

Function Inverses

Name: _____ Date: _____ Score: _____ / 31

Quick Review

An **inverse function** undoes the original. If f takes x to y , then f^{-1} takes y back to x . Officially: $f(f^{-1}(x)) = x$ for every x in the domain of f^{-1} , and $f^{-1}(f(x)) = x$ for every x in the domain of f . (Heads up: $f^{-1}(x)$ means the inverse function, *not* the reciprocal $\frac{1}{f(x)}$.)

To find an inverse algebraically: write $y = f(x)$, swap x and y , then solve for y . Quick check: $f(x) = 3x - 5 \Rightarrow y = 3x - 5 \Rightarrow x = 3y - 5 \Rightarrow y = \frac{x+5}{3}$. So $f^{-1}(x) = \frac{x+5}{3}$.

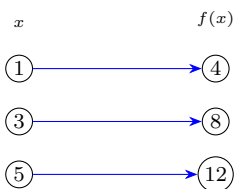
Existence. Only *one-to-one* functions have inverse functions (those passing the *horizontal-line test*). $f(x) = x^2$ on all reals isn't one-to-one (2 and -2 both map to 4), so its inverse isn't a function unless we restrict the domain. Take $f(x) = (x - 4)^2$ with $x \geq 4$: now it's one-to-one, and $f^{-1}(x) = 4 + \sqrt{x}$ (positive root only).

Graphs. f and f^{-1} are reflections across the line $y = x$. If (a, b) is on f , then (b, a) is on f^{-1} . The domain of f^{-1} equals the range of f , and vice versa. **Double inverse** returns the original: $(f^{-1})^{-1} = f$.

PRACTICE

Find the inverse or use inverse-function facts as asked.

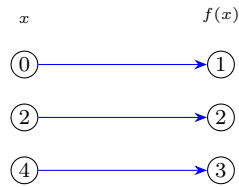
1. Simplify $f(f^{-1}(x))$ for x in the domain of f^{-1} . _____
2. $f(x) = x + 7$; $f^{-1}(x)$ _____
3. $g(x) = 3x - 5$; $g^{-1}(x)$ _____
4. $h(x) = \frac{x - 2}{4}$; $h^{-1}(x)$ _____
5. $f(x) = 2x + 8$; g with $g(f(x)) = x$ _____
6. $f(x) = 5x - 1$. Is its inverse a function? _____
7. $f(x) = \frac{2x + 1}{3}$; $f^{-1}(x)$ _____
8. The mapping diagram shows f . Give one point on f^{-1} . _____



9. $f(x) = (x - 4)^2$, $x \geq 4$; $f^{-1}(x)$ _____
10. $f(x) = \frac{1}{2}x + 1$; $f^{-1}(x)$ _____
11. $(f^{-1})^{-1}(x)$ _____
12. If (a, b) is on f , what point must be on f^{-1} ? _____
13. $f(x) = x^2$. Inverse a function (no restriction)? _____
14. $f(x) = 3x - 2$; $f^{-1}(x)$ _____



15. The mapping diagram shows f . Which point lies on f^{-1} , $(0, 1)$ or $(1, 0)$? _____



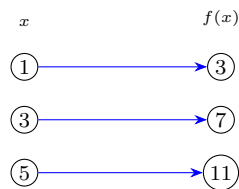
16. $f(x) = \sqrt{x}$, $x \geq 0$; $f^{-1}(x)$ _____

17. $f(x) = \frac{x}{4} - 3$; $f^{-1}(x)$ _____

18. $f^{-1}(x) = \frac{1}{f(x)}$? _____

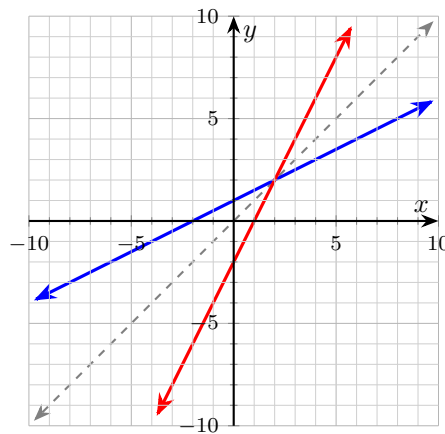
19. $f(x) = x^3 + 1$; $f^{-1}(x)$ _____

20. The mapping diagram shows a one-to-one function f . Find $f^{-1}(7)$. _____

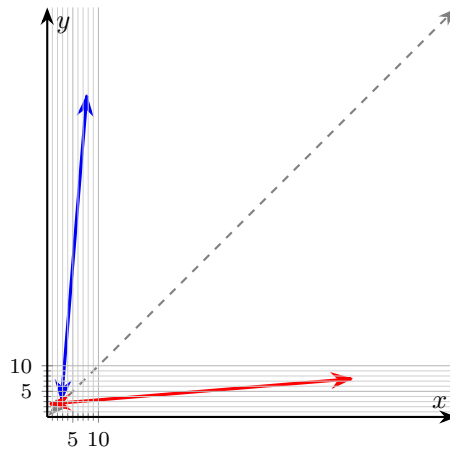


◆ Word Problems

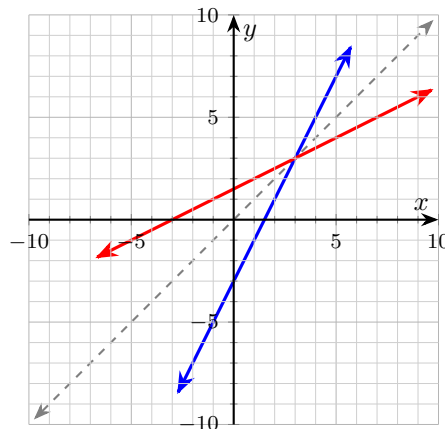
21. The function $C(F) = \frac{5}{9}(F - 32)$ converts Fahrenheit to Celsius. Find $C^{-1}(c)$, then convert 25° Celsius back to Fahrenheit. _____



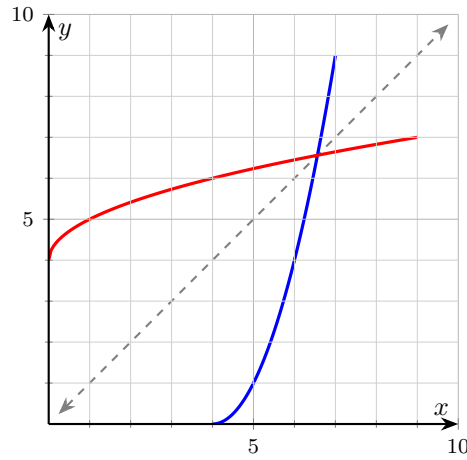
22. A ticket-pricing function is $P(n) = 12n - 30$, where n is the number of tickets bought (group discount factored in) and $P(n)$ is the total bill in dollars. Find $P^{-1}(b)$ and interpret it. Sketch P and P^{-1} on the plane below.



23. Show that $f(x) = 2x - 3$ and $g(x) = \frac{x + 3}{2}$ are inverses. Then use the plane below to sketch both and the line $y = x$.



24. Let $f(x) = (x - 4)^2$ with restricted domain $x \geq 4$. Find $f^{-1}(x)$ and its domain, then verify with one example. Sketch f and f^{-1} over the visible window. _____



Additional Practice

- 25. If $f(x) = 2x - 5$, find $f(4)$. _____
- 26. If $g(x) = x^2 + 1$, find $g(-3)$. _____
- 27. For $f(x) = 3x + 2$, solve $f(x) = 14$. _____
- 28. Find $(f + g)(x)$ if $f = x + 1$, $g = 2x - 5$. _____
- 29. Find $(fg)(x)$ if $f = x - 2$, $g = x + 3$. _____
- 30. Find $f(g(x))$ if $f(x) = 2x$, $g(x) = x + 7$. _____
- 31. Find the inverse of $f(x) = x - 9$. _____



Answer Keys

1. x	13. no
2. $\frac{x-7}{3}$	14. $\frac{x+2}{3}$
3. $\frac{x+5}{3}$	15. $(1, 0)$
4. $4x+2$	16. $x^2, x \geq 0$
5. $g(x) = \frac{x-8}{2}$	17. $4(x+3)$
6. yes	18. no
7. $\frac{3x-1}{2}$	19. $\sqrt[3]{x-1}$
8. $(8, 3)$ (or $(4, 1)$ or $(12, 5)$)	20. 3
9. $4 + \sqrt{x}$	21. $C^{-1}(c) = \frac{9}{5}c + 32, C^{-1}(25) = 77^\circ F$
10. $2x-2$	22. $P^{-1}(b) = \frac{b+30}{12}$; tickets bought for a bill of b
11. $f(x)$	23. $f \circ g = g \circ f = x$
12. (b, a)	24. $f^{-1}(x) = 4 + \sqrt{x}, \text{ domain } x \geq 0$
Additional Practice Answers	
25. 3	29. $x^2 + x - 6$
26. 10	30. $2x + 14$
27. $x = 4$	31. $f^{-1}(x) = x + 9$
28. $3x - 4$	

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

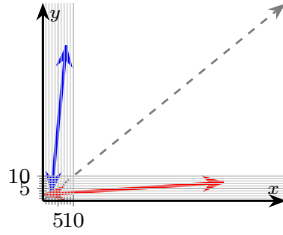
Step-by-Step Explanations

- A careful way to see it: By definition, inverses undo each other: $f \circ f^{-1} = I$. This is the part to check before moving on, because it keeps the answer tied to the original question.
- Write $y = x + 7$, swap to get $x = y + 7$, then solve for y : $y = x - 7$. The inverse undoes the function, and subtracting 7 reverses adding 7. So $f^{-1}(x) = x - 7$.
- Write $y = 3x - 5$ and swap x and y : $x = 3y - 5$. Solve for y —add 5 to get $x + 5 = 3y$, then divide by 3: $y = \frac{x+5}{3}$. So $g^{-1}(x) = \frac{x+5}{3}$.
- Write $y = \frac{x-2}{4}$ and swap: $x = \frac{y-2}{4}$. Clear the fraction by multiplying by 4: $4x = y - 2$, then add 2: $y = 4x + 2$. So $h^{-1}(x) = 4x + 2$.
- The function g that undoes f is its inverse. Write $y = 2x + 8$ and swap: $x = 2y + 8$. Subtract 8 ($x - 8 = 2y$) and divide by 2: $g(x) = \frac{x-8}{2}$.
- Keep the rule visible: Linear with nonzero slope is one-to-one. Passes horizontal-line test. That gives a quick check on the answer.
- Write $y = \frac{2x+1}{3}$ and swap: $x = \frac{2y+1}{3}$. Multiply by 3: $3x = 2y + 1$. Subtract 1 and divide by 2: $y = \frac{3x-1}{2}$. So $f^{-1}(x) = \frac{3x-1}{2}$.
- The inverse reverses every arrow. Reading the 3 → 8 arrow backward gives 8 → 3, so $(8, 3)$ is on f^{-1} . Any reversed pair works.
- A careful way to see it: With $x \geq 4, x - 4 \geq 0$. So $\sqrt{(x-4)^2} = x - 4$, giving $y = 4 + \sqrt{x}$ (positive branch). That gives a quick check on the answer.
- Keep the rule visible: $x = \frac{1}{2}y + 1 \Rightarrow 2x - 2 = y$. Check: $f(2x - 2) = \frac{1}{2}(2x - 2) + 1 = x - 1 + 1 = x$. That gives a quick check on the answer.
- One steady path is: Inverting twice returns the original function. This is the part to check before moving on, because it keeps the answer tied to the original question.
- Start with the key idea: Inverse swaps each input-output pair. This is the part to check before moving on, because it keeps the answer tied to the original question.
- A careful way to see it: f isn't one-to-one ($f(2) = f(-2) = 4$). Inverse relation fails vertical-line test. That gives a quick check on the answer.
- Write $y = 3x - 2$ and swap: $x = 3y - 2$. Add 2 ($x + 2 = 3y$), then divide by 3: $y = \frac{x+2}{3}$. So $f^{-1}(x) = \frac{x+2}{3}$.

- The diagram shows $f(0) = 1$, so $(0, 1)$ is on f . The inverse swaps the pair to $(1, 0)$. So $(1, 0)$ is the point on f^{-1} .
- Start with the key idea: Swap: $x = \sqrt{y} \Rightarrow y = x^2$. Restrict to $x \geq 0$ to match f 's range. That gives a quick check on the answer.
- A careful way to see it: $x = \frac{y}{4} - 3 \Rightarrow x + 3 = \frac{y}{4} \Rightarrow y = 4(x + 3)$, i.e. $4x + 12$. This is the part to check before moving on, because it keeps the answer tied to the original question.
- Notation trap: f^{-1} is the inverse function, not the reciprocal. E.g. for $f(x) = 2x, f^{-1}(x) = \frac{x}{2} \neq \frac{1}{2x}$.
- One steady path is: $x = y^3 + 1 \Rightarrow x - 1 = y^3 \Rightarrow y = \sqrt[3]{x-1}$. Cube roots are one-to-one on all reals, so no domain restriction needed. That gives a quick check on the answer.
- Start with the key idea: f^{-1} reverses the arrows. The arrow into 7 comes from 3, so $f^{-1}(7) = 3$. (Reading the diagram backward is exactly what an inverse does.) That gives a quick check on the answer.
- Set $c = \frac{9}{5}(F - 32)$ and solve for F . Multiply both sides by $\frac{5}{9}$: $\frac{9}{5}c = F - 32$, so $F = \frac{9}{5}c + 32$. At $c = 25$: $\frac{9}{5}(25) + 32 = 45 + 32 = 77$. So $25^\circ C = 77^\circ F$. (Quick sanity-check: freezing $0^\circ C \rightarrow 32^\circ F$. Formula works.) The plot at right shows a generic function-inverse pair $f(x) = \frac{1}{2}x + 1$ and $f^{-1}(x) = 2x - 2$ reflecting across the dashed line $y = x$ —the universal picture of an inverse pair.
- Solve $b = 12n - 30$ for n : $b + 30 = 12n$, so $n = \frac{b+30}{12}$. The inverse takes a bill b and returns the number of tickets. At $b = 90$: $\frac{120}{12} = 10$ tickets. (Bills divisible by 12 after adding 30 give whole-ticket counts, which is what the real context demands.)

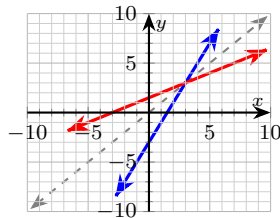
Answer graph





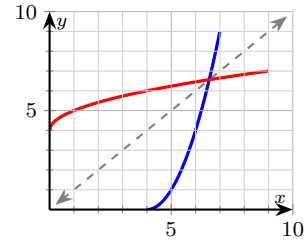
23. One steady path is: $f(g(x)) = 2 \cdot \frac{x+3}{2} - 3 = (x+3) - 3 = x$.
 $g(f(x)) = \frac{(2x-3)+3}{2} = \frac{2x}{2} = x$. Both compositions return the input, so they are inverses. Visually, the two lines reflect across $y = x$ — a 45° flip. The slopes are reciprocals: 2 for f , $\frac{1}{2}$ for g . That's always true of linear inverses. That gives a quick check on the answer.

Answer graph

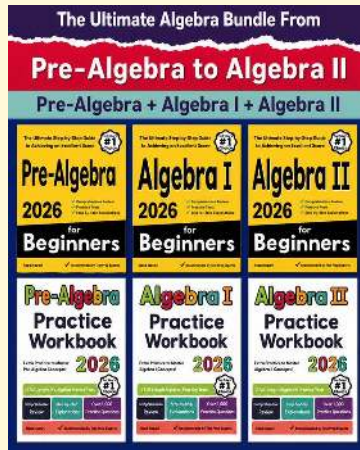


24. Swap variables: $x = (y - 4)^2$. Since the original $x \geq 4$ gives $y - 4 \geq 0$, take the positive square root: $\sqrt{x} = y - 4$, so $y = 4 + \sqrt{x}$. Domain of f^{-1} is the range of f , namely $[0, \infty)$. Check at $x = 5$: $f(5) = 1$ and $f^{-1}(1) = 4 + 1 = 5$. Round trip returns the starting input. The restriction $x \geq 4$ on f was essential — without it, f wouldn't be one-to-one and the inverse wouldn't be a function.

Answer graph



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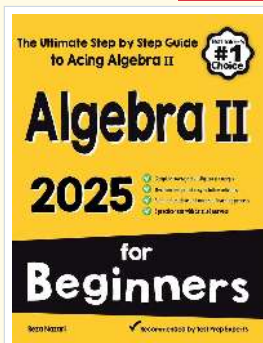
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