

# Graphing the Cosine Function

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Score: \_\_\_\_\_ / 36

## Q Quick Review

Cosine is sine's twin – same shape, same range, same period, just shifted. Where sine starts at  $(0, 0)$  and climbs, cosine starts at the top:  $(0, 1)$ . Plot  $y = \cos x$  and you get a wave that begins at a crest and rolls forever.

**The five anchor points of one cycle.** Start at  $(0, 1)$ , drop to zero at  $(\frac{\pi}{2}, 0)$ , hit the bottom at  $(\pi, -1)$ , climb back to zero at  $(\frac{3\pi}{2}, 0)$ , and return to the top at  $(2\pi, 1)$ .

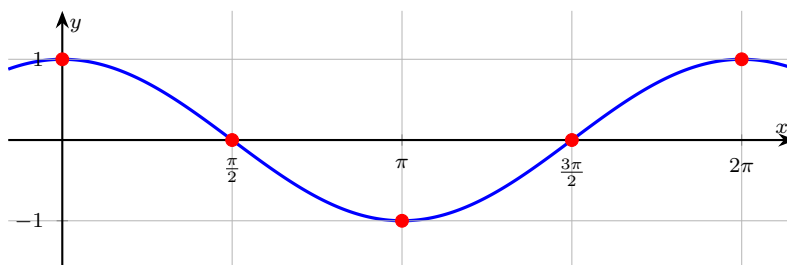
**Key facts.** Domain:  $\mathbb{R}$ . Range:  $[-1, 1]$ . Period:  $2\pi$ . Amplitude: 1. Zeros:  $x = \frac{\pi}{2} + n\pi$  (every half-period from  $\frac{\pi}{2}$ ). Symmetry: about the  $y$ -axis – cosine is *even*,  $\cos(-x) = \cos x$ .

**Sine vs. cosine in one line.** They are the same curve shifted by  $\frac{\pi}{2}$ :  $\cos x = \sin(x + \frac{\pi}{2})$ . That single fact unlocks half the problems in this chapter.

**Transformation read.** In  $y = a \cos(bx)$  the amplitude is  $|a|$  and the period is  $\frac{2\pi}{|b|}$ . A negative  $a$  reflects the graph across the  $x$ -axis – the wave now starts at the bottom  $(0, -|a|)$  instead of the top.

**Common slips.** Confusing the cosine zero ( $x = \frac{\pi}{2}$ ) with the sine maximum – they happen at the same  $x$ , which is why students mix them up. Forgetting that cosine *starts* at the top, so the  $y$ -intercept is 1 (not 0). Calling cosine odd – it's even.

Here's the parent cosine wave from 0 to  $2\pi$ :



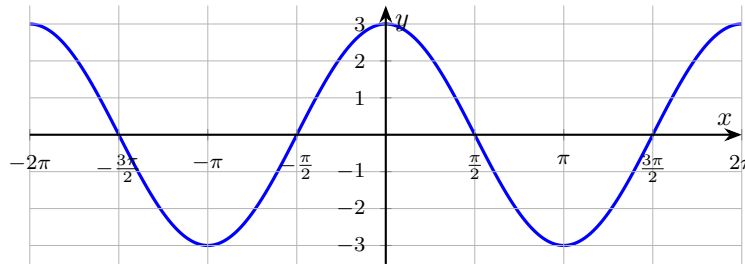
## PRACTICE

Read amplitude, period, and intercepts from each cosine graph. Sketch when asked.

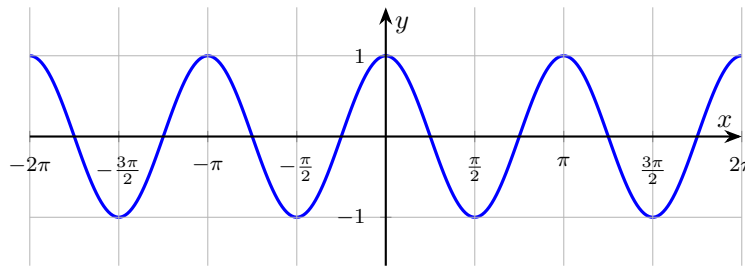
1. State the amplitude and period of  $y = \cos x$ . \_\_\_\_\_
2. State the amplitude and period of  $y = 4 \cos(3x)$ . \_\_\_\_\_
3. What is the  $y$ -intercept of  $y = \cos x$ ? \_\_\_\_\_
4. Where does  $\cos x$  reach its maximum on  $[0, 2\pi]$ ? \_\_\_\_\_
5. At which values of  $x$  does  $\cos x = 0$ ? \_\_\_\_\_
6. How does the graph of  $y = -\cos x$  compare to  $y = \cos x$ ? \_\_\_\_\_
7. What is the range of  $y = \cos x$ ? \_\_\_\_\_
8. Evaluate  $\cos(\frac{\pi}{2})$ . \_\_\_\_\_
9. Evaluate  $\cos(\pi)$ . \_\_\_\_\_
10.  $y = \cos x$  is symmetric about which feature? \_\_\_\_\_



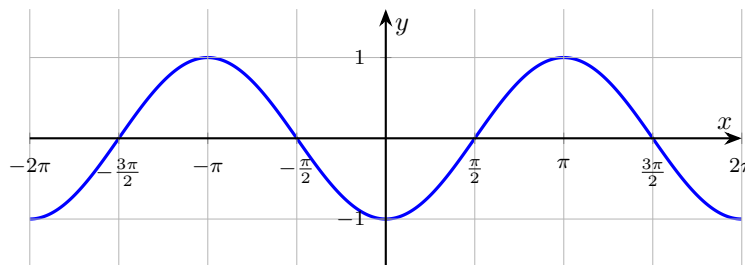
11. Identify the amplitude and period from the graph below. \_\_\_\_\_



12. Identify the period from the graph below. \_\_\_\_\_



13. Identify the amplitude from the graph below. \_\_\_\_\_



14. Write the equation of a cosine function with amplitude 2 and period  $4\pi$ , no shifts. \_\_\_\_\_

15. State the domain and range of  $y = \cos x$ . \_\_\_\_\_

16. Evaluate  $\cos(2\pi)$ . \_\_\_\_\_

17. At what  $x$  does  $\cos x$  first reach its minimum on  $[0, 2\pi)$ ? \_\_\_\_\_

18. True or false:  $\cos x$  and  $\sin x$  have the same period. \_\_\_\_\_

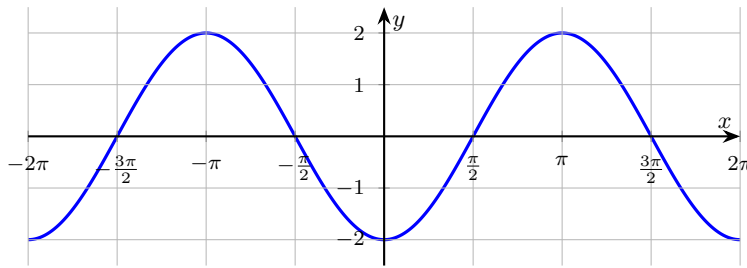
19. How many complete cycles of  $y = \cos(2x)$  appear on  $[0, 2\pi)$ ? \_\_\_\_\_

20. Evaluate  $\cos\left(-\frac{\pi}{3}\right)$ . \_\_\_\_\_



◆ Word Problems

21. A daily-temperature model is  $T(t) = 10 \cos t + 70$ , where  $T$  is in degrees Fahrenheit and  $t$  is in radians (with  $t = 0$  at the warmest moment). State the amplitude, period, and the minimum temperature. \_\_\_\_\_
22. A pendulum's horizontal position is  $x(t) = 0.4 \cos(2t)$  meters. How far does the pendulum swing from center, and how long does one full back-and-forth take? \_\_\_\_\_
23. The graph below shows a transformation of the cosine function. Find the amplitude, period, and equation. \_\_\_\_\_



24. A wheel of radius 1 meter rolls without slipping. The horizontal position of a marker on its rim relative to the wheel's center is  $x(\theta) = \cos \theta$ . At which angle (in  $[0, 2\pi]$ ) is the marker farthest to the left, and how far left is it? \_\_\_\_\_

Additional Practice

25. Amplitude of  $y = 4 \sin x$ . \_\_\_\_\_
26. Period of  $y = \sin(2x)$ . \_\_\_\_\_
27. Amplitude of  $y = -3 \cos x$ . \_\_\_\_\_
28. Period of  $y = \tan(5x)$ . \_\_\_\_\_
29. Midline of  $y = 2 \sin x - 7$ . \_\_\_\_\_
30. Phase shift of  $y = \sin(x - \pi/3)$ . \_\_\_\_\_
31. Range of  $y = 5 \cos x$ . \_\_\_\_\_
32. Range of  $y = 2 \sin x + 1$ . \_\_\_\_\_
33. Asymptotes of  $y = \tan x$  in one period. \_\_\_\_\_
34. Domain of  $y = \sec x$  excludes what? \_\_\_\_\_
35. Range of  $y = \csc x$ . \_\_\_\_\_
36. Range of  $y = \arcsin x$ . \_\_\_\_\_



## Answer Keys

<p>1. amplitude 1, period <math>2\pi</math></p> <p>2. amplitude 4, period <math>\frac{2\pi}{3}</math></p> <p>3. 1</p> <p>4. <math>x = 0</math> and <math>x = 2\pi</math></p> <p>5. <math>x = \frac{\pi}{2} + n\pi, n \in \mathbb{Z}</math></p> <p>6. reflected across the <math>x</math>-axis</p> <p>7. <math>[-1, 1]</math></p> <p>8. 0</p> <p>9. -1</p> <p>10. the <math>y</math>-axis</p> <p>11. amplitude 3, period <math>2\pi</math></p> <p>12. <math>\pi</math></p>	<p>13. 1</p> <p>14. <math>y = 2 \cos\left(\frac{x}{2}\right)</math></p> <p>15. domain <math>\mathbb{R}</math>, range <math>[-1, 1]</math></p> <p>16. 1</p> <p>17. <math>x = \pi</math></p> <p>18. True</p> <p>19. 2</p> <p>20. <math>\frac{1}{2}</math></p> <p>21. amplitude 10, period <math>2\pi</math>, min = <math>60^\circ\text{F}</math></p> <p>22. 0.4 m; <math>\pi</math> s</p> <p>23. amplitude 2, period <math>2\pi</math>, <math>y = -2 \cos x</math></p> <p>24. <math>\theta = \pi</math>; -1 m</p>
<b>Additional Practice Answers</b>	
<p>25. 4</p> <p>26. <math>\pi</math></p> <p>27. 3</p> <p>28. <math>\frac{\pi}{5}</math></p> <p>29. <math>y = -7</math></p> <p>30. <math>\frac{\pi}{3}</math> right</p>	<p>31. <math>[-5, 5]</math></p> <p>32. <math>[-1, 3]</math></p> <p>33. <math>x = \pm \frac{\pi}{2}</math></p> <p>34. <math>\cos x = 0</math></p> <p>35. <math>(-\infty, -1] \cup [1, \infty)</math></p> <p>36. <math>[-\frac{\pi}{2}, \frac{\pi}{2}]</math></p>

**Additional Practice:** Answers for all numbered items, including the added practice, are shown in the grid above.

### Step-by-Step Explanations

1. The parent cosine wave swings 1 unit above and below the midline and finishes a full cycle every  $2\pi$ .
2. Match  $y = a \cos(bt)$ :  $a = 4$ ,  $b = 3$ . Amplitude is  $|a| = 4$ , so the wave reaches 4 above and below the midline. Period is  $\frac{2\pi}{|b|} = \frac{2\pi}{3}$  – always divide  $2\pi$  by  $b$ , so the graph fits three full cycles in the usual  $2\pi$ .
3. One steady path is: Plug  $x = 0$ :  $\cos(0) = 1$ . Cosine starts at the top. This is the part to check before moving on, because it keeps the answer tied to the original question.
4. Cosine hits its max of 1 at the start and end of one cycle. In general,  $x = 2n\pi$ .
5. Cosine is the  $x$ -coordinate on the unit circle;  $x$ -coordinate is zero at the top and bottom of the circle, which is  $\frac{\pi}{2} + n\pi$ .
6. The negative sign sends each output to its opposite. The new wave starts at the bottom:  $(0, -1)$ .
7. Same as sine – cosine is the  $x$ -coordinate on the unit circle, which lives in  $[-1, 1]$ .
8. Start with the key idea: At  $\frac{\pi}{2}$ , the unit-circle point is  $(0, 1)$ . Cosine reads the  $x$ -coordinate: 0. That gives a quick check on the answer.
9. A careful way to see it: At  $\pi$ , the unit-circle point is  $(-1, 0)$ , so  $\cos \pi = -1$ . This is the part to check before moving on, because it keeps the answer tied to the original question.
10. Keep the rule visible: Cosine is even:  $\cos(-x) = \cos x$ . That gives mirror symmetry across the  $y$ -axis. That gives a quick check on the answer.
11. The wave reaches 3 at the peak and -3 at the trough. It starts at  $(0, 3)$  and returns at  $(2\pi, 3)$ , so the period is  $2\pi$ . The equation is  $y = 3 \cos x$ .
12. The cosine starts at the top, dips, and returns to the top after only  $\pi$  units. Period =  $\pi$ , equation  $y = \cos(2x)$ .
13. The wave maxes at 1 and mins at -1, so amplitude = 1. It starts at the bottom  $(0, -1)$ , so the equation is  $y = -\cos x$  – the parent cosine flipped across the  $x$ -axis.
14. Amplitude 2 gives  $a = 2$ . For the period, set  $\frac{2\pi}{b} = 4\pi$  and solve:

- $b = \frac{2\pi}{4\pi} = \frac{1}{2}$ . No shifts, so  $y = 2 \cos\left(\frac{x}{2}\right)$ ; the small  $b$  stretches the wave to twice the usual width.
15. Every angle has a cosine (domain all reals), and the outputs are the  $x$ -coordinates on the unit circle.
  16. Start with the key idea:  $2\pi$  is one full lap – back to the start.  $\cos(2\pi) = \cos(0) = 1$ . This is the part to check before moving on, because it keeps the answer tied to the original question.
  17. A careful way to see it: The minimum is -1, hit at the leftmost point of the circle:  $x = \pi$ . That gives a quick check on the answer.
  18. Keep the rule visible: Both have period  $2\pi$ . Cosine is just sine shifted left by  $\frac{\pi}{2}$ . That gives a quick check on the answer.
  19. Here  $b = 2$ , so the period is  $\frac{2\pi}{|b|} = \frac{2\pi}{2} = \pi$ . The interval  $[0, 2\pi]$  has length  $2\pi$ , and  $\frac{2\pi}{\pi} = 2$ , so exactly two full cycles fit.
  20. Start with the key idea: Cosine is even:  $\cos\left(-\frac{\pi}{3}\right) = \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$ . This is the part to check before moving on, because it keeps the answer tied to the original question.
  21. Match against  $y = a \cos(bt) + d$ :  $a = 10$ ,  $b = 1$ ,  $d = 70$ . Amplitude is  $|a| = 10$  and period is  $\frac{2\pi}{|b|} = 2\pi$ . The minimum sits one amplitude below the midline:  $d - |a| = 70 - 10 = 60^\circ\text{F}$ .
  22. Amplitude = 0.4, so the pendulum swings 0.4 m either side of center. Period =  $\frac{2\pi}{2} = \pi$  seconds, the time for one complete swing.
  23. The wave starts at  $(0, -2)$  – a reflected cosine – and reaches +2 at the trough turned upside down (which is  $x = \pi$ ). So amplitude = 2 and the leading sign is negative. Period is the standard  $2\pi$ . Equation:  $y = -2 \cos x$ .
  24. Start with the key idea:  $\cos \theta$  hits its minimum of -1 at  $\theta = \pi$ . The marker is then 1 meter to the left of the wheel's center. That gives a quick check on the answer.



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