

Finding Zeros of Polynomials

Name: _____ Date: _____ Score: _____ / 28

Quick Review

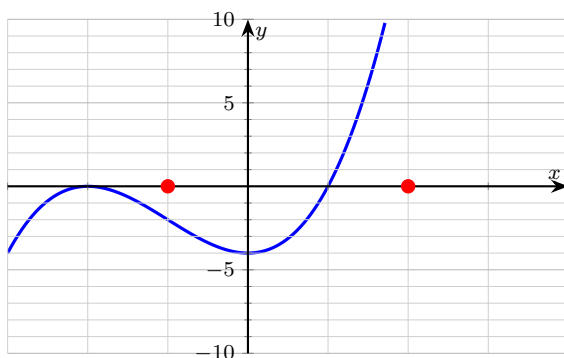
A **zero** of a polynomial $f(x)$ is a value of x that makes $f(x) = 0$. Graphically, zeros are the x -intercepts.

If f is already factored, use the **zero-product property**: a product is zero exactly when at least one factor is zero. So $(x - 3)(x + 5) = 0$ gives $x = 3$ or $x = -5$.

For a quadratic in standard form, factor and apply the zero-product property: $x^2 - 5x + 6 = (x - 2)(x - 3) = 0$ has zeros $x = 2, 3$.

For cubics and higher, use the **Rational Root Theorem**: any rational zero of $f(x)$ has the form $\pm \frac{p}{q}$ where p | constant term and q | leading coefficient. Test each candidate by plugging in. Once you find a zero c , divide out $(x - c)$ (synthetic division) and factor the resulting depressed polynomial.

Multiplicity: if a factor $(x - c)$ appears with exponent k , then c is a zero of multiplicity k . Graphically: multiplicity 1 means the graph crosses; multiplicity 2 means it touches and bounces back (doesn't cross); multiplicity ≥ 3 flattens through. The graph of $f(x) = (x - 2)^2(x + 1)$ below illustrates: $x = 2$ is a double zero (touches), $x = -1$ is a simple zero (crosses).



The **Fundamental Theorem of Algebra** says a degree- n polynomial has exactly n complex zeros (counted with multiplicity). Graphs show only the real ones — complex zeros come in conjugate pairs $a \pm bi$ and don't appear as x -intercepts. So a degree-4 polynomial can have 0, 2, or 4 real zeros (and the rest are complex).

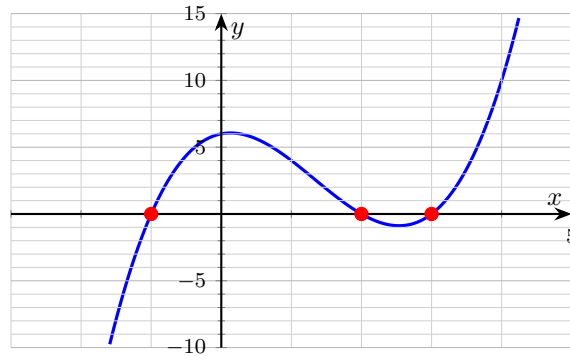
PRACTICE

Find all real zeros of each polynomial, including multiplicities when stated.

1. Zeros of $f(x) = (x - 3)(x + 5)$ _____
2. Zeros of $g(x) = x^2 - 5x + 6$ _____
3. Rational root candidates for $2x^3 + 3x^2 - 8x + 3$ _____
4. Find a zero of $f(x) = x^3 - 3x^2 + 2$ _____
5. Given that $x = 2$ is a zero of $f(x) = x^3 - 6x^2 + 11x - 6$, find the other zeros _____
6. Zeros of $f(x) = (x - 2)^2(x + 1)$, with multiplicity _____



7. All real zeros of $f(x) = x^3 - 4x^2 + x + 6$ from its graph below: _____

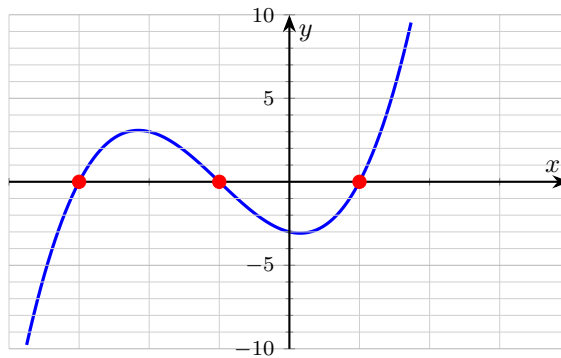


8. If $x = -2$ is a zero of $p(x) = x^3 + kx^2 - 4x - 12$, find k _____

9. Real zeros of $f(x) = x^3 - 2x^2 + 4x - 8$ _____

10. Zeros of $f(x) = x^2(x - 3)(x + 2)$ _____

11. The graph shows a cubic with three distinct real zeros. State its zeros: _____

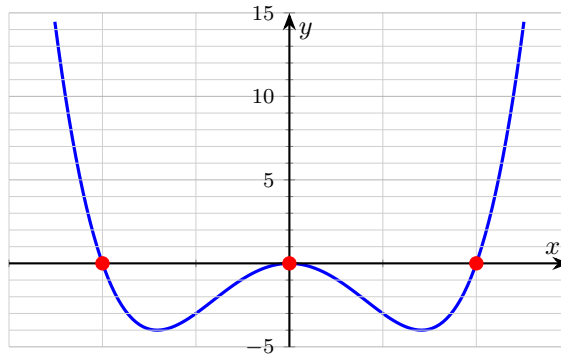


12. Real zero of $f(x) = x^3 + 8$ _____

13. Number of real zeros of $f(x) = x^4 + 1$ _____



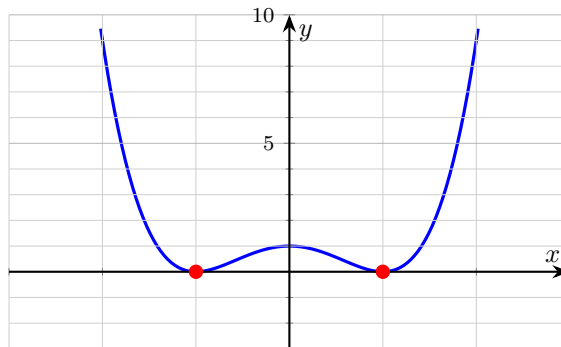
14. The graph below has x-intercepts; list all real zeros: _____



15. Find all zeros of $f(x) = x^3 + x^2 - 4x - 4$ _____

16. Identify the multiplicity of $x = 4$ in $f(x) = (x - 4)^3(x + 1)$ _____

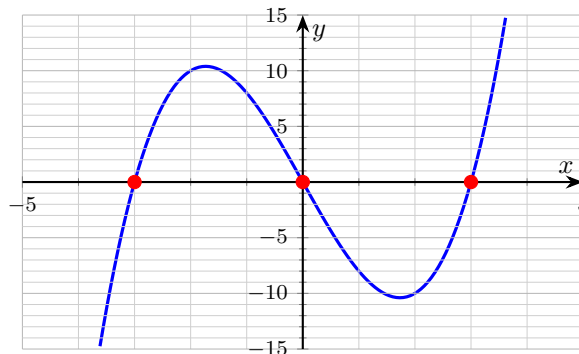
17. The graph below shows a polynomial with a double zero. Identify the zeros and their multiplicities: _____



18. Zeros of $f(x) = 2(x - 1)(x + 3)(x - 5)$ _____

19. Use the Fundamental Theorem to state how many complex zeros (with multiplicity) $x^5 - 7x^2 + 1$ has _____

20. Real zeros of the polynomial shown below: _____



◆ Word Problems

21. A polynomial $p(x) = x^3 + x^2 - 9x - 9$ models the net profit (in thousands) of a startup over time. Find all real values of x where the profit is zero. _____
22. A box's volume polynomial is $V(x) = x^3 - 6x^2 + 11x - 6$, and $V(x) = 0$ corresponds to a degenerate box. Find the values of x where the box collapses. _____
23. A polynomial $p(x)$ has zeros at $x = -2, 1$ (double root), and 4 , with leading coefficient 1 . Write $p(x)$ in factored form and expand to standard form. _____
24. For the polynomial $p(x) = x^4 - 5x^2 + 4$, find all real zeros. _____

Additional Practice

25. Write $3x - 5 + x^3$ in standard form. _____
26. Find the degree of $7x^4 - 2x^2 + 9$. _____
27. Add $(2x^2 + 3x - 1) + (x^2 - 5x + 4)$. _____
28. Subtract $(5x^2 - x + 6) - (2x^2 + 3x - 1)$. _____



Answer Keys

- | | |
|---|---------------------------------------|
| 1. $x = 3, x = -5$ | 13. 0 |
| 2. $x = 2, x = 3$ | 14. $x = -2, x = 0, x = 2$ |
| 3. $\pm 1, \pm 3, \pm \frac{1}{2}, \pm \frac{3}{2}$ | 15. $x = -1, x = 2, x = -2$ |
| 4. $x = 1$ | 16. 3 |
| 5. $x = 1, x = 3$ | 17. $x = -1, 1$ (each multiplicity 2) |
| 6. $x = 2$ (mult. 2), $x = -1$ (mult. 1) | 18. $x = 1, x = -3, x = 5$ |
| 7. $x = -1, x = 2, x = 3$ | 19. 5 |
| 8. $k = 3$ | 20. $x = -3, x = 0, x = 3$ |
| 9. $x = 2$ | 21. $x = -3, x = -1, x = 3$ |
| 10. $x = 0$ (mult. 2), $x = 3, x = -2$ | 22. $x = 1, x = 2, x = 3$ |
| 11. $x = -3, x = -1, x = 1$ | 23. $p(x) = (x + 2)(x - 1)^2(x - 4)$ |
| 12. $x = -2$ | 24. $x = -2, -1, 1, 2$ |
- Additional Practice Answers**
- | | |
|--------------------|---------------------|
| 25. $x^3 + 3x - 5$ | 27. $3x^2 - 2x + 3$ |
| 26. 4 | 28. $3x^2 - 4x + 7$ |

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

- A careful way to see it: Zero-product: $x - 3 = 0$ or $x + 5 = 0$. This is the part to check before moving on, because it keeps the answer tied to the original question.
- Factor the quadratic: two numbers with product 6 and sum -5 are $-2, -3$, so $(x - 2)(x - 3) = 0$. By the zero-product property, $x - 2 = 0$ or $x - 3 = 0$, giving $x = 2, 3$.
- One steady path is: p divides 3: $\pm 1, \pm 3$. q divides 2: $\pm 1, \pm 2$. Candidates: $\pm 1, \pm 3, \pm \frac{1}{2}, \pm \frac{3}{2}$. That gives a quick check on the answer.
- Test $x = 1$: $f(1) = 1 - 3 + 2 = 0 \checkmark$. So $x = 1$ is a zero. Other zeros require dividing out $(x - 1)$ and solving the resulting quadratic.
- Since $x = 2$ is a zero, divide out $(x - 2)$. Synthetic division with $c = 2$ and $1, -6, 11, -6$ gives bottom row $1, -4, 3, 0$ (remainder 0 confirms the zero). The quotient $x^2 - 4x + 3$ factors as $(x - 1)(x - 3)$, so the other zeros are $x = 1$ and $x = 3$.
- Factor $(x - 2)$ appears twice, so $x = 2$ has multiplicity 2. Factor $(x + 1)$ appears once, so $x = -1$ has multiplicity 1. Graphically: $x = 2$ touches the axis but doesn't cross; $x = -1$ crosses straight through.
- Rational candidates $\pm 1, \pm 2, \pm 3, \pm 6$. Test $x = -1$: $f(-1) = -1 - 4 - 1 + 6 = 0 \checkmark$. Synthetic divide by $c = -1$: quotient $x^2 - 5x + 6 = (x - 2)(x - 3)$. So zeros at $-1, 2, 3$ — matching the red dots.
- If $x = -2$ is a zero, then $p(-2) = 0$. Substitute: $(-2)^3 + k(4) - 4(-2) - 12 = -8 + 4k + 8 - 12 = 4k - 12$. Set $4k - 12 = 0$, so $k = 3$.
- Group: $x^2(x - 2) + 4(x - 2) = (x - 2)(x^2 + 4)$. The first factor gives $x = 2$; the second gives $x^2 = -4$, so $x = \pm 2i$ (complex, not real). Only real zero: $x = 2$.
- The factor $x = x^1$ appears twice (from x^2), giving $x = 0$ with multiplicity 2. The others are simple.
- Reading the red dots from the graph: $x = -3, -1, 1$. (The polynomial is $f(x) = x^3 + 3x^2 - x - 3 = (x + 3)(x + 1)(x - 1)$.)
- Sum of cubes: $x^3 + 8 = (x + 2)(x^2 - 2x + 4)$. Real zero: $x = -2$. The quadratic factor has discriminant $4 - 16 = -12 < 0$, so its two zeros are complex.
- A careful way to see it: $x^4 + 1 > 0$ for all real x (sum of a nonneg and a positive). All four zeros are complex. That gives a quick check on the answer.
- Factor: $f(x) = x^4 - 4x^2 = x^2(x^2 - 4) = x^2(x - 2)(x + 2)$. Zeros: $x = 0$ (mult. 2), $x = 2, -2$ (each mult. 1).
- One steady path is: Group: $x^2(x + 1) - 4(x + 1) = (x + 1)(x^2 - 4) = (x + 1)(x - 2)(x + 2)$. Zeros: $-1, 2, -2$. That gives a quick check on the answer.
- Start with the key idea: Factor $(x - 4)$ appears with exponent 3. This is the part to check before moving on, because it keeps the answer tied to the original question.
- The polynomial is $f(x) = x^4 - 2x^2 + 1 = (x^2 - 1)^2 = (x - 1)^2(x + 1)^2$. Both $x = 1$ and $x = -1$ have multiplicity 2 — the graph touches the axis at each but doesn't cross.
- Zero-product: each factor in turn. The leading coefficient 2 doesn't affect zeros.
- Degree $n \Rightarrow$ exactly n complex zeros, counted with multiplicity. Real zeros are a subset.
- The polynomial is $f(x) = x^3 - 9x = x(x - 3)(x + 3)$. Real zeros: $-3, 0, 3$. Notice the odd symmetry of the curve — f is an odd function.
- Group: $x^2(x + 1) - 9(x + 1) = (x + 1)(x^2 - 9) = (x + 1)(x - 3)(x + 3)$. Real zeros: $x = -3, -1, 3$. (In context, only the positive value $x = 3$ might be physically meaningful, depending on the model's domain.)
- Try $x = 1$: $V(1) = 1 - 6 + 11 - 6 = 0 \checkmark$. Synthetic divide by $c = 1$: quotient $x^2 - 5x + 6 = (x - 2)(x - 3)$. So zeros at $1, 2, 3$. All three are positive and physically realizable as degenerate-box dimensions.
- From the zero list: $(x + 2), (x - 1)^2$ (double root), $(x - 4)$. So $p(x) = (x + 2)(x - 1)^2(x - 4)$. Expanded: first multiply $(x - 1)^2 = x^2 - 2x + 1$, then $(x + 2)(x - 4) = x^2 - 2x - 8$. Product of two quadratics: $(x^2 - 2x + 1)(x^2 - 2x - 8)$. Let $u = x^2 - 2x$ to see the pattern: $(u + 1)(u - 8) = u^2 - 7u - 8 = (x^2 - 2x)^2 - 7(x^2 - 2x) - 8 = x^4 - 4x^3 + 4x^2 - 7x^2 + 14x - 8 = x^4 - 4x^3 - 3x^2 + 14x - 8$.
- Treat as a quadratic in $u = x^2$: $u^2 - 5u + 4 = (u - 1)(u - 4) = 0$, so $u = 1$ or $u = 4$. Then $x^2 = 1 \Rightarrow x = \pm 1$; $x^2 = 4 \Rightarrow x = \pm 2$. Four real zeros: $-2, -1, 1, 2$. (All real because both u -values are positive.)



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