

Fundamental Trigonometric Identities

Name: _____ Date: _____ Score: _____ / 31

Q Quick Review

The fundamental identities are the building blocks for everything in this chapter. Memorize them like multiplication facts.

Reciprocal identities.

$$\csc \theta = \frac{1}{\sin \theta}, \quad \sec \theta = \frac{1}{\cos \theta}, \quad \cot \theta = \frac{1}{\tan \theta}.$$

Quotient identities.

$$\tan \theta = \frac{\sin \theta}{\cos \theta}, \quad \cot \theta = \frac{\cos \theta}{\sin \theta}.$$

Pythagorean identities. Divide the first one by $\cos^2 \theta$ to get the second, by $\sin^2 \theta$ to get the third:

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

Even/odd identities. Cosine and secant are even (negative input doesn't change the output). Sine, tangent, cotangent, and cosecant are odd (negative input flips the sign):

$$\cos(-\theta) = \cos \theta, \quad \sec(-\theta) = \sec \theta$$

$$\sin(-\theta) = -\sin \theta, \quad \tan(-\theta) = -\tan \theta$$

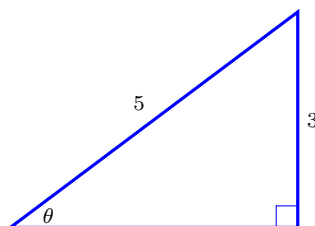
How to verify an identity. Start on one side, replace things using reciprocal / quotient / Pythagorean swaps, simplify, and aim to match the other side. Don't move pieces across the equals sign – that assumes the identity, which is the thing you're trying to prove. Convert everything to $\sin \theta$ and $\cos \theta$ when stuck.

Common slips. Writing $\sin^2 \theta + \cos^2 \theta = \theta$ (no – it's = 1). Distributing across a square: $\sin^2 \theta \neq (\sin \theta) \cdot 2$ – it means $(\sin \theta)^2$. Treating $\sin(-\theta)$ like $\cos(-\theta)$: sine flips, cosine doesn't.

PRACTICE

Simplify each expression using the fundamental identities. Assume every expression is defined.

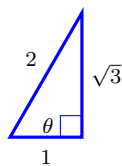
1. Simplify $\sin \theta \cdot \csc \theta$. _____
2. Simplify $\cos \theta \cdot \sec \theta$. _____
3. Simplify $\sec^2 \theta - \tan^2 \theta$. _____
4. Simplify $\csc^2 \theta - \cot^2 \theta$. _____
5. Simplify $\frac{\sin \theta}{\cos \theta}$. _____
6. For the acute angle θ in the triangle below, $\sin \theta = \frac{3}{5}$. Use $\sin^2 \theta + \cos^2 \theta = 1$ to find $\cos \theta$. _____



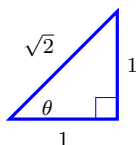
7. Simplify $1 - \cos^2 \theta$. _____
8. Simplify $\frac{\tan \theta}{\sec \theta}$. _____
9. Simplify $\frac{\cos \theta}{1 - \sin^2 \theta}$. _____



10. Simplify $\sin(-\theta) \cos \theta + \cos(-\theta) \sin \theta$. _____
11. Evaluate $(1 - \cos \theta)(1 + \cos \theta)$ at $\theta = \frac{\pi}{3}$. The 30-60-90 triangle below has $\theta = \frac{\pi}{3}$ at the marked corner. _____



12. Evaluate $\tan \theta + \cot \theta$ at $\theta = \frac{\pi}{4}$. The 45-45-90 triangle below sets up $\theta = \frac{\pi}{4}$. _____



13. Simplify $\sin \theta \cos \theta \cdot \tan \theta \cdot \sec \theta$. _____
14. Simplify $\frac{1}{\tan \theta} + \tan \theta$. _____
15. Simplify $\cos^2 \theta(1 + \tan^2 \theta)$. _____
16. Simplify $\sin \theta \cot \theta$. _____
17. Simplify $(\sin \theta + \cos \theta)^2$. _____
18. Simplify $(1 - \sin \theta)(1 + \sin \theta)$. _____
19. Simplify $\frac{\sec \theta}{\csc \theta}$. _____
20. Simplify $\sin^2 \theta + \cos^2 \theta + \tan^2 \theta$. _____

◆ Word Problems

21. Given that $\sin \theta = \frac{3}{5}$ and θ is in Quadrant II, find $\cos \theta$ using a fundamental identity. _____
22. A right-triangle problem gives $\tan \theta = \frac{5}{12}$ with θ acute. Find $\sec \theta$ using a fundamental identity. _____
23. A student claims that $\sin^2 \theta + \cos^2 \theta = 2$ at $\theta = 45^\circ$, because $\sin 45^\circ + \cos 45^\circ = \sqrt{2}$ and $(\sqrt{2})^2 = 2$. Find the actual value of $\sin^2 45^\circ + \cos^2 45^\circ$ and explain the student's mistake. _____
24. A surveyor's instrument reads $\cos \theta = \frac{8}{17}$ for an angle θ in Quadrant I. Find $\csc \theta$ using identities. _____

Additional Practice

25. Find $\sin \theta$ if opposite = 5, hypotenuse = 13. _____
26. Find $\cos \theta$ if adjacent = 12, hypotenuse = 13. _____
27. Find $\tan \theta$ if opposite = 7, adjacent = 4. _____
28. Find $\sin 30^\circ$. _____
29. Find $\cos 60^\circ$. _____
30. Find $\tan 45^\circ$. _____
31. Convert 180° to radians. _____



Answer Keys

<p>1. $\boxed{1}$</p> <p>2. $\boxed{1}$</p> <p>3. $\boxed{1}$</p> <p>4. $\boxed{1}$</p> <p>5. $\boxed{\tan \theta}$</p> <p>6. $\boxed{\frac{4}{5}}$</p> <p>7. $\boxed{\sin^2 \theta}$</p> <p>8. $\boxed{\sin \theta}$</p> <p>9. $\boxed{\sec \theta}$</p> <p>10. $\boxed{0}$</p> <p>11. $\boxed{\frac{3}{4}}$</p> <p>12. $\boxed{2}$</p> <p>Additional Practice Answers</p> <p>25. $\boxed{\frac{5}{13}}$</p> <p>26. $\boxed{\frac{12}{13}}$</p> <p>27. $\boxed{\frac{7}{4}}$</p>	<p>13. $\boxed{\sin \theta \tan \theta}$</p> <p>14. $\boxed{\sec \theta \csc \theta}$</p> <p>15. $\boxed{1}$</p> <p>16. $\boxed{\cos \theta}$</p> <p>17. $\boxed{1 + 2 \sin \theta \cos \theta}$</p> <p>18. $\boxed{\cos^2 \theta}$</p> <p>19. $\boxed{\tan \theta}$</p> <p>20. $\boxed{\sec^2 \theta}$</p> <p>21. $\boxed{-\frac{4}{5}}$</p> <p>22. $\boxed{\frac{13}{12}}$</p> <p>23. $\boxed{1}$; the student squared the sum, not each term</p> <p>24. $\boxed{\frac{17}{15}}$</p> <p>28. $\boxed{\frac{1}{2}}$</p> <p>29. $\boxed{\frac{1}{2}}$</p> <p>30. $\boxed{1}$</p> <p>31. $\boxed{\pi}$</p>
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Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

- A careful way to see it: Cosecant is the reciprocal of sine, so the product is $\sin \theta \cdot \frac{1}{\sin \theta} = 1$ (provided $\sin \theta \neq 0$). That gives a quick check on the answer.
- Keep the rule visible: Same idea: secant is the reciprocal of cosine, so $\cos \theta \cdot \frac{1}{\cos \theta} = 1$. That gives a quick check on the answer.
- Rearrange the Pythagorean identity $1 + \tan^2 \theta = \sec^2 \theta$: subtract $\tan^2 \theta$ from both sides and you get $\sec^2 \theta - \tan^2 \theta = 1$.
- Start with the key idea: From $1 + \cot^2 \theta = \csc^2 \theta$, subtracting $\cot^2 \theta$ gives 1. This is the part to check before moving on, because it keeps the answer tied to the original question.
- This ratio is exactly the quotient identity: $\frac{\sin \theta}{\cos \theta} = \tan \theta$ by definition. Whenever you see sine over cosine, replace it with tangent on sight.
- Rearrange the Pythagorean identity to $\cos^2 \theta = 1 - \sin^2 \theta = 1 - \frac{9}{25} = \frac{16}{25}$, then take the square root: $\cos \theta = \pm \frac{4}{5}$. Because θ is the acute angle in the triangle, cosine is positive, so $\cos \theta = \frac{4}{5}$. (It's the classic 3-4-5 triangle, with adjacent 4 over hypotenuse 5.)
- One steady path is: Mirror image of the previous: $\sin^2 \theta = 1 - \cos^2 \theta$. This is the part to check before moving on, because it keeps the answer tied to the original question.
- Start with the key idea: Write each in sin/cos: $\frac{\sin \theta / \cos \theta}{1 / \cos \theta} = \frac{\sin \theta}{\cos \theta} \cdot \cos \theta = \sin \theta$. This is the part to check before moving on, because it keeps the answer tied to the original question.
- A careful way to see it: Replace $1 - \sin^2 \theta$ with $\cos^2 \theta$: $\frac{\cos \theta}{\cos^2 \theta} = \frac{1}{\cos \theta} = \sec \theta$. This is the part to check before moving on, because it keeps the answer tied to the original question.
- Sine is odd, so $\sin(-\theta) = -\sin \theta$. Cosine is even, so $\cos(-\theta) = \cos \theta$. The expression becomes $-\sin \theta \cos \theta + \cos \theta \sin \theta = 0$.
- Simplify symbolically first: $(1 - \cos \theta)(1 + \cos \theta)$ is a difference of squares equal to $1 - \cos^2 \theta$, which the Pythagorean identity turns into $\sin^2 \theta$. At $\theta = \frac{\pi}{3}$

- the triangle gives $\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$, so $\sin^2 \frac{\pi}{3} = \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{4}$. Multiplying out directly would work too, but the identity is faster.
- Start with the key idea: $\tan \frac{\pi}{4} = 1$ and $\cot \frac{\pi}{4} = 1$, so the sum is 2. (Symbolically the expression equals $\frac{1}{\sin \theta \cos \theta}$; at $\frac{\pi}{4}$ that's $\frac{1}{(1/\sqrt{2})(1/\sqrt{2})} = \frac{1}{1/2} = 2$ ✓.) That gives a quick check on the answer.
 - A careful way to see it: Convert all to sin/cos: $\sin \theta \cos \theta \cdot \frac{\sin \theta}{\cos \theta} \cdot \frac{1}{\cos \theta} = \frac{\sin^2 \theta}{\cos \theta} = \sin \theta \cdot \frac{\sin \theta}{\cos \theta} = \sin \theta \tan \theta$. This is the part to check before moving on, because it keeps the answer tied to the original question.
 - Keep the rule visible: Common denominator: $\frac{1 + \tan^2 \theta}{\tan \theta} = \frac{\sec^2 \theta}{\tan \theta} = \frac{1/\cos^2 \theta}{\sin \theta / \cos \theta} = \frac{1}{\sin \theta \cos \theta} = \sec \theta \csc \theta$. This is the part to check before moving on, because it keeps the answer tied to the original question.
 - One steady path is: $1 + \tan^2 \theta = \sec^2 \theta = \frac{1}{\cos^2 \theta}$. Multiplying by $\cos^2 \theta$ gives 1. This is the part to check before moving on, because it keeps the answer tied to the original question.
 - Start with the key idea: $\sin \theta \cdot \frac{\cos \theta}{\sin \theta} = \cos \theta$ (the sines cancel). This is the part to check before moving on, because it keeps the answer tied to the original question.
 - A careful way to see it: Expand: $\sin^2 \theta + 2 \sin \theta \cos \theta + \cos^2 \theta = 1 + 2 \sin \theta \cos \theta$ (the squared terms combine to 1). That gives a quick check on the answer.
 - Keep the rule visible: Difference of squares: $1 - \sin^2 \theta = \cos^2 \theta$. This is the part to check before moving on, because it keeps the answer tied to the original question.
 - One steady path is: $\frac{1/\cos \theta}{1/\sin \theta} = \frac{\sin \theta}{\cos \theta} = \tan \theta$. This is the part to check before moving on, because it keeps the answer tied to the original question.



20. Start with the key idea: The first two sum to 1, and $1 + \tan^2 \theta = \sec^2 \theta$. Done. This is the part to check before moving on, because it keeps the answer tied to the original question.

21. Use $\sin^2 \theta + \cos^2 \theta = 1$. Then $\cos^2 \theta = 1 - \frac{9}{25} = \frac{16}{25}$, so $\cos \theta = \pm \frac{4}{5}$. In

Q2, cosine is negative, so $\cos \theta = -\frac{4}{5}$.

22. Use $1 + \tan^2 \theta = \sec^2 \theta$: $1 + \frac{25}{144} = \frac{144 + 25}{144} = \frac{169}{144}$. So $\sec \theta = \pm \frac{13}{12}$,

and since θ is acute, secant is positive: $\frac{13}{12}$. (Sanity check: 5-12-13 triangle.)

23. One steady path is: $\sin^2 45^\circ + \cos^2 45^\circ = \left(\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2 = \frac{1}{2} + \frac{1}{2} =$

1. The student computed $(\sin \theta + \cos \theta)^2$ instead of $\sin^2 \theta + \cos^2 \theta$. The Pythagorean identity squares each piece separately, then adds; it doesn't add first and then square. That gives a quick check on the answer.

24. From $\sin^2 \theta + \cos^2 \theta = 1$, $\sin^2 \theta = 1 - \frac{64}{289} = \frac{225}{289}$, so $\sin \theta = \frac{15}{17}$ (positive

in Q1). Then $\csc \theta = \frac{1}{\sin \theta} = \frac{17}{15}$. (Sanity check: 8-15-17 is a Pythagorean triple.)



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