# Focus Vertex and Directrix of a Parabola

| Name: ____________________ | Date: ____________ | Score: ______ / 29 |
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## Quick Review

The locus definition of a parabola: it's the set of points *equidistant* from a fixed point (the **focus**) and a fixed line (the **directrix**). The vertex sits exactly halfway between them, on the axis of symmetry.

**Standard forms.** Two shapes to memorize:

**Vertical axis:** $(x-h)^2 = 4p(y-k)$. Vertex $(h, k)$. Focus $(h,\ k+p)$. Directrix $y = k - p$. Opens up if $p > 0$, down if $p < 0$.

**Horizontal axis:** $(y-k)^2 = 4p(x-h)$. Vertex $(h, k)$. Focus $(h+p,\ k)$. Directrix $x = h - p$. Opens right if $p > 0$, left if $p < 0$.

**The $4p$ trap.** The number on the right side is $4p$, not $p$. If you see $x^2 = 12y$, that's $4p = 12$, so $p = 3$ – not $p = 12$. Always divide by $4$.

**Building from focus and directrix.** The vertex is the midpoint between the focus and the foot of the perpendicular from the focus to the directrix. $|p|$ is the distance from vertex to focus (which equals the distance from vertex to directrix). Sign of $p$: positive if the focus is above/right of the vertex, negative if below/left.

**Focus above/below the directrix? Pick the orientation.** If the focus and directrix have the same $x$-coordinate axis, the parabola is vertical (focus directly above or below). If they share a $y$-coordinate axis, the parabola is horizontal.

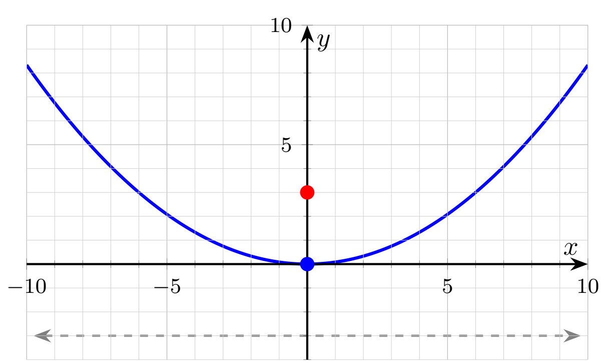
**Common slips.** Forgetting that the directrix is on the *opposite* side of the vertex from the focus – never on the same side. Using $p$ where you needed $4p$ (or vice versa). Putting the directrix through the vertex (impossible – they're never coincident).

## PRACTICE

*For each parabola, identify the vertex, focus, and directrix. The focus is a single point; the directrix is a line. They sit on opposite sides of the vertex at equal distance.*

**1.** For $(x-h)^2 = 4p(y-k)$ with $p > 0$, the focus is at: ________

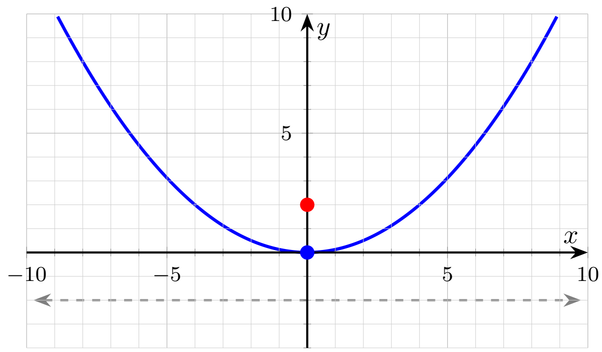
**2.** For the parabola $x^2 = 12y$, find the focus. ________




**3.** For the parabola $y^2 = 8x$, find the directrix. ________

**4.** The parabola $(y+1)^2 = -4(x-3)$ opens which way? ________

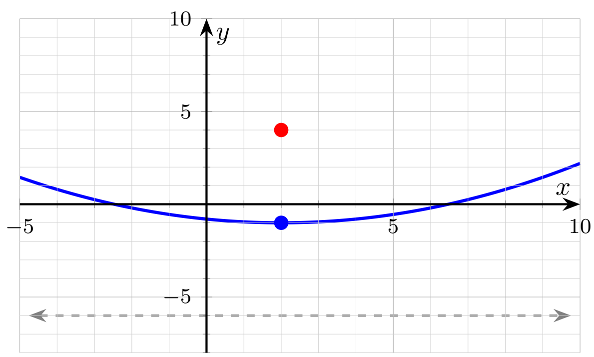
5. A parabola has focus $(0, 2)$ and directrix $y = -2$. Find its equation. ________




6. For the parabola $(x-2)^2 = 12(y+1)$, find the focus. ________

7. For $(y+3)^2 = -16(x-1)$, find the focus. ________

8. A parabola has vertex $(2, -1)$ and focus $(2, 4)$. Find its equation. ________




9. For the parabola $y^2 = 12x$, find the focus. ________

10. A parabola has vertex at the origin and focus at $(0, 4)$. Find the directrix. ________

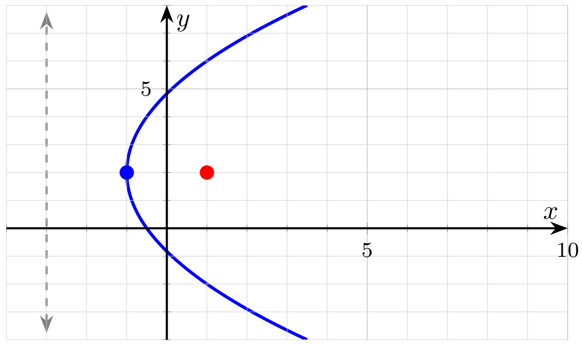
11. Find the directrix of $(x-2)^2 = 12(y+1)$. ________

12. Mark TRUE or FALSE: For any parabola, the focus lies on the directrix. ________

13. Write the equation of the parabola with vertex $(0, 0)$ and focus $(0, -5)$. ________

14. For $(y-2)^2 = 8(x+1)$, find the vertex. ________

**15.** For $(y-2)^2 = 8(x+1)$, find the focus. ________




**16.** A parabola has focus $(3, 0)$ and directrix $x = -3$. Find its equation. ________

**17.** Find the vertex and focus of $y^2 = -8x$. ________

**18.** Mark TRUE or FALSE: The vertex of a parabola is the midpoint of the segment from the focus to the directrix along the axis of symmetry. ________

**19.** For $(x+1)^2 = -8(y-3)$, find the directrix. ________

**20.** A parabola has vertex $(0, 0)$, opens to the right, and its focus is $5$ units from the vertex. Find its equation. ________

## ♦ Word Problems

**21.** A satellite dish's cross-section is a parabola with its vertex at the origin and opens upward. The receiver (which catches the reflected signal) sits at the focus, $6$ inches above the vertex. Find the equation of the cross-section and the equation of the directrix. ____

**22.** A car headlight has a parabolic reflector. The light bulb is placed at the focus so that the rays bounce out parallel to the axis. If the parabola has vertex at the origin and opens to the right, and the bulb is $3$ cm from the vertex, find the equation of the reflector and the equation of the directrix. ____

**23.** A parabolic arch over a roadway has its vertex at the top of the arch. The arch's focus sits $6$ feet below the vertex. Set up coordinates with the vertex at the origin so the parabola opens downward. Find the equation of the arch and its directrix. ____

**24.** A flashlight bulb sits at the focus of a parabolic mirror. Coordinates are set up with the vertex of the mirror at the origin, and the mirror opens to the right (away from the user). The bulb sits at $(2, 0)$. The opening of the flashlight (where the light emerges) is at $x = 10$ inches. How wide is the opening? ____

## Additional Practice

**25.** Center and radius of $(x-3)^2 + (y+2)^2 = 25$. ________

**26.** Write a circle with center $(0, 0)$ and radius $7$. ________

**27.** Find the radius of $x^2 + y^2 = 64$. ________

**28.** Find the center of $(x+5)^2 + (y-1)^2 = 9$. ________

**29.** Vertex of $y = (x-4)^2 + 6$. ________

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# Answer Keys

1. $(h,\ k+p)$
2. $(0,\ 3)$
3. $x=-2$
4. left
5. $x^2=8y$
6. $(2,\ 2)$
7. $(-3,\ -3)$
8. $(x-2)^2=20(y+1)$
9. $(3,\ 0)$
10. $y=-4$
11. $y=-4$
12. FALSE
13. $x^2=-20y$
14. $(-1,\ 2)$
15. $(1,\ 2)$
16. $y^2=12x$
17. vertex $(0,0)$; focus $(-2,0)$
18. TRUE
19. $y=5$
20. $y^2=20x$
21. $x^2=24y$; directrix $y=-6$
22. $y^2=12x$; directrix $x=-3$
23. $x^2=-24y$; directrix $y=6$
24. opening width: $4\sqrt{20}=8\sqrt{5}\approx 17.9$ in

### Additional Practice Answers

25. $(3,-2),\ r=5$
26. $x^2+y^2=49$
27. $8$
28. $(-5,1)$
29. $(4,6)$

**Additional Practice:** Answers for all numbered items, including the added practice, are shown in the grid above.

## Step-by-Step Explanations

**1.** In the vertical-axis form $(x-h)^2=4p(y-k)$, the vertex is $(h,k)$ and the focus sits on the axis of symmetry $p$ units from the vertex. With $p>0$ the parabola opens upward, so the focus is above the vertex at $(h,\ k+p)$.

**2.** Compare to $x^2=4py$: $4p=12$, so $p=3$ (not 12). Vertex at the origin, opens upward. Focus is $p=3$ units above: $(0,3)$. The dashed line at $y=-3$ is the directrix.

**3.** Compare $y^2=8x$ to $y^2=4px$: $4p=8$, so divide by 4 to get $p=2$ (not 8 – the $4p$ trap). The $y$ is squared, so this is a horizontal-axis parabola opening right from the origin. The directrix is the vertical line on the opposite side: $x=-p=-2$.

**4.** Horizontal-axis form $(y-k)^2=4p(x-h)$ with $4p=-4$, so $p=-1<0$. Horizontal parabola, $p<0$: opens to the left. Vertex $(3,-1)$.

**5.** Vertex sits midway: $\left(0,\dfrac{2+(-2)}{2}\right)=(0,0)$. Distance from vertex to focus: $p=2$ (focus above vertex, so $p>0$). Use $(x-h)^2=4p(y-k)$ at origin: $x^2=4(2)y=8y$.

**6.** The form $(x-2)^2=12(y+1)$ matches $(x-h)^2=4p(y-k)$, so the vertex is $(2,-1)$. From $4p=12$, divide by 4 to get $p=3$. The $x$ is squared, so the axis is vertical, and $p>0$ means it opens upward. The focus is $p=3$ units above the vertex: $(2,\ -1+3)=(2,2)$.

**7.** From $(y+3)^2=-16(x-1)$, the vertex is $(1,-3)$. The $y$ is squared, so the axis is horizontal; $4p=-16$ gives $p=-4$. A negative $p$ on a horizontal parabola means it opens left. The focus is $p$ units in the $+x$ direction from the vertex: $(1+(-4),\ -3)=(-3,-3)$, i.e. 4 units to the left.

**8.** Same $x$-coordinate – vertical axis. $p=4-(-1)=5$ (focus above vertex). Vertical form: $(x-2)^2=4(5)(y+1)=20(y+1)$.

**9.** Since $y$ is squared, compare $y^2=12x$ to $y^2=4px$: $4p=12$, so $p=3$. The vertex is at the origin and $p>0$ opens it to the right. The focus is $p=3$ units right of the vertex: $(3,0)$.

**10.** The focus $(0,4)$ is directly above the vertex $(0,0)$, so the axis is vertical and $p$ is the vertical distance from vertex to focus: $p=4$. The directrix is the same distance on the opposite side of the vertex: $y=k-p=0-4=-4$.

**11.** From $(x-2)^2=12(y+1)$, the vertex is $(2,-1)$ and $4p=12$ gives $p=3$; the $x$ is squared and $p>0$, so it opens up. The directrix sits opposite the focus, $p=3$ units *below* the vertex: $y=k-p=-1-3=-4$.

**12.** The focus is a point; the directrix is a line. They are always on opposite sides of the vertex – the vertex sits midway between them. They never touch.

**13.** The focus $(0,-5)$ is directly below the vertex $(0,0)$, so the axis is vertical and $p$ is negative (focus below): $p=-5$. Use $(x-h)^2=4p(y-k)$ at the origin: $x^2=4(-5)y=-20y$. The negative right side confirms it opens downward.

**14.** Match $(y-2)^2=8(x+1)$ to $(y-k)^2=4p(x-h)$. The $(y-2)$ gives $k=2$, and $(x+1)=(x-(-1))$ gives $h=-1$ (sign flip). Vertex $(h,k)=(-1,2)$.

**15.** One steady path is: $4p=8$, so $p=2>0$. Horizontal axis, opens right. Focus is $p=2$ units to the right of vertex $(-1,2)$: $(1,2)$. Directrix is vertical line $x=-1-2=-3$. That gives a quick check on the answer.

**16.** The vertex sits midway between focus $(3,0)$ and directrix $x=-3$: $\left(\dfrac{3+(-3)}{2},0\right)=(0,0)$. The focus is right of the vertex, so the axis is horizontal with $p=3$. Use $(y-k)^2=4p(x-h)$ at the origin: $y^2=4(3)x=12x$.

**17.** Since $y$ is squared, compare $y^2=-8x$ to $y^2=4px$: $4p=-8$, so $p=-2$. The vertex is at the origin. A negative $p$ opens the parabola left, so the focus is 2 units to the left: $(0+(-2),0)=(-2,0)$.

**18.** By definition, the vertex is equidistant from the focus and the directrix. Along the axis of symmetry, that makes it the midpoint of the perpendicular segment between them.

**19.** From $(x+1)^2=-8(y-3)$, the vertex is $(-1,3)$ and $4p=-8$ gives $p=-2$. The $x$ is squared with $p<0$, so it opens down and the directrix is above the vertex: $y=k-p=3-(-2)=5$.

**20.** Opening right means a horizontal axis with positive $p$, and "5 units from the vertex" gives $p=5$. Use $(y-k)^2=4p(x-h)$ at the origin: $y^2=4(5)x=20x$. (Focus $(5,0)$, directrix $x=-5$.)

**21.** Vertical axis, opens upward, $p=6$. Use $(x-0)^2=4(6)(y-0)$: $x^2=24y$. Directrix: $y=k-p=0-6=-6$. **Sanity check:** the point on the parabola directly across from the focus – the "latus rectum" endpoints – should be $|4p|/2=12$ inches on each side. Plug $y=6$ into $x^2=24y$: $x^2=144$, $x=\pm12$ ✓. That's the actual opening width of the dish at receiver height.

**22.** Horizontal axis (opens right), $p=3$. Use $(y-0)^2=4(3)(x-0)$: $y^2=12x$. Directrix: $x=h-p=0-3=-3$. (The reflective property – any ray from the focus bounces off the parabola in a direction parallel to the axis – is why parabolic reflectors are used for headlights, flashlights, and satellite dishes alike. The mirror surface is a paraboloid in 3-D, but each cross-section is a parabola.)

**23.** Vertex at the origin, parabola opens downward (arch shape). Focus sits 6 ft below the vertex at $(0,-6)$, so $p=-6$ (negative because the focus is below). Use $(x-h)^2=4p(y-k)$ at the origin: $x^2=4(-6)y=-24y$. The directrix sits on the opposite side of the vertex from the focus, at the same distance: $y=k-p=0-(-6)=6$. **Sanity check:** pick a point on the arch, say $y=-6$ (level with the focus): $x^2=-24(-6)=144$, so $x=\pm12$. The arch is 24 ft wide at focus-height, a reasonable overpass span. (In a real-world design problem, the road height under the arch would be set separately from the focus position.)

**24.** Horizontal axis, opens right. $p=2$ (focus at $(2,0)$ is 2 units right of vertex). Equation: $y^2=4(2)x=8x$. At $x=10$: $y^2=80$, so $y=\pm\sqrt{80}=\pm4\sqrt{5}$. The opening stretches from $-4\sqrt{5}$ to $+4\sqrt{5}$, giving width $8\sqrt{5}\approx17.9$ inches. (Real flashlights aren't quite this wide, but the principle scales – a deeper parabola gives a narrower, more focused beam, while a shallower one gives a wider flood.)

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