

Graphing Inverse Cosine

Name: _____ Date: _____ Score: _____ / 24

Q Quick Review

$y = \arccos x$ asks “which angle has cosine equal to x ?” Like \arcsin , we restrict cosine’s input to make it one-to-one. The choice for cosine is $[0, \pi]$ – a different interval than \arcsin ’s.

Domain and range.

Domain of \arccos : $[-1, 1]$.

Range of \arccos : $[0, \pi]$.

Graph shape. Arccos’s graph is the reflection of the restricted cosine across $y = x$. It *decreases* from $(-1, \pi)$ through $(0, \frac{\pi}{2})$ down to $(1, 0)$. (Different shape from \arcsin – arccos goes down, not up.)

Key facts.

Symmetry: *no* simple axis or origin symmetry. Arccos is neither even nor odd; instead, $\arccos(-x) = \pi - \arccos x$.

Decreasing on its entire domain.

y -intercept: $(0, \frac{\pi}{2})$.

Composition: $\cos(\arccos x) = x$ for $x \in [-1, 1]$. $\arccos(\cos x) = x$ only when $x \in [0, \pi]$.

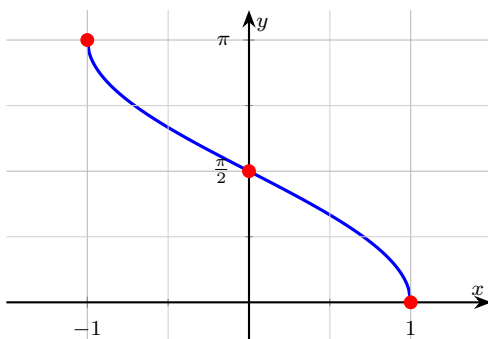
The big difference between arcsin and arccos.

Arcsin range: $[-\pi/2, \pi/2]$ – can be negative.

Arccos range: $[0, \pi]$ – never negative.

Common slips. Reporting a negative angle from arccos (it never returns negatives). Mixing up the principal ranges. Plugging in -1.5 – it’s outside the domain.

Here’s the parent arccos graph – decreasing from $(-1, \pi)$ to $(1, 0)$:



PRACTICE

Use exact values where possible. Outputs of arccos stay in $[0, \pi]$ – never negative.

1. State the domain of $y = \arccos x$. _____
2. State the range of $y = \arccos x$. _____
3. Evaluate $\arccos(0)$. _____
4. Evaluate $\arccos(1)$. _____
5. Evaluate $\arccos(-1)$. _____
6. Evaluate $\arccos\left(\frac{1}{2}\right)$. _____
7. Evaluate $\arccos\left(-\frac{1}{2}\right)$. _____

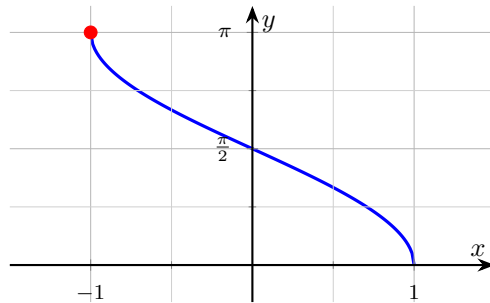


8. Evaluate $\arccos\left(\frac{\sqrt{3}}{2}\right)$.

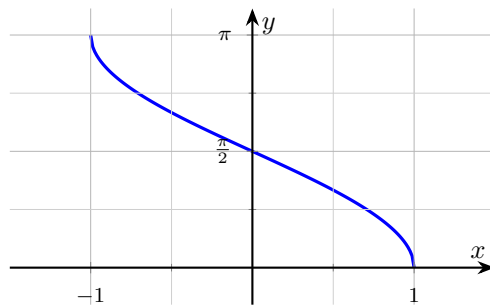
9. Is $\arccos(-2)$ defined?

10. State the y -intercept of $y = \arccos x$.

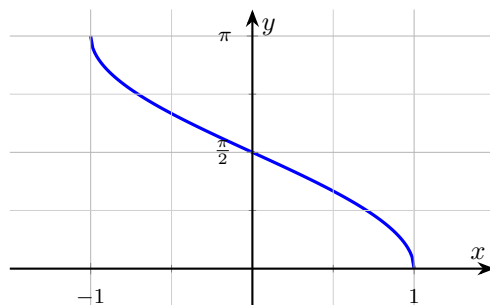
11. Read $\arccos(-1)$ from the graph below.



12. Is $y = \arccos x$ (shown below) increasing or decreasing?



13. From the graph below, state whether $\arccos x$ ever takes negative values.



14. State the maximum value of $y = \arccos x$.

15. State the minimum value of $y = \arccos x$.

16. Evaluate $\cos(\arccos(0.4))$.

17. Evaluate $\arccos\left(\cos\left(\frac{2\pi}{3}\right)\right)$.

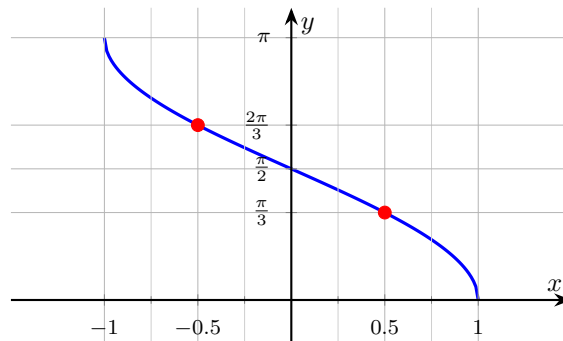
18. Evaluate $\arccos\left(\cos\left(-\frac{\pi}{4}\right)\right)$.



19. Use the identity $\arccos(-x) = \pi - \arccos x$ to compute $\arccos\left(-\frac{\sqrt{3}}{2}\right)$. _____
20. State the slope of $y = \arccos x$ at $x = 0$. _____

◆ Word Problems

21. A box opens to angle θ where $\cos \theta = \frac{1}{4}$. Find θ (in radians, principal range), rounded to two decimals. _____
22. An astronomer's elevation angle satisfies $\cos \theta = \frac{4}{5}$ for an angle in $[0, \pi/2]$. State θ in exact form using \arccos , then state $\sin \theta$. _____
23. The graph below shows $y = \arccos x$. Use it to estimate $\arccos(0.5)$ and $\arccos(-0.5)$. _____



24. A pendulum swings to its maximum angle θ such that $\cos \theta = \frac{7}{8}$. Find θ in degrees, rounded to one decimal. Why does the model reject $\cos \theta = -1.1$? _____



Answer Keys

1. $[-1, 1]$

2. $[0, \pi]$

3. $\frac{\pi}{2}$

4. 0

5. $\frac{\pi}{3}$

6. $\frac{\pi}{3}$

7. $\frac{2\pi}{3}$

8. $\frac{\pi}{6}$

9. No, undefined

10. $(0, \frac{\pi}{2})$

11. π

12. Decreasing

13. Never; range is $[0, \pi]$

14. π

15. 0

16. 0.4

17. $\frac{2\pi}{3}$

18. $\frac{\pi}{4}$

19. $\frac{5\pi}{6}$

20. -1

21. $\theta = \arccos(0.25) \approx 1.32 \text{ rad}$

22. $\theta = \arccos\left(\frac{4}{5}\right); \sin \theta = \frac{3}{5}$

23. $\arccos(0.5) = \frac{\pi}{3}; \arccos(-0.5) = \frac{2\pi}{3}$

24. $\theta \approx 28.96^\circ; -1.1 \text{ outside } [-1, 1]$

Step-by-Step Explanations

1. Cosine's outputs live in $[-1, 1]$, and arccos can only accept what cosine produces.

2. To invert cosine we restrict its inputs to $[0, \pi]$, and that interval becomes arccos's output range. Notice it starts at 0, so arccos is *never* negative – a common place to slip if you expect symmetry like arcsin's.

3. One steady path is: $\cos(\pi/2) = 0$ and $\pi/2$ is in $[0, \pi]$. This is the part to check before moving on, because it keeps the answer tied to the original question.

4. Start with the key idea: $\cos 0 = 1$ and 0 is the left endpoint of $[0, \pi]$. This is the part to check before moving on, because it keeps the answer tied to the original question.

5. A careful way to see it: $\cos \pi = -1$ and π is the right endpoint of $[0, \pi]$. This is the part to check before moving on, because it keeps the answer tied to the original question.

6. Keep the rule visible: $\cos(\pi/3) = \frac{1}{2}$ and $\pi/3$ is in $[0, \pi]$. This is the part to check before moving on, because it keeps the answer tied to the original question.

7. We need the angle in $[0, \pi]$ whose cosine is $-\frac{1}{2}$. The reference angle is $\frac{\pi}{3}$, and cosine is negative in Q2, so the answer is $\pi - \frac{\pi}{3} = \frac{2\pi}{3}$ – which is safely inside $[0, \pi]$.

8. Start with the key idea: $\cos(\pi/6) = \frac{\sqrt{3}}{2}$. This is the part to check before moving on, because it keeps the answer tied to the original question.

9. A careful way to see it: -2 is outside the domain $[-1, 1]$. This is the part to check before moving on, because it keeps the answer tied to the original question.

10. Keep the rule visible: $\arccos(0) = \frac{\pi}{2}$. This is the part to check before moving on, because it keeps the answer tied to the original question.

11. One steady path is: The leftmost point of the graph is $(-1, \pi)$. This is the part to check before moving on, because it keeps the answer tied to the original question.

12. As x goes from -1 to 1 , the output drops from π to 0 – a steady slide downward.

13. The graph stays at or above the x -axis throughout its domain. The output range is $[0, \pi]$ – arccos is never negative.

14. Keep the rule visible: Reached at $x = -1$. This is the part to check before moving on, because it keeps the answer tied to the original question.

15. One steady path is: Reached at $x = 1$. This is the part to check before moving on, because it keeps the answer tied to the original question.

16. Start with the key idea: Cosine undoes arccos for inputs in the domain $[-1, 1]$: $\cos(\arccos x) = x$. That gives a quick check on the answer.

17. A careful way to see it: $2\pi/3$ is already inside $[0, \pi]$, so arccos and cosine fully undo each other. That gives a quick check on the answer.

18. Keep the rule visible: $\cos(-\pi/4) = \cos(\pi/4) = \frac{\sqrt{2}}{2}$. Then $\arccos(\frac{\sqrt{2}}{2}) = \frac{\pi}{4}$, in the principal range. (Not $-\pi/4$ – arccos never returns negatives.) That gives a quick check on the answer.

19. One steady path is: $\arccos(\frac{\sqrt{3}}{2}) = \pi/6$, so $\arccos(-\frac{\sqrt{3}}{2}) = \pi - \pi/6 = \frac{5\pi}{6}$. This is the part to check before moving on, because it keeps the answer tied to the original question.

20. The derivative of arccos is $-\frac{1}{\sqrt{1-x^2}}$; at $x = 0$ that's -1 . The graph descends at a 45° angle near the y -axis.

21. A careful way to see it: $\theta = \arccos(1/4)$. Since cosine of $\pi/3$ is 0.5 and cosine of $\pi/2$ is 0, $\arccos(0.25)$ falls between – closer to $\pi/2$. Calculator gives ≈ 1.318 rad. The angle is in $[0, \pi]$, in fact in $[\pi/3, \pi/2]$. That gives a quick check on the answer.

22. Since $\theta \in [0, \pi/2]$, $\sin \theta \geq 0$. From the 3-4-5 triangle: cosine adj/hyp = $4/5$, so opposite = 3 and $\sin \theta = \frac{3}{5}$.

23. Read the y -values at $x = 0.5$ and $x = -0.5$ off the marked dots. Note that the two answers sum to π , matching the identity $\arccos(-x) = \pi - \arccos x$.

24. Start with the key idea: $\theta = \arccos(7/8) \approx 0.5054 \text{ rad} \approx 28.96^\circ$. The input -1.1 is outside the domain $[-1, 1]$ – no real angle has cosine less than -1 , so the model rightly refuses to compute it. That gives a quick check on the answer.



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