Fundamental Trigonometric Identities

Name: ________________	Date: ____________	Score: ______ / 31

Quick Review

The fundamental identities are the building blocks for everything in this chapter. Memorize them like multiplication facts.

Reciprocal identities.

$\csc\theta = \dfrac{1}{\sin\theta}, \quad \sec\theta = \dfrac{1}{\cos\theta}, \quad \cot\theta = \dfrac{1}{\tan\theta}.$

Quotient identities.

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}, \quad \cot\theta = \dfrac{\cos\theta}{\sin\theta}.$

Pythagorean identities. Divide the first one by $\cos^2\theta$ to get the second, by $\sin^2\theta$ to get the third:

$\sin^2\theta + \cos^2\theta = 1$

$1 + \tan^2\theta = \sec^2\theta$

$1 + \cot^2\theta = \csc^2\theta$

Even/odd identities. Cosine and secant are even (negative input doesn't change the output). Sine, tangent, cotangent, and cosecant are odd (negative input flips the sign):

$\cos(-\theta) = \cos\theta, \quad \sec(-\theta) = \sec\theta$

$\sin(-\theta) = -\sin\theta, \quad \tan(-\theta) = -\tan\theta$

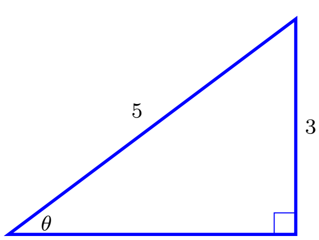
How to verify an identity. Start on one side, replace things using reciprocal / quotient / Pythagorean swaps, simplify, and aim to match the other side. Don't move pieces across the equals sign – that assumes the identity, which is the thing you're trying to prove. Convert everything to $\sin\theta$ and $\cos\theta$ when stuck.

Common slips. Writing $\sin^2\theta + \cos^2\theta = \theta$ (no – it's $= 1$). Distributing across a square: $\sin^2\theta \neq (\sin\theta)\cdot 2$ – it means $(\sin\theta)^2$. Treating $\sin(-\theta)$ like $\cos(-\theta)$: sine flips, cosine doesn't.

PRACTICE

Simplify each expression using the fundamental identities. Assume every expression is defined.

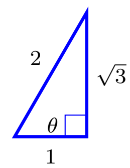
1. Simplify $\sin\theta \cdot \csc\theta$. ________
2. Simplify $\cos\theta \cdot \sec\theta$. ________
3. Simplify $\sec^2\theta - \tan^2\theta$. ________
4. Simplify $\csc^2\theta - \cot^2\theta$. ________
5. Simplify $\dfrac{\sin\theta}{\cos\theta}$. ________
6. For the acute angle θ in the triangle below, $\sin\theta = \dfrac{3}{5}$. Use $\sin^2\theta + \cos^2\theta = 1$ to find $\cos\theta$. ________



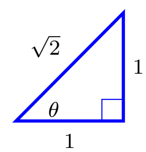
7. Simplify $1 - \cos^2\theta$. ________
8. Simplify $\dfrac{\tan\theta}{\sec\theta}$. ________
9. Simplify $\dfrac{\cos\theta}{1 - \sin^2\theta}$. ________

10. Simplify $\sin(-\theta)\cos\theta + \cos(-\theta)\sin\theta$. ______

11. Evaluate $(1-\cos\theta)(1+\cos\theta)$ at $\theta = \frac{\pi}{3}$. The 30-60-90 triangle below has $\theta = \frac{\pi}{3}$ at the marked corner. ______



12. Evaluate $\tan\theta + \cot\theta$ at $\theta = \frac{\pi}{4}$. The 45-45-90 triangle below sets up $\theta = \frac{\pi}{4}$. ______



13. Simplify $\sin\theta\cos\theta \cdot \tan\theta \cdot \sec\theta$. ______

14. Simplify $\frac{1}{\tan\theta} + \tan\theta$. ______

15. Simplify $\cos^2\theta(1+\tan^2\theta)$. ______

16. Simplify $\sin\theta\cot\theta$. ______

17. Simplify $(\sin\theta + \cos\theta)^2$. ______

18. Simplify $(1-\sin\theta)(1+\sin\theta)$. ______

19. Simplify $\frac{\sec\theta}{\csc\theta}$. ______

20. Simplify $\sin^2\theta + \cos^2\theta + \tan^2\theta$. ______

♦ Word Problems

21. Given that $\sin\theta = \frac{3}{5}$ and θ is in Quadrant II, find $\cos\theta$ using a fundamental identity. ___

22. A right-triangle problem gives $\tan\theta = \frac{5}{12}$ with θ acute. Find $\sec\theta$ using a fundamental identity. ___

23. A student claims that $\sin^2\theta + \cos^2\theta = 2$ at $\theta = 45°$, because $\sin 45° + \cos 45° = \sqrt{2}$ and $(\sqrt{2})^2 = 2$. Find the actual value of $\sin^2 45° + \cos^2 45°$ and explain the student's mistake. ___

24. A surveyor's instrument reads $\cos\theta = \frac{8}{17}$ for an angle θ in Quadrant I. Find $\csc\theta$ using identities. ___

Additional Practice

25. Find $\sin\theta$ if opposite $= 5$, hypotenuse $= 13$. ______

26. Find $\cos\theta$ if adjacent $= 12$, hypotenuse $= 13$. ______

27. Find $\tan\theta$ if opposite $= 7$, adjacent $= 4$. ______

28. Find $\sin 30°$. ______

29. Find $\cos 60°$. ______

30. Find $\tan 45°$. ______

31. Convert $180°$ to radians. ______

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Answer Keys

1. $\boxed{1}$
2. $\boxed{1}$
3. $\boxed{1}$
4. $\boxed{1}$
5. $\boxed{\tan\theta}$
6. $\boxed{\frac{4}{5}}$
7. $\boxed{\sin^2\theta}$
8. $\boxed{\sin\theta}$
9. $\boxed{\sec\theta}$
10. $\boxed{0}$
11. $\boxed{\frac{3}{4}}$
12. $\boxed{2}$
13. $\boxed{\sin\theta\tan\theta}$
14. $\boxed{\sec\theta\csc\theta}$
15. $\boxed{1}$
16. $\boxed{\cos\theta}$
17. $\boxed{1+2\sin\theta\cos\theta}$
18. $\boxed{\cos^2\theta}$
19. $\boxed{\tan\theta}$
20. $\boxed{\sec^2\theta}$
21. $\boxed{-\frac{4}{5}}$
22. $\boxed{\frac{13}{12}}$
23. $\boxed{1;\text{ the student squared the sum, not each term}}$
24. $\boxed{\frac{17}{15}}$

Additional Practice Answers

25. $\boxed{\frac{5}{13}}$
26. $\boxed{\frac{12}{13}}$
27. $\boxed{\frac{7}{4}}$
28. $\boxed{\frac{1}{2}}$
29. $\boxed{\frac{1}{2}}$
30. $\boxed{1}$
31. $\boxed{\pi}$

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. A careful way to see it: Cosecant is the reciprocal of sine, so the product is $\sin\theta\cdot\frac{1}{\sin\theta}=1$ (provided $\sin\theta\neq0$). That gives a quick check on the answer.

2. Keep the rule visible: Same idea: secant is the reciprocal of cosine, so $\cos\theta\cdot\frac{1}{\cos\theta}=1$. That gives a quick check on the answer.

3. Rearrange the Pythagorean identity $1+\tan^2\theta=\sec^2\theta$: subtract $\tan^2\theta$ from both sides and you get $\sec^2\theta-\tan^2\theta=1$.

4. Start with the key idea: From $1+\cot^2\theta=\csc^2\theta$, subtracting $\cot^2\theta$ gives 1. This is the part to check before moving on, because it keeps the answer tied to the original question.

5. This ratio is exactly the quotient identity: $\frac{\sin\theta}{\cos\theta}=\tan\theta$ by definition. Whenever you see sine over cosine, replace it with tangent on sight.

6. Rearrange the Pythagorean identity to $\cos^2\theta=1-\sin^2\theta=1-\frac{9}{25}=\frac{16}{25}$, then take the square root: $\cos\theta=\pm\frac{4}{5}$. Because θ is the acute angle in the triangle, cosine is positive, so $\cos\theta=\frac{4}{5}$. (It's the classic 3-4-5 triangle, with adjacent 4 over hypotenuse 5.)

7. One steady path is: Mirror image of the previous: $\sin^2\theta=1-\cos^2\theta$. This is the part to check before moving on, because it keeps the answer tied to the original question.

8. Start with the key idea: Write each in sin/cos: $\frac{\sin\theta/\cos\theta}{1/\cos\theta}=\frac{\sin\theta}{\cos\theta}\cdot\cos\theta=\sin\theta$. This is the part to check before moving on, because it keeps the answer tied to the original question.

9. A careful way to see it: Replace $1-\sin^2\theta$ with $\cos^2\theta$: $\frac{\cos\theta}{\cos^2\theta}=\frac{1}{\cos\theta}=\sec\theta$. This is the part to check before moving on, because it keeps the answer tied to the original question.

10. Sine is odd, so $\sin(-\theta)=-\sin\theta$. Cosine is even, so $\cos(-\theta)=\cos\theta$. The expression becomes $-\sin\theta\cos\theta+\cos\theta\sin\theta=0$.

11. Simplify symbolically first: $(1-\cos\theta)(1+\cos\theta)$ is a difference of squares equal to $1-\cos^2\theta$, which the Pythagorean identity turns into $\sin^2\theta$. At $\theta=\frac{\pi}{3}$ the triangle gives $\sin\frac{\pi}{3}=\frac{\sqrt{3}}{2}$, so $\sin^2\frac{\pi}{3}=\left(\frac{\sqrt{3}}{2}\right)^2=\frac{3}{4}$. Multiplying out directly would work too, but the identity is faster.

12. Start with the key idea: $\tan\frac{\pi}{4}=1$ and $\cot\frac{\pi}{4}=1$, so the sum is 2. (Symbolically the expression equals $\frac{1}{\sin\theta\cos\theta}$; at $\frac{\pi}{4}$ that's $\frac{1}{(1/\sqrt{2})(1/\sqrt{2})}=\frac{1}{1/2}=2$ ✓.) That gives a quick check on the answer.

13. A careful way to see it: Convert all to sin/cos: $\sin\theta\cos\theta\cdot\frac{\sin\theta}{\cos\theta}\cdot\frac{1}{\cos\theta}=\frac{\sin^2\theta}{\cos\theta}=\sin\theta\cdot\frac{\sin\theta}{\cos\theta}=\sin\theta\tan\theta$. This is the part to check before moving on, because it keeps the answer tied to the original question.

14. Keep the rule visible: Common denominator: $\frac{1+\tan^2\theta}{\tan\theta}=\frac{\sec^2\theta}{\tan\theta}=\frac{1/\cos^2\theta}{\sin\theta/\cos\theta}=\frac{1}{\sin\theta\cos\theta}=\sec\theta\csc\theta$. This is the part to check before moving on, because it keeps the answer tied to the original question.

15. One steady path is: $1+\tan^2\theta=\sec^2\theta=\frac{1}{\cos^2\theta}$. Multiplying by $\cos^2\theta$ gives 1. This is the part to check before moving on, because it keeps the answer tied to the original question.

16. Start with the key idea: $\sin\theta\cdot\frac{\cos\theta}{\sin\theta}=\cos\theta$ (the sines cancel). This is the part to check before moving on, because it keeps the answer tied to the original question.

17. A careful way to see it: Expand: $\sin^2\theta+2\sin\theta\cos\theta+\cos^2\theta=1+2\sin\theta\cos\theta$ (the squared terms combine to 1). That gives a quick check on the answer.

18. Keep the rule visible: Difference of squares: $1-\sin^2\theta=\cos^2\theta$. This is the part to check before moving on, because it keeps the answer tied to the original question.

19. One steady path is: $\frac{1/\cos\theta}{1/\sin\theta}=\frac{\sin\theta}{\cos\theta}=\tan\theta$. This is the part to check before moving on, because it keeps the answer tied to the original question.

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20. Start with the key idea: The first two sum to 1, and $1+\tan^2\theta=\sec^2\theta$. Done. This is the part to check before moving on, because it keeps the answer tied to the original question.

21. Use $\sin^2\theta+\cos^2\theta=1$. Then $\cos^2\theta=1-\frac{9}{25}=\frac{16}{25}$, so $\cos\theta=\pm\frac{4}{5}$. In Q2, cosine is negative, so $\cos\theta=-\frac{4}{5}$.

22. Use $1+\tan^2\theta=\sec^2\theta$: $1+\frac{25}{144}=\frac{144+25}{144}=\frac{169}{144}$. So $\sec\theta=\pm\frac{13}{12}$, and since θ is acute, secant is positive: $\frac{13}{12}$. (Sanity check: 5-12-13 triangle.)

23. One steady path is: $\sin^2 45^\circ+\cos^2 45^\circ=\left(\frac{\sqrt{2}}{2}\right)^2+\left(\frac{\sqrt{2}}{2}\right)^2=\frac{1}{2}+\frac{1}{2}=1$. The student computed $(\sin\theta+\cos\theta)^2$ instead of $\sin^2\theta+\cos^2\theta$. The Pythagorean identity squares each piece separately, then adds; it doesn't add first and then square. That gives a quick check on the answer.

24. From $\sin^2\theta+\cos^2\theta=1$, $\sin^2\theta=1-\frac{64}{289}=\frac{225}{289}$, so $\sin\theta=\frac{15}{17}$ (positive in Q1). Then $\csc\theta=\frac{1}{\sin\theta}=\frac{17}{15}$. (Sanity check: 8-15-17 is a Pythagorean triple.)