Graphing Quadratic Inequalities

Name: ______	Date: ______	Score: ____ / 34

Quick Review

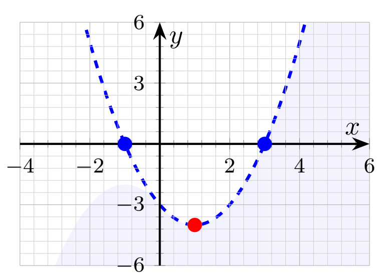
Graphing a quadratic inequality in two variables works just like graphing a linear one: **draw the boundary curve**, then **shade the correct region**. The boundary is the parabola you'd get by replacing the inequality symbol with $=$. Use a **solid** curve for $\leq$ or $\geq$ (boundary included) and a **dashed** curve for strict $<$ or $>$ (boundary excluded). For the shading: pick a test point not on the parabola — $(0,0)$ if the curve misses the origin — and plug into the inequality. If it makes the statement true, shade the side that contains the test point.

A handy shortcut once the inequality is written with y isolated: $y >$ stuff $\Rightarrow$ shade *above* the parabola, $y <$ stuff $\Rightarrow$ shade *below*. That works whether the parabola opens up or down — "above" and "below" refer to the curve, not to the x-axis. The vertex is always on the boundary, not in the interior of the shaded region. And changing the parabola's direction (sign of a) doesn't change the shading rule — only the test point or the y-isolated form does.

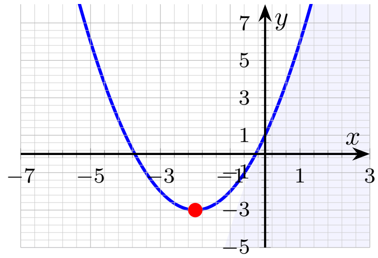
PRACTICE

Identify boundary type, shading direction, or matching inequality.

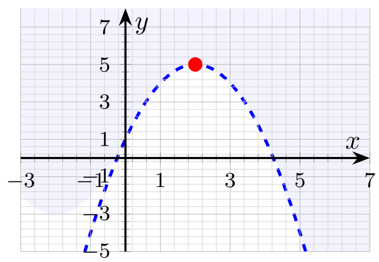
1. Boundary for $y \leq x^2 + 1$: solid or dashed? ________
2. Shading for $y > x^2$: above or below? ________
3. Is $(0,0)$ a solution of $y \geq x^2 - 4$? ________
4. Dashed curve $y = x^2 - 1$, shaded below. Which inequality? ________
5. Find the vertex of the boundary for $y \geq (x-2)^2 - 3$. ________
6. Test $(0,0)$ in $y < -x^2 + 4$. Is it in the region? ________
7. Find the vertex of the boundary for $y \leq -x^2 + 2x + 3$. ________
8. Describe the graph of $y > -(x+1)^2 + 4$ (boundary, vertex, shading). ________
9. Identify the inequality (dashed, shade below). ________



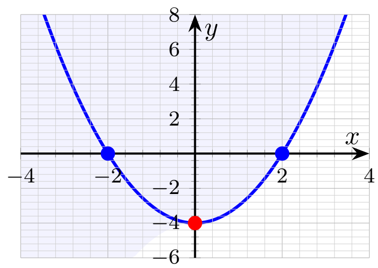
10. Identify the inequality (solid, shade below). ________



11. Identify the inequality (dashed, shade above). ________



12. Identify the inequality (solid, shade above). ________



13. Is $(2, 0)$ in the solution region of $y > x^2 - 4$? ________

14. Is $(1, -5)$ in the solution region of $y > x^2 - 4$? ________

15. Is $(-3, 4)$ in the solution region of $y > x^2 - 4$? ________

16. Direction of opening for boundary $y \le -2(x-1)^2 + 3$? ________

17. Solid or dashed for $y > x^2 + 2x$? ________

18. Solid or dashed for $y \ge -x^2 + 5$? ________

19. Shading region for $y < (x-3)^2 + 1$? ________

20. Vertex of the boundary for $y \le 2(x+1)^2 - 5$? ________

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Word Problems

21. A landscape architect's lawn must satisfy $y \geq x^2 - 4$ (in meters), where x is the distance from a fence and y is the distance to the back property line. Is the point $(3, 2)$ inside the allowed region? ____

22. A signal-strength region is described by $y \leq -x^2 + 9$, where x and y are in km. The vertex of the boundary parabola sits at the strongest point. Find the vertex and the boundary's x-intercepts. ____

23. Describe the graph of $y \geq (x-1)^2 - 4$: boundary shape, solid or dashed, where to shade, and a test point that confirms the shading. ____

24. A factory's daily output must satisfy $y > -(x-3)^2 + 8$, where x is hours of operation and y is widget count. Identify the boundary's vertex and the direction of opening. ____

Additional Practice

25. Solve $x^2 - 5x + 6 = 0$. ________

26. Solve $x^2 = 49$. ________

27. Find the vertex of $y = (x-3)^2 - 4$. ________

28. Find the axis of symmetry of $y = x^2 + 6x + 1$. ________

29. Factor $x^2 + 7x + 10$. ________

30. Find the discriminant of $x^2 - 4x + 8 = 0$. ________

31. Solve $2x^2 - 8 = 0$. ________

32. Write roots -1 and 6 as a quadratic. ________

33. Find the y-intercept of $y = x^2 - 3x - 10$. ________

34. Find zeros of $y = (x-4)(x+2)$. ________

Answer Keys

1. solid
2. above
3. yes
4. $y < x^2 - 1$
5. $(2, -3)$
6. yes
7. $(1, 4)$
8. dashed, opens down, vertex $(-1, 4)$, shade above
9. $y < (x-1)^2 - 4$
10. $y \leq (x+2)^2 - 3$
11. $y > -(x-2)^2 + 5$
12. $y \geq x^2 - 4$
13. no
14. no
15. no
16. down
17. dashed
18. solid
19. below the parabola
20. $(-1, -5)$
21. no
22. vertex $(0, 9)$; x-int $(\pm 3, 0)$
23. solid parabola, vertex $(1, -4)$; shade above
24. vertex $(3, 8)$, opens down

Additional Practice Answers

25. $x = 2,\ 3$
26. $x = -7,\ 7$
27. $(3, -4)$
28. $x = -3$
29. $(x+5)(x+2)$
30. -16
31. $x = -2,\ 2$
32. $(x+1)(x-6)$
33. $(0, -10)$
34. $x = 4,\ -2$

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. A non-strict symbol ($\leq$ or $\geq$) includes the boundary itself, so draw the parabola solid. Dashed is reserved for strict $<$ or $>$.
2. With y already isolated, $y >$ stuff means shade above the curve. Confirm with a test point like $(0, 1)$: is $1 > 0^2$? Yes, and $(0, 1)$ sits above the parabola. ✓
3. Substitute $x = 0, y = 0$ into the inequality: is $0 \geq 0^2 - 4 = -4$? Yes, $0 \geq -4$ is true, so $(0, 0)$ is in the region.
4. Read the two clues: dashed boundary means a strict symbol, and shading below the curve means $y <$. Put them together: $y < x^2 - 1$.
5. The boundary $y = (x-2)^2 - 3$ is in vertex form $a(x-h)^2 + k$. Read off $h = 2$ (number subtracted inside) and $k = -3$, so the vertex is $(2, -3)$.
6. Plug in $x = 0, y = 0$: is $0 < -0^2 + 4 = 4$? Yes, $0 < 4$ is true, so the origin lies in the solution region.
7. Boundary is standard form with $a = -1, b = 2$. Vertex x: $-\dfrac{b}{2a} = -\dfrac{2}{2(-1)} = 1$. Plug back: $y = -1 + 2 + 3 = 4$, so the vertex is $(1, 4)$.
8. Strict $> \Rightarrow$ dashed. Negative leading coefficient $\Rightarrow$ opens down. Vertex $(-1, 4)$. $y >$ stuff $\Rightarrow$ shade above the curve.
9. Vertex $(1, -4)$, opens up. Dashed $\Rightarrow$ strict. Shaded below $\Rightarrow y <$ form. So $y < (x-1)^2 - 4$.
10. Keep the rule visible: Vertex $(-2, -3)$, opens up. Solid $\Rightarrow \leq$ or $\geq$. Shaded below $\Rightarrow \leq$. So $y \leq (x+2)^2 - 3$. That gives a quick check on the answer.
11. Vertex $(2, 5)$, opens down. Dashed $\Rightarrow$ strict. Shaded above $\Rightarrow y >$. So $y > -(x-2)^2 + 5$.
12. Start with the key idea: Vertex $(0, -4)$, opens up. Solid $\Rightarrow \leq$ or $\geq$. Shaded above $\Rightarrow \geq$. So $y \geq x^2 - 4$. That gives a quick check on the answer.
13. Plug in $x = 2, y = 0$: is $0 > 2^2 - 4 = 0$? No — $0 > 0$ is false. The point sits exactly on the boundary, and a strict $>$ excludes the boundary.
14. Plug in $x = 1, y = -5$: is $-5 > 1^2 - 4 = -3$? No, since $-5 < -3$. The point sits below the parabola, but the inequality wants the region above.
15. Plug in $x = -3, y = 4$: is $4 > (-3)^2 - 4 = 5$? No, $4 < 5$. At $x = -3$ the boundary is at height 5, and the point sits below it.
16. The opening direction is set by the sign of the leading coefficient. Here it's $-2 < 0$, so the parabola opens downward.
17. The symbol is a strict $>$, so the boundary is not part of the solution — draw it dashed.
18. The $\geq$ is non-strict, so the boundary points satisfy the inequality and the curve is drawn solid.
19. One steady path is: $y <$ stuff $\Rightarrow$ below the curve. (The parabola's vertex is $(3, 1)$, but that doesn't change the rule.) That gives a quick check on the answer.
20. Vertex form $a(x-h)^2 + k$. Since $(x+1) = (x-(-1))$, we get $h = -1$, and $k = -5$. Vertex $(-1, -5)$ — the 2 only sets the steepness.
21. Plug into the inequality: $y = 2$ and $x^2 - 4 = 9 - 4 = 5$. Is $2 \geq 5$? No. So $(3, 2)$ is *not* in the allowed region — it sits below the boundary parabola at that x-value.
22. Boundary $y = -x^2 + 9$. Vertex: $h = 0$ (the x-term is missing), $k = 9$. x-intercepts: $0 = -x^2 + 9 \Rightarrow x^2 = 9 \Rightarrow x = \pm 3$.
23. One steady path is: $\geq \Rightarrow$ solid boundary. Vertex $(1, -4)$, opens up. $y \geq$ stuff $\Rightarrow$ shade above the curve. Test $(0, 0)$: $0 \geq (0-1)^2 - 4 = -3$. True. The origin is above the parabola, so the shading is on the side containing it. That gives a quick check on the answer.
24. Vertex form gives $h = 3$, $k = 8$. The leading -1 means the boundary parabola opens downward. The strict $>$ makes the boundary dashed, and $y >$ shades above the curve.

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