Graphing Inverse Tangent

Name: _____ Date: _____ Score: _____ / 33

Quick Review

$y = \arctan x$ answers "which angle has tangent equal to x?" Tangent's domain is restricted to $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (an open interval – the asymptotes get cut out), which becomes the range of arctan.

Domain and range.

Domain of $\arctan$: $\mathbb{R}$ – arctan accepts *every* real number, no restriction. (Unlike arcsin and arccos.)

Range of $\arctan$: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ – open interval.

Graph shape. A smooth, increasing S-curve through $(0, 0)$. It flattens out toward two *horizontal asymptotes*: $y = \frac{\pi}{2}$ as $x \to +\infty$, and $y = -\frac{\pi}{2}$ as $x \to -\infty$. The graph *never reaches* those values – that's why arcsin includes its endpoints but arctan doesn't.

Key facts.

Symmetry: about the origin. Arctan is odd, $\arctan(-x) = -\arctan x$.

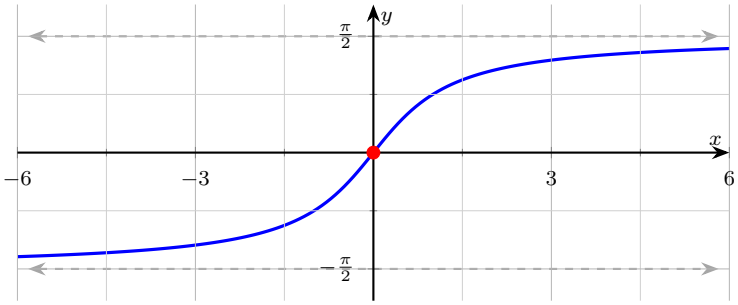
Increasing on $\mathbb{R}$.

y-intercept: $(0, 0)$.

Composition: $\tan(\arctan x) = x$ for all real x. $\arctan(\tan x) = x$ only when $x \in (-\pi/2, \pi/2)$.

Common slips. Reporting $\frac{3\pi}{4}$ for $\arctan(-1)$ – principal range gives $-\frac{\pi}{4}$ instead. Confusing arctan with $\frac{1}{\tan x} = \cot x$. Saying arctan "reaches" $\pm\pi/2$ – it only approaches those values.

Parent arctan graph – S-curve approaching dashed horizontal asymptotes at $y = \pm\frac{\pi}{2}$:



PRACTICE

Use exact values where possible. Arctan accepts any real input.

1. State the domain of $y = \arctan x$. _____
2. State the range of $y = \arctan x$. _____
3. Evaluate $\arctan(0)$. _____
4. Evaluate $\arctan(1)$. _____
5. Evaluate $\arctan(-1)$. _____
6. Evaluate $\arctan(\sqrt{3})$. _____
7. Evaluate $\arctan\left(\frac{1}{\sqrt{3}}\right)$. _____
8. $y = \arctan x$ is symmetric about which feature? _____
9. State the horizontal asymptotes of $y = \arctan x$. _____
10. State the y-intercept of $y = \arctan x$. _____

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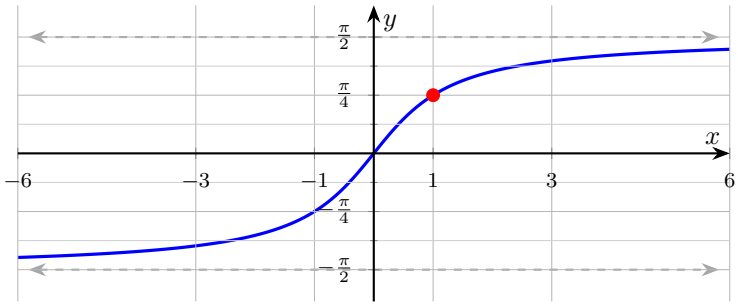
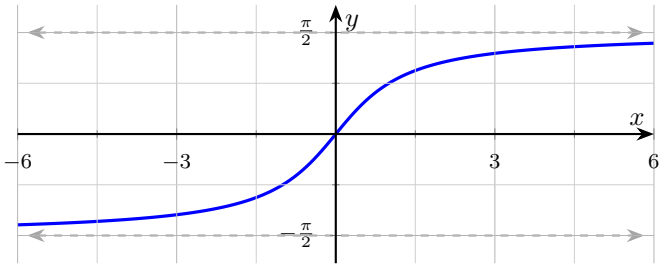
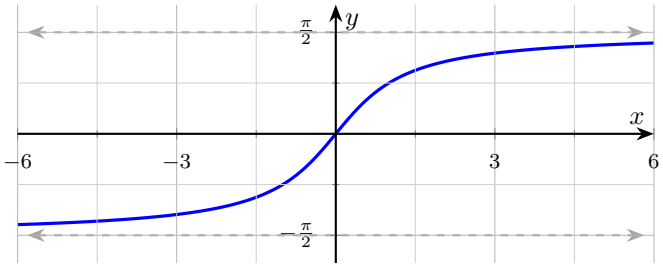
11. Read $\arctan(1)$ off the graph below. _____



12. From the graph of $y = \arctan x$ below, state whether the function has any vertical asymptotes. _____



13. Is $y = \arctan x$ (shown below) increasing or decreasing? _____



14. Evaluate $\tan(\arctan(7))$. _____

15. Evaluate $\arctan\left(\tan\left(\frac{\pi}{4}\right)\right)$. _____

16. Evaluate $\arctan\left(\tan\left(\frac{3\pi}{4}\right)\right)$. _____

17. State the slope of $y = \arctan x$ at $x = 0$. _____

18. Find $\lim_{x \to +\infty} \arctan x$. _____

19. Find $\lim_{x \to -\infty} \arctan x$. _____

20. State the range of $y = 2 \arctan x$. _____

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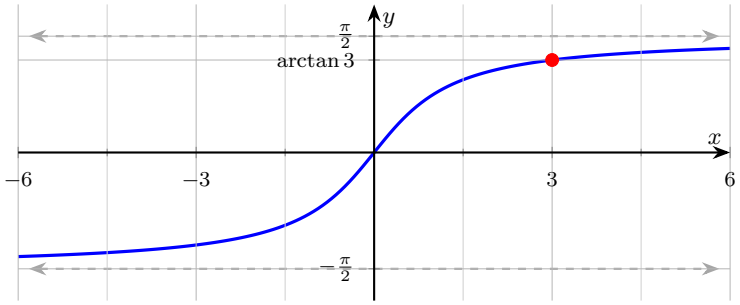
Word Problems

21. A boat travels 40 km east and then 30 km north. The bearing from the starting point satisfies $\tan\theta = \dfrac{40}{30}$, where θ is east of north. Find θ (degrees, one decimal). ____

22. A camera mounted on a stage rotates to track an actor walking horizontally. The horizontal distance from straight-ahead is x meters and the camera-to-stage distance is 5 m. The rotation angle is $\theta = \arctan(x/5)$. State the range of θ as x varies over all real values. ____

23. The graph below shows $y = \arctan x$. Use it to estimate $\arctan(3)$ in degrees. _____



24. A drone's flight path makes the relationship $\tan\theta = v_y/v_x$, where v_x and v_y are horizontal and vertical speed. If $v_x = 10$ m/s and $v_y = 10$ m/s, find θ exactly. As v_y grows huge while v_x stays at 10, what happens to θ? ____

Additional Practice

25. Amplitude of $y = 4\sin x$. _____

26. Period of $y = \sin(2x)$. _____

27. Amplitude of $y = -3\cos x$. _____

28. Period of $y = \tan(5x)$. _____

29. Midline of $y = 2\sin x - 7$. _____

30. Phase shift of $y = \sin(x - \pi/3)$. _____

31. Range of $y = 5\cos x$. _____

32. Range of $y = 2\sin x + 1$. _____

33. Asymptotes of $y = \tan x$ in one period. _____

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Answer Keys

1. $\mathbb{R}$
2. $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
3. 0
4. $\frac{\pi}{4}$
5. $-\frac{\pi}{4}$
6. $\frac{\pi}{3}$
7. $\frac{\pi}{6}$
8. The origin
9. $y = \pm\frac{\pi}{2}$
10. $(0, 0)$
11. $\frac{\pi}{4}$
12. No vertical asymptotes; horizontal asymptotes $y = \pm\frac{\pi}{2}$
13. Increasing
14. 7
15. $\frac{\pi}{4}$
16. $-\frac{\pi}{4}$
17. 1
18. $\frac{\pi}{2}$
19. $-\frac{\pi}{2}$
20. $(-\pi, \pi)$
21. $\theta = \arctan\left(\frac{4}{3}\right) \approx 53.1°$
22. $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
23. $\arctan(3) \approx 71.6°$
24. $\theta = \frac{\pi}{4};\ \theta \to \frac{\pi}{2}$

Additional Practice Answers

25. 4
26. π
27. 3
28. $\frac{\pi}{5}$
29. $y = -7$
30. $\frac{\pi}{3}$ right
31. $[-5, 5]$
32. $[-1, 3]$
33. $x = \pm\frac{\pi}{2}$

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. A careful way to see it: Every real number is a valid tangent value – arctan accepts everything. That gives a quick check on the answer.

2. Tangent is restricted to the open interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (its asymptotes are cut out), and that becomes arctan's output range. The parentheses are open because arctan only *approaches* $\pm\frac{\pi}{2}$ – it never quite reaches them.

3. One steady path is: $\tan 0 = 0$ and 0 is dead-center in the principal range. This is the part to check before moving on, because it keeps the answer tied to the original question.

4. Start with the key idea: $\tan(\pi/4) = 1$ and $\pi/4$ is in $(-\pi/2, \pi/2)$. This is the part to check before moving on, because it keeps the answer tied to the original question.

5. Arctan is odd, so the answer is $-\pi/4$ – *not* $3\pi/4$ (which is outside the principal range).

6. We want the angle in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ with tangent $\sqrt{3}$. Since $\tan\frac{\pi}{3} = \sqrt{3}$ and $\frac{\pi}{3}$ is in that range, the answer is $\frac{\pi}{3}$.

7. Find the angle in the principal range $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ whose tangent is $\frac{1}{\sqrt{3}}$. That is $\frac{\pi}{6}$, since $\tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$.

8. Start with the key idea: Arctan is odd: $\arctan(-x) = -\arctan x$. This is the part to check before moving on, because it keeps the answer tied to the original question.

9. A careful way to see it: As $x \to +\infty$, $\arctan x \to \pi/2$. As $x \to -\infty$, $\arctan x \to -\pi/2$. The graph never touches those lines. That gives a quick check on the answer.

10. Keep the rule visible: $\arctan(0) = 0$. This is the part to check before moving on, because it keeps the answer tied to the original question.

11. One steady path is: The marked dot at $(1, \pi/4)$ shows the value directly. This is the part to check before moving on, because it keeps the answer tied to the original question.

12. Arctan is defined for every real x – no vertical blow-ups. Instead, the graph flattens out toward the horizontal lines $y = \pm\frac{\pi}{2}$ (dashed).

13. A careful way to see it: As x slides from $-\infty$ to $+\infty$, y climbs from $-\frac{\pi}{2}$ up to $\frac{\pi}{2}$ without reversing. That gives a quick check on the answer.

14. Keep the rule visible: Tangent undoes arctan for every real input: $\tan(\arctan x) = x$. This is the part to check before moving on, because it keeps the answer tied to the original question.

15. One steady path is: $\pi/4$ is inside the principal range, so the composition is identity. That gives a quick check on the answer.

16. Start with the key idea: $\tan(3\pi/4) = -1$. Then $\arctan(-1) = -\frac{\pi}{4}$ (principal-range answer). $3\pi/4$ itself is outside the principal range. That gives a quick check on the answer.

17. A careful way to see it: Derivative of arctan is $\frac{1}{1+x^2}$; at $x = 0$ that's 1. This is the part to check before moving on, because it keeps the answer tied to the original question.

18. Keep the rule visible: The graph flattens against the upper asymptote. This is the part to check before moving on, because it keeps the answer tied to the original question.

19. One steady path is: Lower asymptote. This is the part to check before moving on, because it keeps the answer tied to the original question.

20. The factor of 2 stretches every output vertically, so the range $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ doubles to $\left(2\cdot-\frac{\pi}{2}, 2\cdot\frac{\pi}{2}\right) = (-\pi, \pi)$. It stays open because arctan never reaches its asymptotes.

21. A careful way to see it: $\arctan(4/3) \approx 0.9273$ rad $\approx 53.13°$, which rounds to $53.1°$. (Classic 3-4-5 triangle bearing.) That gives a quick check on the answer.

22. Arctan accepts every real input and returns a value in its principal range $(-\pi/2, \pi/2)$. The camera can rotate almost $180°$ in total – $90°$ each way – but never completely sideways.

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23. One steady path is: $\arctan(3) \approx 1.249$ rad $\approx 71.57^\circ$. Close to but below the 90° asymptote – the curve is flattening out. That gives a quick check on the answer.

24. Start with the key idea: $\tan\theta = 1 \Rightarrow \theta = \frac{\pi}{4}$ (the drone climbs at 45°). As $v_y \to \infty$ with v_x fixed, $\tan\theta \to \infty$, so $\theta \to \frac{\pi}{2}$ – the drone approaches a vertical climb but never quite gets there. That gives a quick check on the answer.

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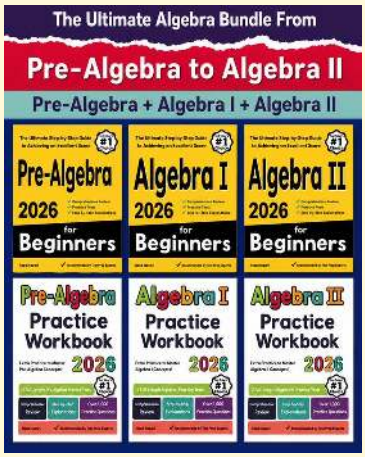

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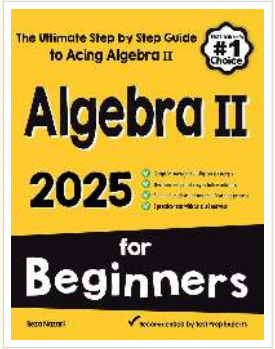

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