Graphing the Cosecant Function

Name: ______________	Date: ______________	Score: _____ / 32

Quick Review

Cosecant is the reciprocal of sine: $\csc x = \dfrac{1}{\sin x}$. Wherever sine is small, cosecant is huge. Wherever sine is exactly zero, cosecant is undefined and the graph has a vertical asymptote.

The U-and-upside-down-U shape. Picture the sine wave faintly underneath. Between each pair of x-intercepts of sine (which are π apart), the cosecant graph is a single U (above the x-axis where sine is positive) or an upside-down U (below where sine is negative).

Key facts.

Domain: all x except $n\pi$ (where $\sin x = 0$).

Range: $(-\infty, -1] \cup [1, \infty)$ – cosecant never enters $(-1, 1)$.

Period: 2π, same as sine.

Vertical asymptotes: $x = n\pi$.

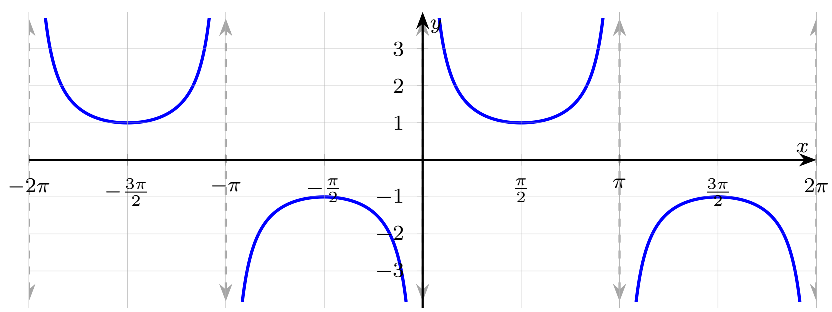
Symmetry: about the origin – cosecant is odd, $\csc(-x) = -\csc x$.

No zeros. A reciprocal of something is never zero, so the cosecant graph never crosses the x-axis.

Transformations. For $y = a\csc(bx)$, period $= \dfrac{2\pi}{|b|}$, range $= (-\infty, -|a|] \cup [|a|, \infty)$. The asymptotes sit wherever $\sin(bx) = 0$, i.e. $bx = n\pi \Rightarrow x = \dfrac{n\pi}{b}$.

Common slips. Mixing up cosecant and secant. Cosecant pairs with *sine* (asymptotes at $n\pi$); secant pairs with cosine (asymptotes at $\frac{\pi}{2} + n\pi$). Drawing cosecant inside $(-1, 1)$ – it never goes there. Forgetting that $|a|$ is the closest cosecant gets to 0, not the amplitude.

The parent cosecant graph (dashed) sits inside the U's of the sine wave (gray for reference):



PRACTICE

Find period, asymptotes, range; sketch cosecant transformations.

1. State the period of $y = \csc x$. ________
2. Where are the vertical asymptotes of $y = \csc x$? ________
3. State the range of $y = \csc x$. ________
4. Evaluate $\csc\left(\dfrac{\pi}{2}\right)$. ________
5. Evaluate $\csc\left(\dfrac{\pi}{6}\right)$. ________
6. Is the graph of $y = \csc x$ symmetric about the y-axis or the origin? ________
7. Does $y = \csc x$ have any x-intercepts? ________
8. What is the period of $y = \csc(2x)$? ________
9. Where are the vertical asymptotes of $y = \csc(2x)$ on $[0, 2\pi)$? ________

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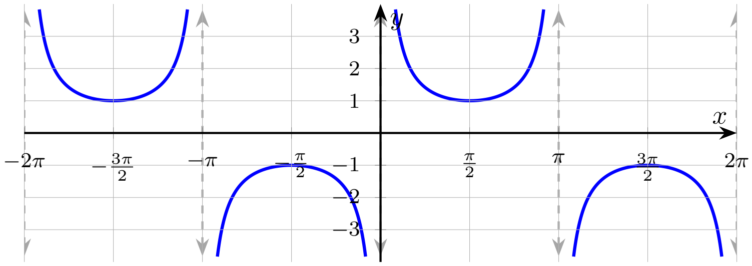
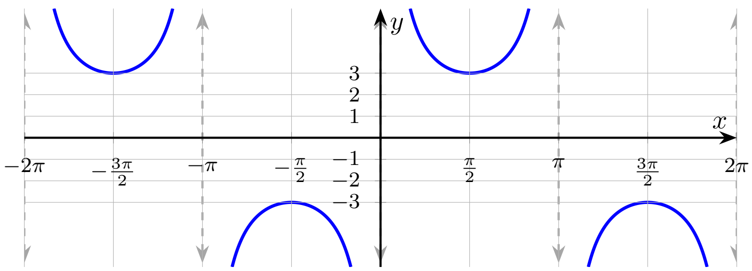
10. Identify the period and one asymptote of the graph below. ________

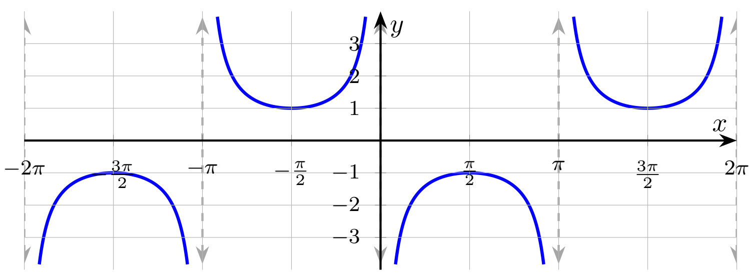


11. What is the minimum positive value of $y = \csc x$? ________

12. State the range of $y = 3\csc x$ shown below. ________



13. How does the graph of $y = -\csc x$ (shown below) compare to $y = \csc x$? ________



14. Evaluate $\csc\left(\frac{3\pi}{2}\right)$. ________

15. Does $\csc x$ have any horizontal asymptotes? ________

16. True or false: $\csc x$ has period π. ________

17. For $y = \csc x$, list the asymptotes on $[-2\pi, 2\pi]$. ________

18. Evaluate $\csc\left(-\frac{\pi}{2}\right)$. ________

19. For what range of $\sin x$-values is $\csc x$ positive? ________

20. Write the period of $y = \csc(\pi x)$. ________

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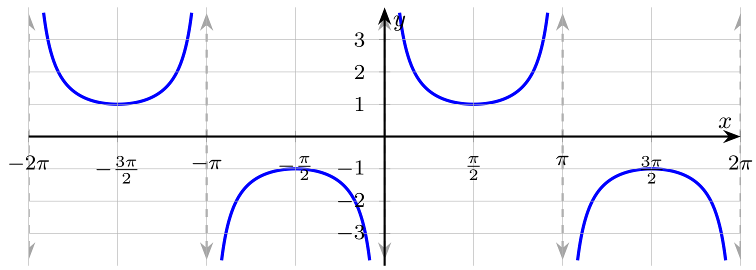
Word Problems

21. In a swing-rope analysis, the tension force has the form $T(\theta) = \dfrac{W}{\sin\theta}$ where W is the weight in pounds and θ is the angle from horizontal. Why does the tension blow up as $\theta \to 0$, and what cosecant value gives $T = 2W$? ____

22. The graph below shows $y = \csc(x)$. From the graph, state the smallest positive x where $y = 2$, and the closest asymptote. ________



23. A ramp of length L feet supports a fixed vertical rise of $h = 12$ feet. The ramp length is $L(\theta) = 12\csc\theta$ feet, where θ is the ramp angle. Find L when $\theta = \dfrac{\pi}{6}$ and explain why the model rejects $\theta = 0$. ____

24. A signal's intensity satisfies $I(t) = \dfrac{4}{\sin t}$ in some pulse model on $(0, \pi)$. List the asymptotes inside the larger interval $(-2\pi, 2\pi)$ and state the minimum positive value of I. ____

Additional Practice

25. Amplitude of $y = 4\sin x$. ________

26. Period of $y = \sin(2x)$. ________

27. Amplitude of $y = -3\cos x$. ________

28. Period of $y = \tan(5x)$. ________

29. Midline of $y = 2\sin x - 7$. ________

30. Phase shift of $y = \sin(x - \pi/3)$. ________

31. Range of $y = 5\cos x$. ________

32. Range of $y = 2\sin x + 1$. ________

Answer Keys

1. $\boxed{2\pi}$
2. $\boxed{x = n\pi,\ n \in \mathbb{Z}}$
3. $\boxed{(-\infty, -1] \cup [1, \infty)}$
4. $\boxed{1}$
5. $\boxed{2}$
6. The origin
7. No
8. $\boxed{\pi}$
9. $\boxed{x = 0,\ \frac{\pi}{2},\ \pi,\ \frac{3\pi}{2}}$
10. period 2π; $x = 0$
11. $\boxed{1}$
12. $\boxed{(-\infty, -3] \cup [3, \infty)}$
13. Reflected across the x-axis
14. $\boxed{-1}$
15. No
16. False
17. $\boxed{x = -2\pi, -\pi, 0, \pi, 2\pi}$
18. $\boxed{-1}$
19. $\sin x > 0$, i.e. Q1, Q2
20. $\boxed{2}$
21. $\boxed{\csc\theta = 2 \Rightarrow \theta = \frac{\pi}{6}}$
22. $x = \frac{\pi}{6}$; asymptote $x = 0$
23. $L = 24$ ft; $\theta = 0$ gives infinite length
24. $\boxed{x = -2\pi, -\pi, 0, \pi, 2\pi;\ \min I = 4}$

Additional Practice Answers

25. $\boxed{4}$
26. $\boxed{\pi}$
27. $\boxed{3}$
28. $\boxed{\frac{\pi}{5}}$
29. $\boxed{y = -7}$
30. $\frac{\pi}{3}$ right
31. $\boxed{[-5, 5]}$
32. $\boxed{[-1, 3]}$

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. A careful way to see it: Cosecant inherits sine's period: 2π. This is the part to check before moving on, because it keeps the answer tied to the original question.

2. Keep the rule visible: $\csc x = \frac{1}{\sin x}$ is undefined where sine is zero – at every integer multiple of π. That gives a quick check on the answer.

3. One steady path is: Since $|\sin x| \le 1$, $|\csc x| \ge 1$. The reciprocal of a small thing is a big thing. That gives a quick check on the answer.

4. Start with the key idea: $\sin(\pi/2) = 1$, so its reciprocal is 1. This is the part to check before moving on, because it keeps the answer tied to the original question.

5. A careful way to see it: $\sin(\pi/6) = \frac{1}{2}$, so $\csc(\pi/6) = 2$. This is the part to check before moving on, because it keeps the answer tied to the original question.

6. Keep the rule visible: Cosecant is odd, just like sine. This is the part to check before moving on, because it keeps the answer tied to the original question.

7. One steady path is: A reciprocal is never zero – the graph never touches the x-axis. That gives a quick check on the answer.

8. Cosecant keeps the same period rule as sine: $\frac{2\pi}{|b|}$. With $b = 2$, period $= \frac{2\pi}{2} = \pi$.

9. Cosecant blows up wherever its sine denominator is zero. Set $\sin(2x) = 0$: $2x = n\pi$, so $x = \frac{n\pi}{2}$. On $[0, 2\pi)$ that lists $0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$.

10. Dashed lines (asymptotes) are π apart, but each U plus the next upside-down U forms one period of 2π. The asymptote at $x = 0$ is the simplest to name.

11. One steady path is: $|\csc x| \ge 1$, with equality at $\sin x = 1$ (i.e. $x = \frac{\pi}{2}$). This is the part to check before moving on, because it keeps the answer tied to the original question.

12. Multiply the parent cosecant range by 3: $|y| \ge 3$. The graph stays out of the strip $-3 < y < 3$.

13. Each upward U becomes downward and vice versa. Asymptote locations don't move.

14. Keep the rule visible: $\sin(3\pi/2) = -1$, so its reciprocal is -1. This is the part to check before moving on, because it keeps the answer tied to the original question.

15. One steady path is: The graph shoots to $\pm\infty$ near every $x = n\pi$ – horizontal lines aren't approached. That gives a quick check on the answer.

16. Start with the key idea: It has period 2π. The U-and-flipped-U pattern repeats every 2π. That gives a quick check on the answer.

17. A careful way to see it: Every integer multiple of π in the interval. This is the part to check before moving on, because it keeps the answer tied to the original question.

18. Keep the rule visible: Use odd-symmetry: $\csc(-\pi/2) = -\csc(\pi/2) = -1$. This is the part to check before moving on, because it keeps the answer tied to the original question.

19. One steady path is: Reciprocals share the sign of the original. Sine positive $\Leftrightarrow$ cosecant positive. That gives a quick check on the answer.

20. Here $b = \pi$. Period $= \frac{2\pi}{|b|} = \frac{2\pi}{\pi} = 2$. The π's cancel, leaving a clean period of 2.

21. The tension is $T = W\csc\theta$, and $\csc\theta \to \infty$ as $\theta \to 0$ – sine approaches zero, the reciprocal explodes. For $T = 2W$, $\csc\theta = 2 \Rightarrow \sin\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}$ (in the first quadrant).

22. Keep the rule visible: $\csc x = 2 \Rightarrow \sin x = \frac{1}{2}$, so the smallest positive solution is $x = \frac{\pi}{6}$. The closest asymptote is $x = 0$, to the left. That gives a quick check on the answer.

23. One steady path is: $L = 12\csc(\pi/6) = 12 \cdot 2 = 24$ ft. At $\theta = 0$, the ramp is horizontal so it can never reach the 12-ft height – the model says $L = \infty$, matching the cosecant asymptote at $\theta = 0$. That gives a quick check on the answer.

24. Start with the key idea: $I = 4\csc t$ has asymptotes wherever $\sin t = 0$ – at $t = n\pi$. Inside $(-2\pi, 2\pi)$ those are $-\pi, 0, \pi$ (and the endpoints $\pm 2\pi$ are right on the edge). The minimum positive I is 4, reached when $\sin t = 1$. That gives a quick check on the answer.

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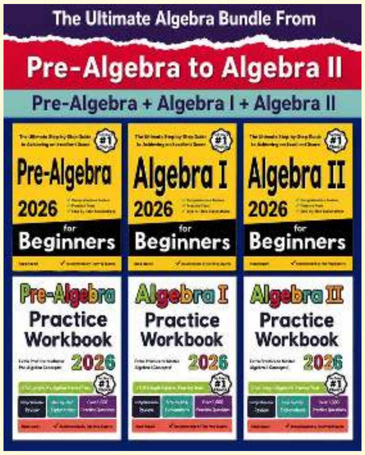

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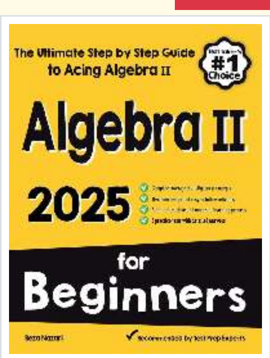

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