Graphing the Cotangent Function

Name: ______________ Date: ______________ Score: ______ / 32

Quick Review

Cotangent is tangent's flip-mate: $\cot x = \dfrac{\cos x}{\sin x} = \dfrac{1}{\tan x}$. Same kind of branch-and-asymptote graph as tangent, but it slopes the *other* way and its asymptotes sit at different places.

The branches go down. Between any two consecutive asymptotes, cotangent slides from $+\infty$ down to $-\infty$ (tangent does the opposite – it climbs).

Key facts.

Domain: all x except $n\pi$ (where $\sin x = 0$).

Range: $\mathbb{R}$.

Period: π.

Vertical asymptotes: $x = n\pi$.

Zeros: $x = \dfrac{\pi}{2} + n\pi$ – wherever cosine is zero.

Symmetry: about the origin – cotangent is odd, $\cot(-x) = -\cot x$.

Tangent vs. cotangent at a glance.

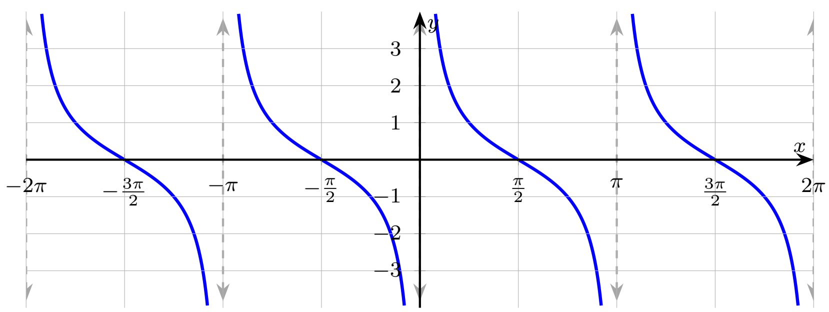
Tangent: zeros at $n\pi$, asymptotes at $\dfrac{\pi}{2} + n\pi$, climbs left-to-right.

Cotangent: zeros at $\dfrac{\pi}{2} + n\pi$, asymptotes at $n\pi$, descends left-to-right.

Transformations. For $y = a\cot(bx)$, period $= \dfrac{\pi}{|b|}$. Asymptotes wherever $\sin(bx) = 0$, i.e. $x = \dfrac{n\pi}{b}$.

Common slips. Putting cotangent's asymptotes at tangent's locations. Drawing cotangent as a climber – it's a descender. Forgetting that the period is π, not 2π.

Parent cotangent graph – descending branches with asymptotes at integer multiples of π:



PRACTICE

Find period, asymptotes, zeros; sketch cotangent transformations.

1. State the period of $y = \cot x$. ________
2. Where are the vertical asymptotes of $y = \cot x$? ________
3. Where are the zeros of $y = \cot x$? ________
4. State the range of $y = \cot x$. ________
5. Evaluate $\cot\left(\dfrac{\pi}{4}\right)$. ________
6. Evaluate $\cot\left(\dfrac{\pi}{3}\right)$. ________
7. Is $y = \cot x$ symmetric about the y-axis or the origin? ________
8. What is the period of $y = \cot(2x)$? ________

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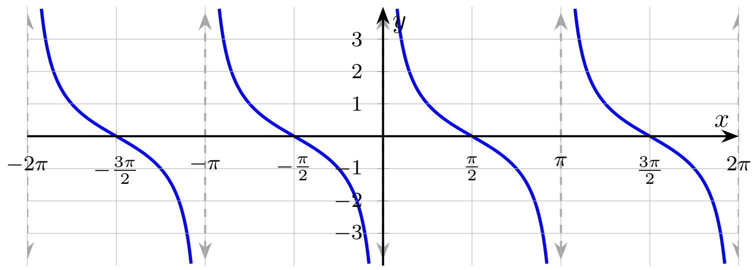
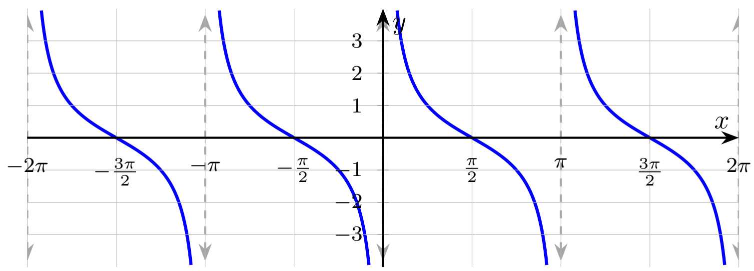
9. Where are the vertical asymptotes of $y = \cot(2x)$ on $[0, 2\pi)$? ________

10. Identify the period and one asymptote of the graph below. ________

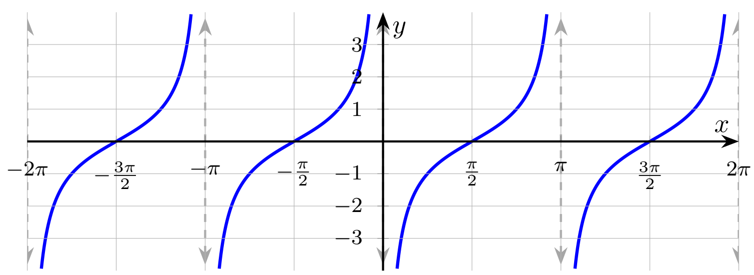


11. Does $y = \cot x$ (shown below) increase or decrease as x moves left to right within one branch? ________



12. True or false: $\cot x$ has the same zeros as $\sin x$. ________

13. How does $y = -\cot x$ (shown below) compare to $y = \cot x$? ________



14. Evaluate $\cot\left(\frac{\pi}{2}\right)$. ________

15. State the asymptotes of $y = \cot x$ on $[-2\pi, 2\pi]$. ________

16. What is the value of $\cot(\pi)$? ________

17. For what x-values is $\cot x > 0$ on $[0, 2\pi)$? ________

18. What is the period of $y = \cot\left(\frac{x}{4}\right)$? ________

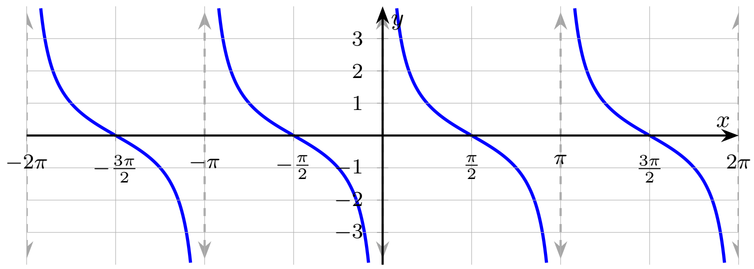
19. State the y-axis intercept of $y = \cot x$. ________

20. Evaluate $\cot\left(-\frac{\pi}{4}\right)$. ________

Word Problems

21. In a flagpole problem, an observer stands horizontal distance x from the base of a 30-ft flagpole and looks up at angle θ of elevation. Then $\cot\theta = \frac{x}{30}$. Find the angle when the observer is 30 ft from the base, and explain why $\theta = 0$ gives $x = \infty$. ____

22. The graph below shows $y = \cot(x)$. Read one zero and one asymptote from the graph. ________



23. In ladder safety, the horizontal distance from a 12-ft ladder's base to a wall is $d(\theta) = 12\cos\theta$, while the rise is $h(\theta) = 12\sin\theta$. The ratio $\frac{d}{h} = \cot\theta$. Compute $\frac{d}{h}$ at $\theta = \frac{\pi}{3}$ and explain why a small θ ratio is *unsafe*. ____

24. A model gives the ratio of side b to side a in a triangle as $\frac{b}{a} = \cot\theta$. Find the first positive θ where $\frac{b}{a} = 1$ and the first positive θ where the ratio is undefined. ____

Additional Practice

25. Amplitude of $y = 4\sin x$. ________

26. Period of $y = \sin(2x)$. ________

27. Amplitude of $y = -3\cos x$. ________

28. Period of $y = \tan(5x)$. ________

29. Midline of $y = 2\sin x - 7$. ________

30. Phase shift of $y = \sin(x - \pi/3)$. ________

31. Range of $y = 5\cos x$. ________

32. Range of $y = 2\sin x + 1$. ________

Answer Keys

1. $\boxed{\pi}$
2. $\boxed{x = n\pi,\ n \in \mathbb{Z}}$
3. $\boxed{x = \frac{\pi}{2} + n\pi}$
4. $\boxed{\mathbb{R}}$
5. $\boxed{1}$
6. $\boxed{\frac{\sqrt{3}}{3}\ \left(\text{or } \frac{1}{\sqrt{3}}\right)}$
7. Origin
8. $\boxed{\frac{\pi}{2}}$
9. $\boxed{x = 0,\ \frac{\pi}{2},\ \pi,\ \frac{3\pi}{2}}$
10. period π; $x = 0$
11. Decrease
12. False
13. Reflected across the x-axis
14. $\boxed{0}$
15. $\boxed{x = -2\pi, -\pi, 0, \pi, 2\pi}$
16. Undefined
17. $\boxed{\left(0, \frac{\pi}{2}\right) \cup \left(\pi, \frac{3\pi}{2}\right)}$
18. $\boxed{4\pi}$
19. None (asymptote at $x = 0$)
20. $\boxed{-1}$
21. $\theta = \frac{\pi}{4}$; $\theta = 0 \Rightarrow \cot\theta \to \infty$
22. zero $x = \frac{\pi}{2}$; asymptote $x = 0$
23. $\cot(\pi/3) = \frac{\sqrt{3}}{3}$; $\theta \to 0$ means ladder lying flat
24. $\theta = \frac{\pi}{4}$; $\theta = \pi$

Additional Practice Answers

25. $\boxed{4}$
26. $\boxed{\pi}$
27. $\boxed{3}$
28. $\boxed{\frac{\pi}{5}}$
29. $\boxed{y = -7}$
30. $\frac{\pi}{3}$ right
31. $\boxed{[-5, 5]}$
32. $\boxed{[-1, 3]}$

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. A careful way to see it: Cotangent (like tangent) repeats every π. This is the part to check before moving on, because it keeps the answer tied to the original question.

2. Keep the rule visible: $\cot x$ has $\sin x$ in the denominator. Sine is zero at every integer multiple of π. That gives a quick check on the answer.

3. One steady path is: $\cot x = 0 \Leftrightarrow \cos x = 0$, which happens at $\frac{\pi}{2} + n\pi$. This is the part to check before moving on, because it keeps the answer tied to the original question.

4. Between asymptotes, cotangent slides from $+\infty$ down to $-\infty$, covering every real value.

5. A careful way to see it: $\cot(\pi/4) = \frac{\cos(\pi/4)}{\sin(\pi/4)} = 1$ (equal numerator and denominator). That gives a quick check on the answer.

6. Keep the rule visible: $\cot(\pi/3) = \frac{1/2}{\sqrt{3}/2} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$. This is the part to check before moving on, because it keeps the answer tied to the original question.

7. One steady path is: Cotangent is odd, like tangent. This is the part to check before moving on, because it keeps the answer tied to the original question.

8. Like tangent, cotangent uses period $\frac{\pi}{|b|}$ – π on top, not 2π. With $b = 2$, period $= \frac{\pi}{2}$.

9. Cotangent has $\sin(2x)$ in its denominator, so asymptotes sit where $\sin(2x) = 0$: $2x = n\pi$, giving $x = \frac{n\pi}{2}$. On $[0, 2\pi)$ that lists $0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$.

10. Each descending branch spans one period of π. $x = 0$ is the asymptote at the origin.

11. Cotangent descends. Each branch starts at $+\infty$ just after an asymptote on the left and ends at $-\infty$ just before the next asymptote on the right.

12. Start with the key idea: $\cot x = 0$ where $\cos x = 0$, not where $\sin x = 0$. $\sin x = 0$ is where cotangent has asymptotes. That gives a quick check on the answer.

13. A careful way to see it: Branches that descended now climb from $-\infty$ to $+\infty$. Asymptotes don't move. That gives a quick check on the answer.

14. Keep the rule visible: $\cot(\pi/2) = \frac{\cos(\pi/2)}{\sin(\pi/2)} = \frac{0}{1} = 0$. This is the part to check before moving on, because it keeps the answer tied to the original question.

15. One steady path is: Every integer multiple of π. This is the part to check before moving on, because it keeps the answer tied to the original question.

16. Start with the key idea: $\sin\pi = 0$, so cotangent's denominator is zero. The graph has an asymptote at $x = \pi$. That gives a quick check on the answer.

17. Since $\cot x = \frac{\cos x}{\sin x}$, it is positive exactly when sine and cosine share a sign – in Q1 $\left(0, \frac{\pi}{2}\right)$ and Q3 $\left(\pi, \frac{3\pi}{2}\right)$.

18. Here $b = \frac{1}{4}$. For cotangent, period $= \frac{\pi}{|b|} = \frac{\pi}{1/4} = 4\pi$ – the small b spreads the branches far apart.

19. One steady path is: There's an asymptote at $x = 0$, so the graph doesn't touch the y-axis. That gives a quick check on the answer.

20. Start with the key idea: Odd-symmetry: $\cot(-\pi/4) = -\cot(\pi/4) = -1$. This is the part to check before moving on, because it keeps the answer tied to the original question.

21. A careful way to see it: $\cot\theta = \frac{30}{30} = 1$, so $\theta = \frac{\pi}{4}$. At $\theta = 0$, $\sin\theta \to 0$ and $\cot\theta \to \infty$ – you'd have to stand infinitely far away to make the pole look horizontal. That gives a quick check on the answer.

22. The graph crosses the x-axis between the asymptotes at $x = 0$ and $x = \pi$ – at $x = \frac{\pi}{2}$ (where cosine is zero). $x = 0$ is the nearest asymptote on the left.

23. One steady path is: $\cot(\pi/3) = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} \approx 0.577$. Small θ makes the ladder nearly horizontal – the base is far from the wall and the ratio of run to rise explodes ($\cot\theta \to \infty$). For safety, θ should be near $\frac{\pi}{3}$ (about $60°$). That gives a quick check on the answer.

24. Start with the key idea: $\cot\theta = 1$ at $\theta = \frac{\pi}{4}$ (where $\cos = \sin$). The ratio is undefined at the first asymptote $\theta > 0$, which is $\theta = \pi$ (where $\sin\theta = 0$ again). That gives a quick check on the answer.

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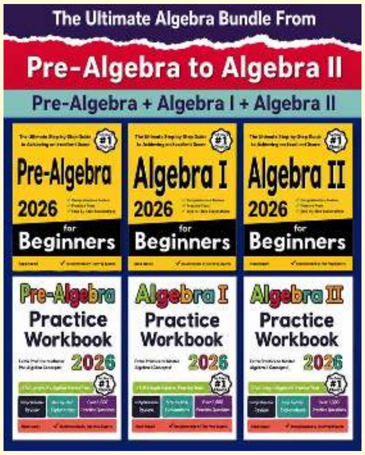

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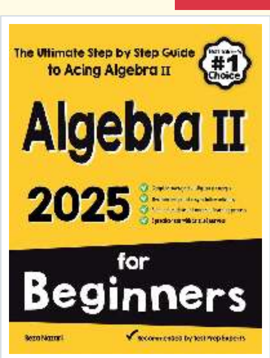

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