Graphing Rational Expressions

Name: ________________ Date: ____________ Score: _____ / 41

Quick Review

Graphing a rational function $f(x) = \frac{p(x)}{q(x)}$ is a four-checklist exercise: *vertical asymptotes, horizontal asymptotes (or slants), holes, intercepts*. Hit all four and the graph practically draws itself.

Vertical asymptote. After reducing f to lowest terms, set the new denominator equal to zero. Each solution is a vertical line the graph runs along but never crosses.

Horizontal asymptote. Compare degrees of numerator (n) and denominator (d). $n < d$: $y = 0$. $n = d$: $y = \frac{\text{leading coeff of } p}{\text{leading coeff of } q}$. $n > d$ by exactly 1: no horizontal asymptote, but a slant $y = mx + b$ found by polynomial division.

Holes. If a factor cancels from top and bottom, the x-value where that factor was zero gives a *hole*, not an asymptote. The y-coordinate of the hole comes from the simplified expression evaluated at that x.

Intercepts. y-intercept: $f(0)$ when defined. x-intercept(s): solve the reduced numerator $= 0$.

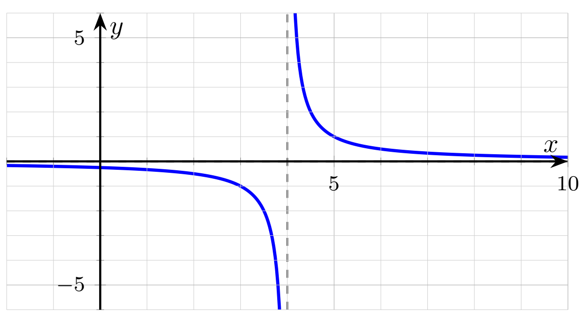
Crossing a horizontal asymptote – yes, it can happen. A horizontal asymptote is end behavior, not a fence. The graph may cross it in the middle. Vertical asymptotes are different – the function is undefined there, so the curve can never touch the line.

Reading the graph. On each rational curve drawn below, the dashed gray lines are the asymptotes. Open white circles are holes. Red dots are intercepts.

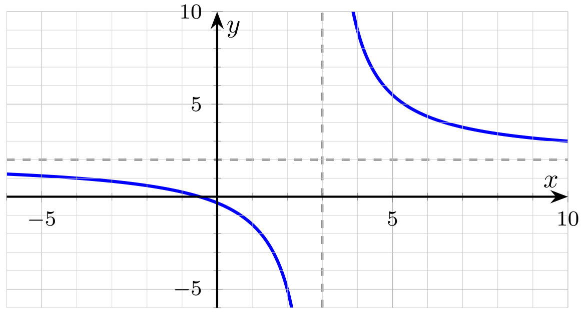
PRACTICE

For each function, read off asymptotes, holes, and intercepts. State the equation of every dashed line you see.

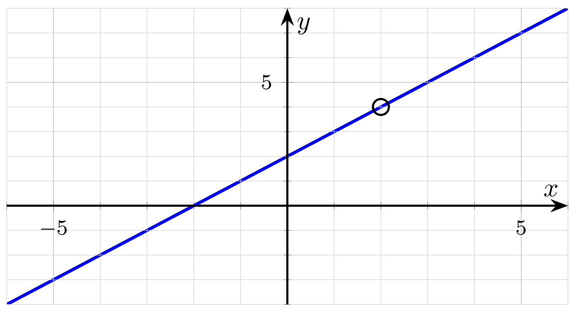
1. For $f(x) = \frac{1}{x-4}$, the vertical asymptote is at $x = ?$. The graph below confirms it. ________



2. For $g(x) = \dfrac{2x+1}{x-3}$, identify the horizontal asymptote shown in the graph. ________

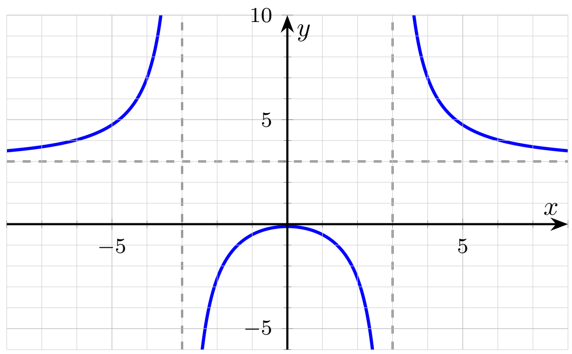


3. For $h(x) = \dfrac{x^2-4}{x-2}$, identify the hole shown in the graph below. ________



4. For $f(x) = \dfrac{2x-6}{x+1}$, find the y-intercept. ________

5. For $f(x) = \dfrac{3x^2+1}{x^2-9}$, the graph has vertical asymptotes at $x = \pm 3$ and a horizontal asymptote at $y = 3$. Confirm the horizontal asymptote from the formula. ________



6. For $f(x) = \dfrac{x^2+1}{x-2}$, what is the end behavior – a horizontal asymptote, a slant asymptote, or neither? ________

7. For $f(x) = \dfrac{(x-1)(x+4)}{(x-1)(x-3)}$, identify both the hole and the vertical asymptote. ________

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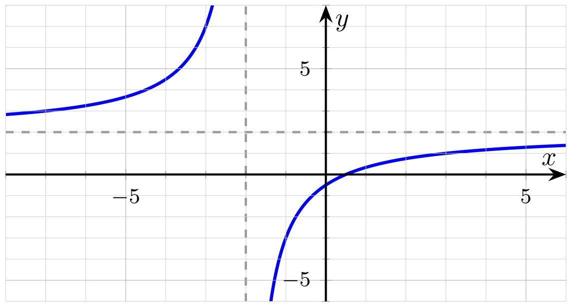
8. For $f(x) = \dfrac{2x-1}{x+2}$, the graph below shows both asymptotes. Write the equation of the vertical asymptote. ________

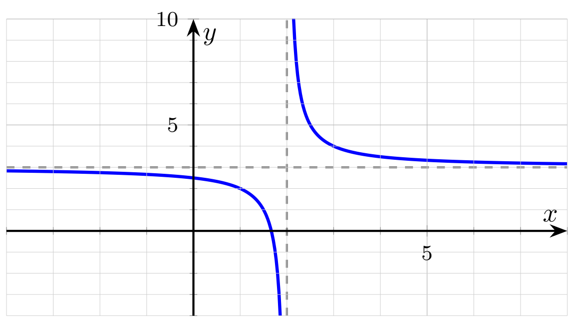


9. Mark TRUE or FALSE: A rational function can cross its horizontal asymptote. ________

10. For $g(x) = \dfrac{(x+2)(x-5)}{(x-5)(x+1)}$, identify the vertical asymptote. ________

11. For $f(x) = \dfrac{x^2+3x-10}{x+5}$, find the hole. ________

12. For $f(x) = \dfrac{1}{x-2} + 3$, the graph (below) shows the asymptotes of the parent $\dfrac{1}{x}$ shifted right 2 and up 3. State both asymptotes. ________



13. For $f(x) = \dfrac{x}{x-3}$, find the x-intercept. ________

14. What is the domain of $f(x) = \dfrac{2x}{x^2-1}$? ________

15. For $g(x) = \dfrac{x-1}{x^2-4}$, identify all vertical asymptotes. ________

16. For $f(x) = \dfrac{4}{x^2+1}$, identify the horizontal asymptote. ________

17. Mark TRUE or FALSE: For $f(x) = \dfrac{3x-1}{x+5}$, the horizontal asymptote is $y = -\dfrac{1}{5}$ because that's the ratio of constants. ________

18. For $f(x) = \dfrac{2}{x-1} - 3$, identify the horizontal asymptote. ________

19. For $f(x) = \dfrac{x^2-1}{x^2+1}$, identify the horizontal asymptote and any vertical asymptotes. ________

20. Mark TRUE or FALSE: A factor that cancels from top and bottom always becomes a hole on the graph. ________

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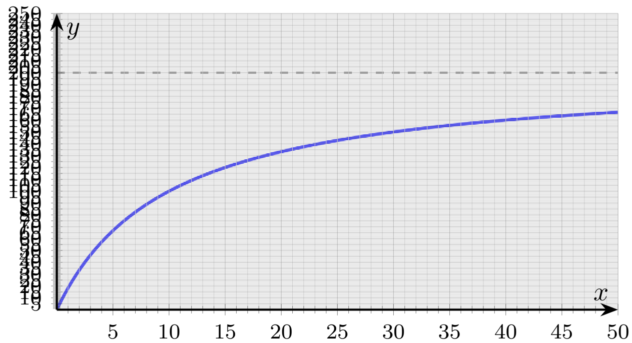
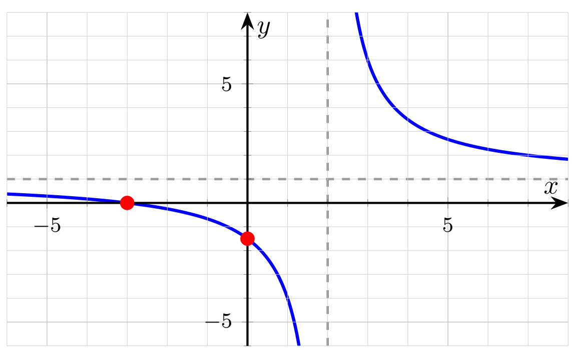
♦ Word Problems

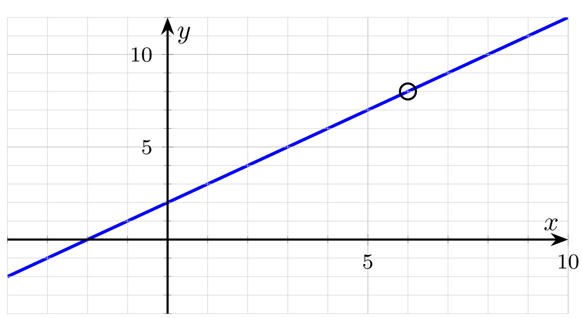
21. A revenue function for a small business is $R(x) = \dfrac{200x}{x+10}$ thousand dollars, where x is the number of months since launch. Identify the horizontal asymptote and explain what it means in context. ____



22. Sketch (or describe) the key features of $f(x) = \dfrac{x+3}{x-2}$: vertical asymptote, horizontal asymptote, x-intercept, y-intercept. The graph is below for confirmation. ____



23. For $f(x) = \dfrac{x^2 - 4x - 12}{x - 6}$, find the hole and explain why there is no vertical asymptote at the same x-value. ____



24. Concentration of a chemical solution decays according to $C(t) = \dfrac{50}{t+2}$ mg/L, with t in minutes. Find the horizontal asymptote and the y-intercept; interpret each in context. ____

Additional Practice

25. Simplify $\frac{x^2-9}{x-3}$. ________

26. Excluded value of $\frac{1}{x+4}$. ________

27. Domain of $f(x)=\frac{x}{x-5}$. ________

28. Multiply $\frac{x}{3}\cdot\frac{6}{x}$. ________

29. Divide $\frac{x^2}{5}\div\frac{x}{10}$. ________

30. Add $\frac{3}{x}+\frac{5}{x}$. ________

31. Subtract $\frac{7}{x-1}-\frac{2}{x-1}$. ________

32. Solve $\frac{1}{x}=4$. ________

33. Solve $\frac{x+2}{x-1}=3$. ________

34. Vertical asymptote of $y=\frac{4}{x+8}$. ________

35. Horizontal asymptote of $y=\frac{3x+1}{x-2}$. ________

36. Simplify complex fraction $\frac{1/x}{3}$. ________

37. Solve $\frac{x}{x+2}=0$. ________

38. Is $x=-2$ allowed in $\frac{x}{x+2}$? ________

39. Simplify $\frac{x^2+5x+6}{x+2}$. ________

40. Find $f(3)$ for $f(x)=\frac{2}{x-1}$. ________

41. Excluded value for $\frac{2}{x+9}$. ________

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Answer Keys

1. $x = 4$
2. $y = 2$
3. $(2, 4)$
4. $(0, -6)$
5. $y = 3$
6. slant asymptote $y = x + 2$
7. hole at $x = 1$, VA at $x = 3$
8. $x = -2$
9. TRUE
10. $x = -1$
11. $(-5, -7)$
12. $x = 2,\ y = 3$
13. $(0, 0)$
14. $x \neq \pm 1$
15. $x = \pm 2$
16. $y = 0$
17. FALSE
18. $y = -3$
19. $y = 1$, no VA
20. TRUE
21. $y = 200$; revenue approaches $200,000/month long-term
22. VA: $x = 2$; HA: $y = 1$; xint: $(-3, 0)$; yint: $(0, -1.5)$
23. Hole at $(6, 8)$; no VA because the $(x - 6)$ factor cancels
24. HA: $y = 0$; yint: $C(0) = 25$ mg/L

Additional Practice Answers

25. $x + 3,\ x \neq 3$
26. $x = -4$
27. $x \neq 5$
28. 2
29. $2x$
30. $\frac{8}{x}$
31. $\frac{5}{x-1}$
32. $x = \frac{1}{4}$
33. $x = \frac{5}{2}$
34. $x = -8$
35. $y = 3$
36. $\frac{1}{3x}$
37. $x = 0$
38. no
39. $x + 3,\ x \neq -2$
40. 1
41. $x = -9$

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. Set $x - 4 = 0$: $x = 4$. The horizontal asymptote is $y = 0$ because the numerator's degree (0) is less than the denominator's (1).
2. Equal degrees – the horizontal asymptote is the ratio of leading coefficients: $\frac{2}{1} = 2$.
3. Factor: $\frac{(x-2)(x+2)}{x-2} = x + 2$ for $x \neq 2$. The canceled $(x - 2)$ leaves a hole; at $x = 2$ the simplified form gives $y = 4$, so the hole is $(2, 4)$.
4. Evaluate $f(0) = \frac{-6}{1} = -6$. (The y-intercept comes from $x = 0$; don't confuse with setting $y = 0$, which gives the x-intercept.)
5. A careful way to see it: Equal degrees (2 and 2); leading coefficients 3 and 1. So $y = \frac{3}{1} = 3$. That gives a quick check on the answer.
6. Numerator's degree exceeds the denominator's by exactly 1, so there's a slant asymptote. Long division: $\frac{x^2+1}{x-2} = x + 2 + \frac{5}{x-2}$. The slant is $y = x + 2$; the remainder term vanishes at large $|x|$.
7. The factor $x - 1$ cancels: hole at $x = 1$. The simplified function is $\frac{x+4}{x-3}$, with the y-value of the hole given by $\frac{1+4}{1-3} = -\frac{5}{2}$. The remaining $x - 3$ in the denominator is a vertical asymptote.
8. Set $x + 2 = 0$: $x = -2$. (The horizontal asymptote is $y = 2$ from the equal-degree rule.)
9. A horizontal asymptote describes end behavior, not a barrier. The graph may dip across it in the middle. (Vertical asymptotes are different – the function is undefined there.)
10. The $(x - 5)$ cancels (hole at $x = 5$). The remaining denominator gives the vertical asymptote $x = -1$.
11. Factor: $x^2 + 3x - 10 = (x + 5)(x - 2)$. Cancel $x + 5$: simplified is $x - 2$. Hole at $x = -5$: $y = -5 - 2 = -7$, so $(-5, -7)$.
12. Start with the key idea: $\frac{1}{x-2} + 3$ has the same shape as $\frac{1}{x}$ but with $x = 2$ as the vertical asymptote and $y = 3$ as the horizontal asymptote. That gives a quick check on the answer.
13. Set the numerator to zero: $x = 0$. The denominator at $x = 0$ is $-3 \neq 0$, so $x = 0$ is a legitimate x-intercept. (Always confirm the denominator doesn't also vanish there – if it did, you'd have a hole, not an intercept.)
14. The domain is everything except where the denominator is zero. Solve $x^2 - 1 = 0$: $(x - 1)(x + 1) = 0$, so $x = \pm 1$. Neither factor cancels with the numerator $2x$, so both are genuine exclusions; everywhere else the function is defined.
15. Factor the denominator: $x^2 - 4 = (x - 2)(x + 2)$. Neither factor cancels with the numerator $(x - 1)$, so both give vertical asymptotes.
16. Numerator degree 0, denominator degree 2. Lower-degree top means horizontal asymptote $y = 0$. (The denominator $x^2 + 1$ is never zero for real x, so there are no vertical asymptotes at all – the graph is a smooth bell.)
17. For end behavior you compare *leading* coefficients, not constants. Here both degrees are 1 with leading coefficients 3 and 1, so the horizontal asymptote is $y = 3$, not $-\frac{1}{5}$.
18. The term $\frac{2}{x-1} \to 0$ at large $|x|$, so $f(x)$ approaches -3. (Vertical shifts change the horizontal asymptote; horizontal shifts change the vertical one.)
19. Both top and bottom have degree 2 with leading coefficient 1, so the horizontal asymptote is $\frac{1}{1} = 1$. For vertical asymptotes you'd need real zeros of the denominator, but $x^2 + 1 > 0$ always – it never hits zero – so there are none.
20. A canceled factor signals a removable discontinuity at the value that made it zero. The y-coordinate comes from the simplified expression at that point.

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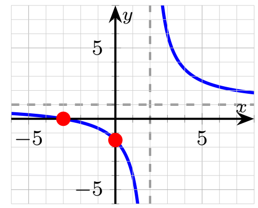
21. Numerator degree 1, denominator degree 1, leading coefficients 200 and 1. So the horizontal asymptote is $y = \frac{200}{1} = 200$. In context: as x grows large (many months in), monthly revenue approaches \$200,000 but never quite reaches it. The asymptote represents the theoretical ceiling. (At $x = 10$ the revenue is already $\frac{2000}{20} = 100$ thousand – halfway to the cap. At $x = 90$, it's $\frac{18000}{100} = 180$ thousand – 90% of the way.)

22. Vertical asymptote: $x - 2 = 0$, so $x = 2$. **Horizontal asymptote:** equal degrees, ratio $\frac{1}{1} = 1$, so $y = 1$. **x-intercept:** numerator zero at $x = -3$, so $(-3, 0)$. **y-intercept:** $f(0) = \frac{3}{-2} = -1.5$, so $(0, -1.5)$. All four features show up on the plot exactly as the algebra predicts – this is the standard workflow for a simple linear-over-linear rational function.

Answer graph



23. Factor the numerator: $x^2 - 4x - 12 = (x - 6)(x + 2)$. Then $f(x) = \frac{(x-6)(x+2)}{x-6} = x + 2$ for $x \neq 6$. The factor $x - 6$ cancels completely. At $x = 6$ the simplified form gives $y = 8$, so the hole is $(6, 8)$. There's no vertical asymptote because the $(x-6)$ doesn't survive simplification – a vertical asymptote requires a denominator factor that the numerator doesn't share. The open circle on the plot marks the missing point.

24. Horizontal asymptote: numerator's degree (0) is less than the denominator's (1), so $y = 0$. In context: as time grows large, the concentration approaches 0 mg/L but never actually reaches it – typical for a passive decay model. **y-intercept:** $C(0) = \frac{50}{2} = 25$ mg/L. That's the starting concentration at $t = 0$. Two minutes later, $C(2) = \frac{50}{4} = 12.5$ mg/L – already half the starting value. (Algebraically there's also a vertical asymptote at $t = -2$, but it has no physical meaning – negative time is outside the model's domain.)

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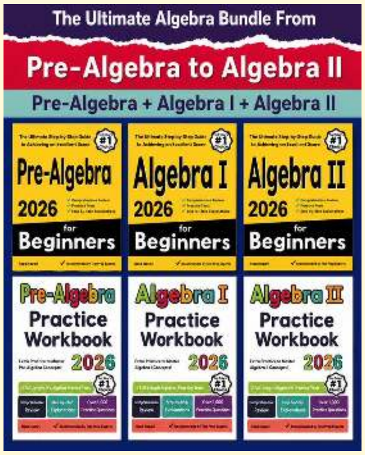

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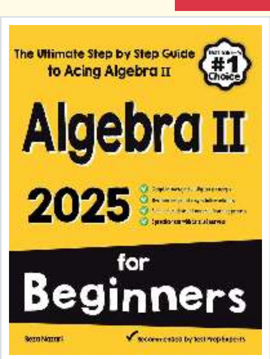

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