pi}{2}$ and $\frac{3\pi}{2}$. That gives a quick check on the answer.

18. Secant is even because cosine is even. So $\sec(-\pi/3) = \sec(\pi/3)$. Since $\cos(\pi/3) = \frac{1}{2}$, the secant value is 2.

19. Here $b = \frac{1}{2}$. Period $= \frac{2\pi}{|b|} = \frac{2\pi}{1/2} = 4\pi$ – dividing by a fraction stretches the graph wider.

20. Secant equals 1 at $x = 0$ and grows as you move toward the asymptote at $\frac{\pi}{2}$. So 1 is the floor.

21. A careful way to see it: $L = 10\sec(\pi/3) = 10 \cdot 2 = 20$ ft. As $\theta \to \pi/2$, the wire goes horizontal and can't anchor to the ground – the model gives infinite length, matching the secant asymptote at $\theta = \pi/2$. That gives a quick check on the answer.

22. Keep the rule visible: $\sec 0 = 1$ (light entering perpendicular). $\sec\theta = 2 \Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$. This is the part to check before moving on, because it keeps the answer tied to the original question.

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23. With $|a| = 2$, range is $|y| \geq 2$. Asymptotes don't move when you scale vertically – still at $\frac{\pi}{2} + n\pi$. First two positive are $\frac{\pi}{2}$ and $\frac{3\pi}{2}$.

24. Start with the key idea: $s = 6\sec(\pi/4) = 6\sqrt{2} \approx 8.49$ ft. At $\theta = \pi/2$ the sight line is vertical – pointing straight down – and $\sec$ is undefined. The model breaks because we can't measure a horizontal distance with a vertical line of sight. That gives a quick check on the answer.

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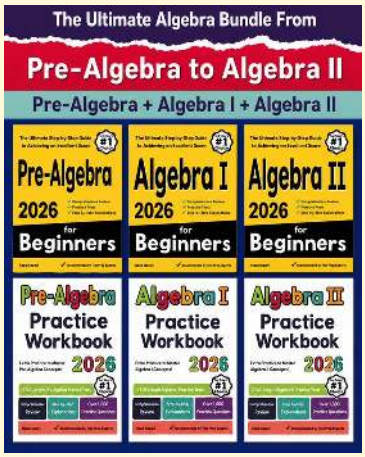

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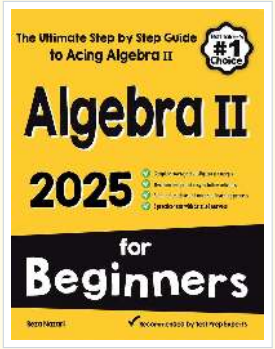

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