Graphing Inverse Sine

Name: ________________ Date: ____________ Score: ______ / 28

Quick Review

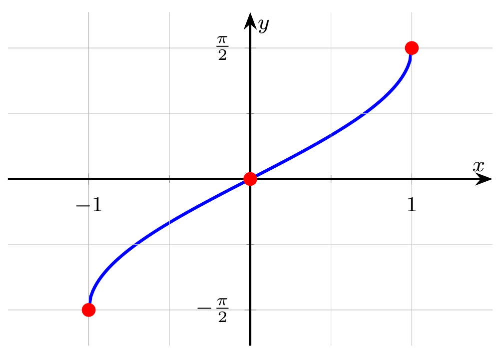
$y = \arcsin x$ (also written $\sin^{-1} x$) answers the question *"which angle has sine equal to x?"* Because sine repeats forever, we restrict its inputs to $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ before we can flip it – and that restricted range becomes the *output range* of arcsin.

Domain and range.
Domain of arcsin: $[-1,\ 1]$. (Sine never outputs more than 1 or less than -1, so arcsin can't accept anything else.)
Range of arcsin: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

Graph shape. Arcsin's graph is the reflection of the restricted sine across the line $y = x$. It rises smoothly from $\left(-1, -\frac{\pi}{2}\right)$ through $(0, 0)$ up to $\left(1, \frac{\pi}{2}\right)$. It's bounded on top and bottom by horizontal lines $y = \pm\frac{\pi}{2}$ – not asymptotes, the graph actually *reaches* those values.

Key facts.
Symmetry: about the origin. Arcsin is odd, $\arcsin(-x) = -\arcsin x$.
Increasing on its entire domain.
y-intercept: $(0, 0)$.
Composition: $\sin(\arcsin x) = x$ for $x \in [-1, 1]$. $\arcsin(\sin x) = x$ only when $x \in [-\pi/2, \pi/2]$.

Common slips. Confusing $\sin^{-1} x$ with $\frac{1}{\sin x} = \csc x$. Plugging in 1.2 – it's outside the domain. Reporting $\frac{3\pi}{4}$ as $\arcsin\left(\frac{\sqrt{2}}{2}\right)$ – the correct answer is $\frac{\pi}{4}$, the principal-range value.
Here's the parent arcsin graph: it lives in a $[-1, 1] \times [-\pi/2, \pi/2]$ window and rises smoothly:



PRACTICE

Use exact values where possible. Mark undefined inputs as undefined.

1. State the domain of $y = \arcsin x$. ________
2. State the range of $y = \arcsin x$. ________
3. Evaluate $\arcsin(0)$. ________
4. Evaluate $\arcsin(1)$. ________
5. Evaluate $\arcsin(-1)$. ________
6. Evaluate $\arcsin\left(\frac{1}{2}\right)$. ________
7. Evaluate $\arcsin\left(-\frac{\sqrt{3}}{2}\right)$. ________

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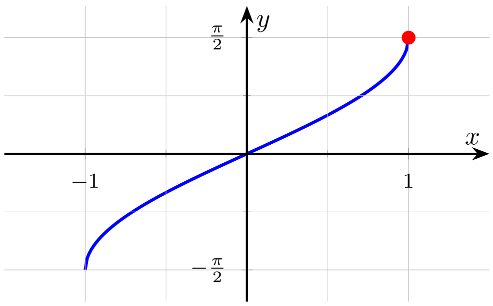
8. Is $\arcsin(2)$ defined? ________

9. $y = \arcsin x$ is symmetric about which feature? ________

10. State the y-intercept of $y = \arcsin x$. ________

11. Read $\arcsin(1)$ from the graph below. ________

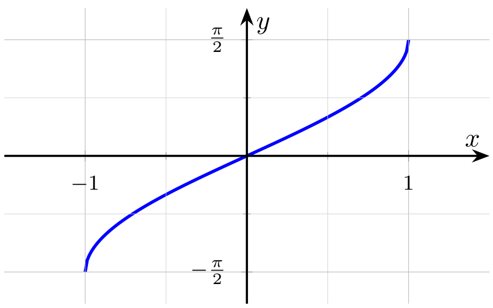


12. True or false: $\arcsin x$ is the same as $\frac{1}{\sin x}$. ________

13. State the maximum value of $y = \arcsin x$. ________

14. State the minimum value of $y = \arcsin x$. ________

15. Is $y = \arcsin x$ (shown below) increasing or decreasing? ________

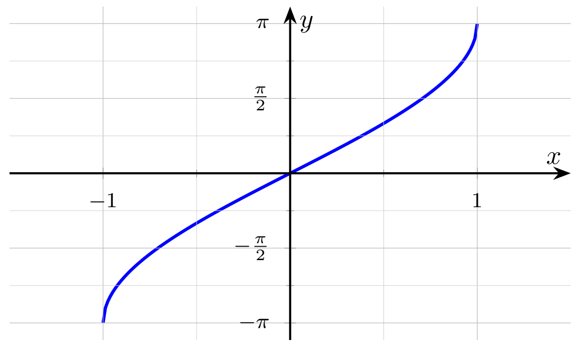


16. Evaluate $\sin(\arcsin(0.7))$. ________

17. Evaluate $\arcsin\left(\sin\left(\frac{3\pi}{4}\right)\right)$. ________

18. What is $\arcsin\left(\sin\left(\frac{\pi}{6}\right)\right)$? ________

19. The graph below is $y = 2\arcsin x$. How does it differ from $y = \arcsin x$? ________



20. State the slope of $y = \arcsin x$ at $x = 0$. ________

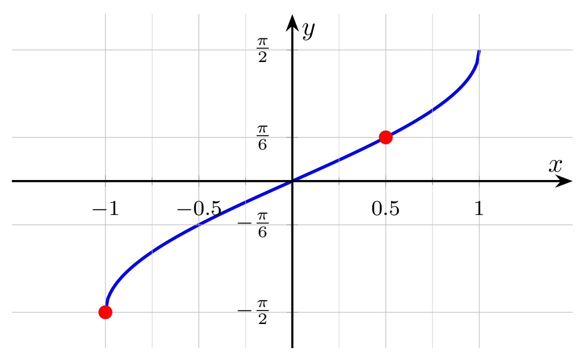
♦ Word Problems

21. A wheelchair ramp rises 1 foot over a 12-foot horizontal run, and the ramp angle θ satisfies $\sin\theta = \dfrac{1}{\sqrt{145}}$ (approximating the hypotenuse). Express θ as an arcsin and approximate it in degrees. ____

22. A swimmer's stroke causes their arm angle (in radians) to follow $\theta = \arcsin(s)$ where s is the sine of the angle, $s \in [-1, 1]$. State the swimmer's full range of arm angles. ____

23. A satellite observation requires the angle $\theta = \arcsin\left(\dfrac{h}{r}\right)$, where h is the satellite's altitude (km) and r is the distance from the observer's eye. State the domain of valid h/r values, and compute θ when $h = r$. ____

24. The graph below shows $y = \arcsin x$. Use it to estimate $\arcsin(-1)$ and $\arcsin(0.5)$. ____



Additional Practice

25. Amplitude of $y = 4\sin x$. ________

26. Period of $y = \sin(2x)$. ________

27. Amplitude of $y = -3\cos x$. ________

28. Period of $y = \tan(5x)$. ________

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Answer Keys

1. $[-1, 1]$
2. $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
3. 0
4. $\frac{\pi}{2}$
5. $-\frac{\pi}{2}$
6. $\frac{\pi}{6}$
7. $-\frac{\pi}{3}$
8. No, undefined
9. The origin
10. $(0, 0)$
11. $\frac{\pi}{2}$
12. False
13. $\frac{\pi}{2}$
14. $-\frac{\pi}{2}$
15. Increasing
16. 0.7
17. $\frac{\pi}{4}$
18. $\frac{\pi}{6}$
19. Vertical stretch by 2; new range $[-\pi, \pi]$
20. 1
21. $\theta = \arcsin\left(\frac{1}{\sqrt{145}}\right) \approx 4.76°$
22. $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
23. $-1 \le h/r \le 1;\ \theta = \frac{\pi}{2}$
24. $\arcsin(-1) = -\frac{\pi}{2};\ \arcsin(0.5) = \frac{\pi}{6}$

Additional Practice Answers

25. 4
26. π
27. 3
28. $\frac{\pi}{5}$

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. Sine outputs are in $[-1, 1]$, so its inverse can only accept inputs from that interval.

2. To invert sine we first restrict its inputs to $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ so it passes the horizontal-line test. That restricted input interval becomes the output range of arcsin – the closed bracket means the endpoints $\pm\frac{\pi}{2}$ are actually reached.

3. One steady path is: $\sin 0 = 0$ and 0 is inside the principal range. This is the part to check before moving on, because it keeps the answer tied to the original question.

4. Start with the key idea: $\sin(\pi/2) = 1$ and $\pi/2$ is the top of the principal range. This is the part to check before moving on, because it keeps the answer tied to the original question.

5. A careful way to see it: $\sin(-\pi/2) = -1$, in range. This is the part to check before moving on, because it keeps the answer tied to the original question.

6. Keep the rule visible: $\sin(\pi/6) = \frac{1}{2}$ and $\pi/6$ is in $[-\pi/2, \pi/2]$. This is the part to check before moving on, because it keeps the answer tied to the original question.

7. One steady path is: Use odd-symmetry: $\arcsin(-\sqrt{3}/2) = -\arcsin(\sqrt{3}/2) = -\pi/3$. This is the part to check before moving on, because it keeps the answer tied to the original question.

8. Start with the key idea: 2 is outside the domain $[-1, 1]$. This is the part to check before moving on, because it keeps the answer tied to the original question.

9. A careful way to see it: Arcsin is odd: $\arcsin(-x) = -\arcsin x$. This is the part to check before moving on, because it keeps the answer tied to the original question.

10. Keep the rule visible: $\arcsin(0) = 0$. This is the part to check before moving on, because it keeps the answer tied to the original question.

11. One steady path is: The graph reaches its top at $x = 1$, where the y-value is $\frac{\pi}{2}$. That gives a quick check on the answer.

12. Start with the key idea: $\arcsin x = \sin^{-1} x$ is the inverse function. $\frac{1}{\sin x} = \csc x$ is the reciprocal – a different thing entirely. That gives a quick check on the answer.

13. A careful way to see it: The graph tops out at $\frac{\pi}{2}$ when $x = 1$. This is the part to check before moving on, because it keeps the answer tied to the original question.

14. Keep the rule visible: Hit at $x = -1$. This is the part to check before moving on, because it keeps the answer tied to the original question.

15. One steady path is: As x runs from -1 to 1, y climbs from $-\frac{\pi}{2}$ up to $\frac{\pi}{2}$ without reversing. That gives a quick check on the answer.

16. Start with the key idea: Inside the domain $[-1, 1]$, sine undoes arcsin directly: $\sin(\arcsin x) = x$. That gives a quick check on the answer.

17. A careful way to see it: $\sin(3\pi/4) = \frac{\sqrt{2}}{2}$. Then $\arcsin(\sqrt{2}/2) = \frac{\pi}{4}$, the principal-range answer (not $3\pi/4$, which is outside the principal range). That gives a quick check on the answer.

18. Keep the rule visible: $\pi/6$ is already inside the principal range, so arcsin and sine fully undo each other. That gives a quick check on the answer.

19. Each y-value doubles. The new range is $[-\pi, \pi]$ – the graph reaches $\pm\pi$ at $x = \pm 1$. Domain unchanged at $[-1, 1]$.

20. At $x = 0$, the tangent line has slope $\frac{1}{\sqrt{1 - 0^2}} = 1$. (Calculus aside – the derivative of $\arcsin$ is $\frac{1}{\sqrt{1 - x^2}}$.)

21. From a 1-ft rise and 12-ft run, the hypotenuse is $\sqrt{145}$, so $\sin\theta = \frac{1}{\sqrt{145}} \approx 0.0831$. Then $\theta = \arcsin(0.0831) \approx 0.0831$ rad, about $4.76°$.

22. As s varies over the full domain $[-1, 1]$, arcsin sweeps its entire range $[-\pi/2, \pi/2]$. The swimmer's arm covers a $180°$ arc.

23. Arcsin only accepts inputs in $[-1, 1]$. Physically, the height can't exceed the slant distance. At $h = r$, $\arcsin(1) = \frac{\pi}{2}$ – looking straight up.

24. The graph reaches its minimum $-\pi/2$ at $x = -1$ and passes through $(0.5, \pi/6)$. Both are exact values you can read off the marked dots.

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