Geometric Sequences

Name: ____________	Date: ____________	Score: ______ / 28

Quick Review

A **geometric sequence** multiplies by the same factor every step. That factor is the **common ratio** r. Test for it by dividing any term by the one before it – $r = a_{n+1}/a_n$, and the answer has to be the same every time you check. (Arithmetic added; geometric multiplies. Don't mix them up.)

Explicit formula. $a_n = a_1 \cdot r^{n-1}$. The exponent is $n-1$, not n, for the same reason arithmetic uses $(n-1)$: term 1 is the starting point – zero multiplications spent. Reaching term n requires $n-1$ multiplies by r.

Recursive form. a_1 is given and $a_n = r \cdot a_{n-1}$.

Two terms given, find r**.** If you know a_p and a_q with $p < q$, then $a_q = a_p \cdot r^{q-p}$. Solve for r by taking the $(q-p)$th root. If the problem says "positive ratio," keep only the positive root.

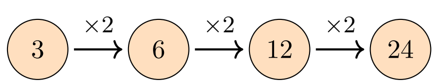
Ratio signs. r can be positive (all terms keep the starting sign), negative (signs flip each step – this is an alternating sequence), or a fraction (terms shrink toward zero). $r = 1$ means everything stays at a_1. $r = 0$ kills the sequence after the first term.

Common slips. Using r^n instead of r^{n-1} – off by one ratio. Subtracting consecutive terms (that finds d, the wrong thing). Forgetting that a negative ratio raised to an even power comes out positive. Confusing geometric growth with exponential functions written $f(x) = ab^x$ – they're the same idea, just continuous instead of discrete.

PRACTICE

Find common ratios, terms, and explicit rules. Trust the formula $a_n = a_1 \cdot r^{n-1}$ and read patterns off the dot diagrams.

1. Find the common ratio of the sequence shown. ________



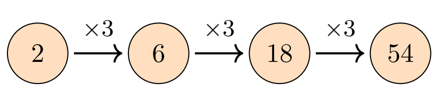
2. Find a_5 for the geometric sequence with $a_1 = 4$, $r = 3$. ________

3. A geometric sequence has $a_1 = 100$ and $r = \frac{1}{2}$. Find a_4. ________

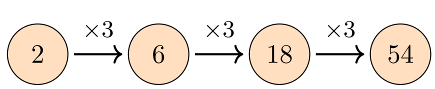
4. Which sequence is geometric? $A: 4, 12, 36, 108, \ldots$ $B: 4, 12, 20, 28, \ldots$ $C: 1, 4, 9, 16, \ldots$ ________

5. A geometric sequence has $a_2 = 6$ and $a_5 = 162$. Find r (positive). ________

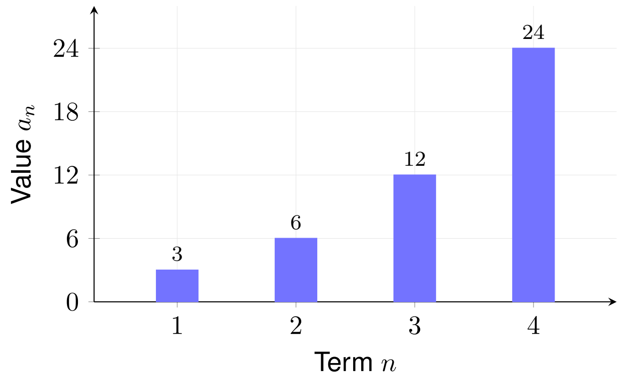
6. Find the explicit formula for the sequence shown. ________



7. Compute a_7 for the sequence shown. ________

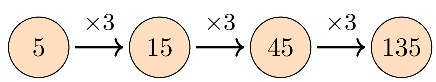


8. The bar chart shows the first four terms of a geometric sequence. Find a_7. ________



9. A geometric sequence has $a_3 = 20$ and $a_6 = 160$. Find r (positive) and write the explicit formula. ________

10. Find a_9 for the sequence shown. ________



11. A geometric sequence has $a_1 = 5$ and $r = 2$. Find a_6. ________

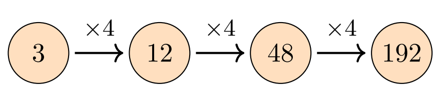
12. True or False: if $|r| > 1$, every term in a geometric sequence must be positive. ________

13. Find the first four terms of the geometric sequence with $a_1 = 6$ and $r = -2$. ________

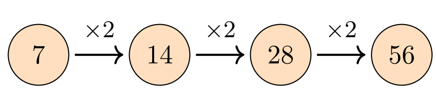
14. For the table shown, write the explicit formula. ________

Geometric Sequence	
n	a_n
1	4
2	12
3	36
4	108

15. Find a_8 for the sequence shown. ________



16. A geometric sequence has $a_n = 7 \cdot 2^{n-1}$. Find a_1 and r, then list the first four terms. ________



17. Find a_7 for the sequence shown.

$$2 \xrightarrow{\times 5} 10 \xrightarrow{\times 5} 50 \xrightarrow{\times 5} 250$$

18. A geometric sequence has $a_1 = 64$ and $r = \frac{1}{4}$. Find a_5.

$$64 \xrightarrow{\times \frac{1}{4}} 16 \xrightarrow{\times \frac{1}{4}} 4 \xrightarrow{\times \frac{1}{4}} 1$$

19. Find a_6 for the sequence shown.

$$4 \xrightarrow{\times 3} 12 \xrightarrow{\times 3} 36 \xrightarrow{\times 3} 108$$

20. Convert from recursive to explicit: $a_1 = 8$ and $a_n = \frac{1}{2}a_{n-1}$ for $n \geq 2$.

$$8 \xrightarrow{\times \frac{1}{2}} 4 \xrightarrow{\times \frac{1}{2}} 2 \xrightarrow{\times \frac{1}{2}} 1$$

♦ Word Problems

21. A population of bacteria starts at 240 and triples every hour. Find the population after 5 hours.

22. A car's value drops by 20% each year. If it's worth \$30,000 new, what is its value after 4 years?

23. A geometric sequence describes weekly social-media followers. Week 1: 50 followers. Week 4: 1350. If the growth stays geometric, find the common ratio (positive) and the explicit formula.

24. A medicine dose in a patient's bloodstream halves every 6 hours. If the initial dose is 400 mg, how much remains after 24 hours?

Additional Practice

25. Find the next term: $4, 9, 14, 19, \ldots$.

26. Find a_{10} if $a_1 = 3$ and $d = 5$.

27. Find the next term: $2, 6, 18, 54, \ldots$.

28. Find a_6 if $a_1 = 5$ and $r = 2$.

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Answer Keys

1. $\boxed{r=2}$
2. $\boxed{a_5=324}$
3. $\boxed{a_4=12.5}$
4. $\boxed{A}$
5. $\boxed{r=3}$
6. $\boxed{a_n=2\cdot 3^{n-1}}$
7. $\boxed{a_7=1458}$
8. $\boxed{a_7=192}$
9. $\boxed{a_n=5\cdot 2^{n-1}}$
10. $\boxed{a_9=32805}$
11. $\boxed{a_6=160}$
12. False
13. $\boxed{6,\ -12,\ 24,\ -48}$
14. $\boxed{a_n=4\cdot 3^{n-1}}$
15. $\boxed{a_8=49152}$
16. $\boxed{a_1=7,\ r=2,\ \text{terms: } 7,14,28,56}$
17. $\boxed{a_7=31250}$
18. $\boxed{a_5=\frac{1}{4}}$
19. $\boxed{a_6=972}$
20. $\boxed{a_n=8\cdot\left(\frac{1}{2}\right)^{n-1}}$
21. $\boxed{58{,}320}$
22. $\boxed{\$12{,}288}$
23. $\boxed{r=3;\ a_n=50\cdot 3^{n-1}}$
24. 25 mg

Additional Practice Answers

25. $\boxed{24}$
26. $\boxed{48}$
27. $\boxed{162}$
28. $\boxed{160}$

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. Divide neighbors: $6/3=2$, $12/6=2$, $24/12=2$. The labeled arrows confirm it – each term doubles. So $r=2$.

2. Keep the rule visible: $a_n=a_1\cdot r^{n-1}$ gives $a_5=4\cdot 3^4=4\cdot 81=324$. Watch the exponent: four multiplications by 3 from the start, not five. That gives a quick check on the answer.

3. One steady path is: $a_4=100\cdot(\frac{1}{2})^3=100\cdot\frac{1}{8}=12.5$. Track it term by term as a check: $100\to 50\to 25\to 12.5$ – three halvings, matching the exponent. That gives a quick check on the answer.

4. Start with the key idea: A has ratio $12/4=3$, $36/12=3$, $108/36=3$. Constant ratio, so A is geometric with $r=3$. B adds 8 each time (arithmetic). C is the squares. That gives a quick check on the answer.

5. From term 2 to term 5 is 3 ratio steps, so $a_5=a_2\cdot r^3$. That gives $162=6r^3$, so $r^3=27$ and $r=3$. (Odd powers preserve sign, so 27 has a unique real cube root; the "positive" qualifier is just a reminder, not a tiebreaker.)

6. Keep the rule visible: $a_1=2$ and $r=3$, so $a_n=2\cdot 3^{n-1}$. Verify: $a_1=2\cdot 3^0=2$ ✓, $a_2=2\cdot 3^1=6$ ✓. That gives a quick check on the answer.

7. The arrows multiply by 3, so $r=3$, and the first dot gives $a_1=2$. Use $a_n=a_1\cdot r^{n-1}$ with $n=7$: $a_7=2\cdot 3^{7-1}=2\cdot 3^6=2\cdot 729=1458$. The exponent is 6, not 7 – six multiplications carry you from term 1 to term 7.

8. Each bar is twice the one before $(3,6,12,24)$, so $r=2$ and $a_1=3$. Then $a_7=a_1\cdot r^{7-1}=3\cdot 2^6=3\cdot 64=192$. Use r^{n-1}, not r^n – six doublings reach term 7.

9. Ratio steps: $6-3=3$. So $a_6=a_3\cdot r^3\Rightarrow 160=20r^3\Rightarrow r^3=8\Rightarrow r=2$. Then $a_3=a_1r^2=4a_1\Rightarrow a_1=5$. Formula: $a_n=5\cdot 2^{n-1}$.

10. The arrows triple each term, so $r=3$ and $a_1=5$. Then $a_9=a_1\cdot r^{9-1}=5\cdot 3^8=5\cdot 6561=32{,}805$. The exponent is 8 (eight multiplications), not 9.

11. One steady path is: $a_6=5\cdot 2^5=5\cdot 32=160$. (Five multiplications from a_1 to a_6.) That gives a quick check on the answer.

12. Sign depends on a_1 and on r. A negative a_1 keeps every term negative when $r>0$; a negative r flips signs every step. $|r|>1$ only controls magnitude, not sign.

13. Multiply by -2 each step: $6\to -12\to 24\to -48$. Signs alternate because r is negative – this is what makes a sequence alternating.

14. From the table, $a_1=4$ and $r=12/4=3$. So $a_n=4\cdot 3^{n-1}$. Sanity check: $a_4=4\cdot 27=108$ ✓.

15. The arrows multiply by 4, so $r=4$ and $a_1=3$. Then $a_8=a_1\cdot r^{8-1}=3\cdot 4^7=3\cdot 16{,}384=49{,}152$. Seven multiplications by 4 reach term 8.

16. Match against $a_n=a_1r^{n-1}$. The coefficient out front is $a_1=7$, and the base of the exponential is $r=2$. (The $n-1$ is just the standard offset.) First four terms: $7,14,28,56$, matching the diagram.

17. The arrows multiply by 5, so $r=5$ and $a_1=2$. Then $a_7=a_1\cdot r^{7-1}=2\cdot 5^6=2\cdot 15{,}625=31{,}250$. Six multiplications by 5 land on term 7.

18. Keep the rule visible: $a_5=64\cdot(\frac{1}{4})^4=64\cdot\frac{1}{256}=\frac{64}{256}=\frac{1}{4}$. Or just track it past the diagram: $64\to 16\to 4\to 1\to\frac{1}{4}$. That gives a quick check on the answer.

19. The arrows triple each term, so $r=3$ and $a_1=4$. Then $a_6=a_1\cdot r^{6-1}=4\cdot 3^5=4\cdot 243=972$. Five multiplications reach term 6.

20. Each step multiplies by $\frac{1}{2}$, so $r=\frac{1}{2}$. $a_n=8\cdot(\frac{1}{2})^{n-1}$. (Or write it as $a_n=\frac{8}{2^{n-1}}$ – same thing, just no fractional base.) First four terms match the diagram: $8,4,2,1$.

21. Tripling each hour from a start of 240 is geometric. Two equivalent setups: (1) call the start $a_0=240$ and use $P=240\cdot 3^t$ – then $P(5)=240\cdot 243=58{,}320$. (2) Or call hour 0 "term 1," so $a_1=240$ and $r=3$, with $a_n=240\cdot 3^{n-1}$; hour 5 is term $n=6$, giving $a_6=240\cdot 3^5=58{,}320$. Both routes land on $58{,}320$ bacteria.

22. A 20% drop multiplies the previous value by 0.8. So $r=0.8$ and $a_1=30{,}000$. After 4 years (term 5): $30{,}000\cdot(0.8)^4=30{,}000\cdot 0.4096=12{,}288$, so \$12,288. (Don't subtract 80% – a 20% drop leaves 80% of what was there.)

23. Three ratio steps from week 1 to week 4, so $1350=50\cdot r^3\Rightarrow r^3=27\Rightarrow r=3$. Explicit: $a_n=50\cdot 3^{n-1}$. **Check:** week 4 predicts $50\cdot 3^3=50\cdot 27=1350$ ✓.

24. Each 6-hour block multiplies by $\frac{1}{2}$. In 24 hours, that's $24/6=4$ halvings: $400\cdot(\frac{1}{2})^4=400\cdot\frac{1}{16}=25$ mg. (Reality check: positive and shrinking – both make sense for a metabolizing dose.)

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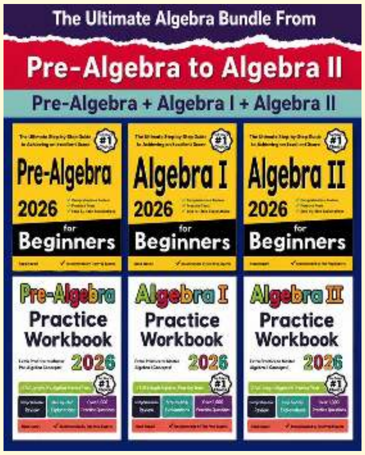

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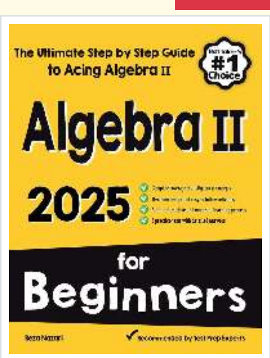

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