Finding Determinants of a Matrix

Name: ______________________	Date: ______________	Score: _____ / 33

Quick Review

The **determinant** is a single number computed from a *square* matrix — and only a square matrix. It tells you whether the matrix is invertible (nonzero determinant ⇒ invertible) and, geometrically, gives the factor by which the matrix scales area (in 2D) or volume (in 3D).

2×2 **formula:** for $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $\det(A) = ad - bc$. Main diagonal product minus off-diagonal product. Two traps: signs (especially when entries are already negative — two negatives on the main diagonal multiply to a *positive*), and order (the subtraction $ad - bc$ is fixed; $bc - ad$ flips the sign).

3×3 **by cofactor expansion** (along row 1): $\det(A) = a_{1,1}\, M_{1,1} - a_{1,2}\, M_{1,2} + a_{1,3}\, M_{1,3}$, where each $M_{1,j}$ is the 2×2 minor (the matrix left after crossing out row 1 and column j). The signs alternate $+, -, +$ across the row.

Triangular shortcut: if a matrix is upper or lower triangular (zeros above or below the main diagonal), its determinant is just the product of the diagonal entries. **Singular matrix:** $\det(A) = 0$. Singular matrices have no inverse and represent systems without a unique solution. The condition $\det(A) = 0$ also signals that the rows (and columns) are linearly dependent — one is a scalar multiple of another, or some combination of others.

PRACTICE

Find each determinant. Use the 2×2 formula or cofactor expansion as appropriate.

1. Find the determinant of the matrix laid out below: $\det \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$. ________

	Col 1	Col 2
Row 1	3	2
Row 2	1	4

2. $\det \begin{bmatrix} 5 & 0 \\ 7 & 3 \end{bmatrix}$ ________

3. $\det \begin{bmatrix} -2 & 5 \\ 3 & -4 \end{bmatrix}$ ________

4. $\det \begin{bmatrix} 6 & -1 \\ 2 & 5 \end{bmatrix}$ ________

5. k such that $\det \begin{bmatrix} 2 & 4 \\ k & 6 \end{bmatrix} = 0$ ________

6. $x > 0$ such that $\det \begin{bmatrix} x & 3 \\ 2 & x \end{bmatrix} = 10$ ________

7. $\det \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$ ________

Scan Me

8. Find the determinant of the 3×3 matrix laid out below: $\det \begin{bmatrix} 2 & 1 & 0 \\ 3 & -1 & 4 \\ 1 & 2 & 1 \end{bmatrix}$. ________

	Col 1	Col 2	Col 3
Row 1	2	1	0
Row 2	3	-1	4
Row 3	1	2	1

9. Find $\det(M)$ where $M = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$ and interpret as an area scale factor. ________

	Col 1	Col 2
Row 1	2	1
Row 2	3	4

10. t such that $\det \begin{bmatrix} t & 6 \\ 2 & 5 \end{bmatrix} = -7$ ________

11. $\det \begin{bmatrix} 0 & 5 \\ -2 & 3 \end{bmatrix}$ ________

12. $\det \begin{bmatrix} 7 & 14 \\ 3 & 6 \end{bmatrix}$ ________

13. $\det \begin{bmatrix} -3 & -1 \\ -5 & -2 \end{bmatrix}$ ________

14. $\det \begin{bmatrix} 1 & 0 & 0 \\ 4 & 2 & 0 \\ -3 & 5 & 7 \end{bmatrix}$ ________

15. $\det \begin{bmatrix} 3 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 0 & 5 \end{bmatrix}$ ________

16. $\det \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \det \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$ ________

17. Find $\det(2A)$ if A is 2×2 with $\det(A) = 5$. ________

18. $\det \begin{bmatrix} 2 & 0 & 1 \\ 0 & 3 & 0 \\ 4 & 0 & 5 \end{bmatrix}$ ________

19. $\det(A^T)$ if $\det(A) = 7$ ________

20. Is $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ invertible? ________

Scan Me

◆ Word Problems

21. A linear transformation $M = \begin{bmatrix} 3 & 1 \\ 2 & 5 \end{bmatrix}$ maps the unit square to a parallelogram. By what factor does M scale area, and is M invertible?

22. A 2×2 matrix $C = \begin{bmatrix} k & 2 \\ 6 & k+1 \end{bmatrix}$ models the costs in a system. For what value(s) of k does the system fail to have a unique solution? (Singular matrix means $\det = 0$.)

23. A network engineer represents three connected systems with $\det(A)$, $\det(B)$, and the combined transformation AB. Given $\det(A) = 4$ and $\det(B) = -3$, find $\det(AB)$. (Use the identity $\det(AB) = \det(A)\det(B)$.)

24. A triangular matrix arises naturally after Gaussian elimination. If row-reduction turns a 3×3 matrix A into an upper-triangular matrix with diagonal entries 2, -5, and 3 (with no row swaps), what is $\det(A)$?

Additional Practice

25. State the dimensions of $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$.

26. Add $\begin{bmatrix} 1 & 4 \\ 2 & 0 \end{bmatrix} + \begin{bmatrix} 3 & -1 \\ 5 & 6 \end{bmatrix}$.

27. Subtract $\begin{bmatrix} 7 & 2 \\ 1 & 5 \end{bmatrix} - \begin{bmatrix} 3 & 4 \\ -2 & 1 \end{bmatrix}$.

28. Find $\det \begin{bmatrix} 3 & 2 \\ 5 & 4 \end{bmatrix}$.

29. Find entry a_{21} in $\begin{bmatrix} 8 & 9 \\ -3 & 4 \end{bmatrix}$.

30. Can a 2×3 matrix multiply a 3×4 matrix?

31. Product size: $(2 \times 3)(3 \times 4)$.

32. Multiply $\begin{bmatrix} 2 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \end{bmatrix}$.

33. Find the identity matrix of order 2.

Answer Keys

1. $\boxed{10}$
2. $\boxed{15}$
3. $\boxed{-7}$
4. $\boxed{32}$
5. $\boxed{k=3}$
6. $\boxed{x=4}$
7. $\boxed{24}$
8. $\boxed{-17}$
9. $\boxed{5}$
10. $\boxed{t=1}$
11. $\boxed{10}$
12. $\boxed{0}$

13. $\boxed{1}$
14. $\boxed{14}$
15. $\boxed{30}$
16. $\boxed{-4}$
17. $\boxed{20}$
18. $\boxed{18}$
19. $\boxed{7}$
20. $\boxed{\text{no } (\det = 0)}$
21. $\boxed{\text{factor } 13, \text{ invertible}}$
22. $\boxed{k = -4 \text{ or } k = 3}$
23. $\boxed{-12}$
24. $\boxed{-30}$

Additional Practice Answers

25. $\boxed{2 \times 3}$
26. $\boxed{\begin{bmatrix} 4 & 3 \\ 7 & 6 \end{bmatrix}}$
27. $\boxed{\begin{bmatrix} 4 & -2 \\ 3 & 4 \end{bmatrix}}$
28. $\boxed{2}$

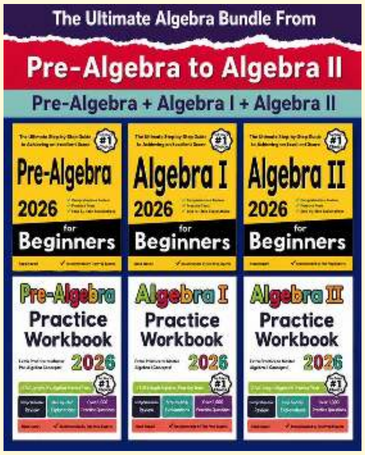
29. $\boxed{-3}$
30. $\boxed{\text{yes}}$
31. $\boxed{2 \times 4}$
32. $\boxed{13}$
33. $\boxed{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}$

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. For a 2×2 matrix, $\det = ad - bc$: multiply the main diagonal, then subtract the off-diagonal product. Here $(3)(4) - (2)(1) = 12 - 2 = 10$. Always do the main diagonal first so the subtraction order stays $ad - bc$, not $bc - ad$.
2. Keep the rule visible: $(5)(3) - (0)(7) = 15 - 0 = 15$. A zero entry doesn't make the determinant zero — the formula still applies. That gives a quick check on the answer.
3. One steady path is: $(-2)(-4) - (5)(3) = 8 - 15 = -7$. Two negatives on the main diagonal multiply to $+8$ — this sign step is where students slip. Then subtract $bc = 15$ to land at -7. That gives a quick check on the answer.
4. Start with the key idea: $(6)(5) - (-1)(2) = 30 - (-2) = 30 + 2 = 32$. The minus-a-negative in the second term flips to a plus. That gives a quick check on the answer.
5. A careful way to see it: $\det = 12 - 4k = 0$, so $4k = 12$ and $k = 3$. When $k = 3$, rows $(2, 4)$ and $(3, 6)$ are scalar multiples ($\times 1.5$) — linearly dependent, which is exactly what $\det = 0$ signals. That gives a quick check on the answer.
6. Apply $ad - bc$: $(x)(x) - (3)(2) = x^2 - 6$. Set $x^2 - 6 = 10$, so $x^2 = 16$ and $x = \pm 4$. A squared variable always gives two roots — the constraint $x > 0$ picks $x = 4$.
7. Upper triangular, so determinant is the product of diagonal entries: $1 \cdot 4 \cdot 6 = 24$. (No cofactor work needed — the shortcut earns its keep.)
8. Expand along row 1: $2 \cdot \det[-1, 4; 2, 1] - 1 \cdot \det[3, 4; 1, 1] + 0 \cdot (\ldots)$. The first minor: $(-1)(1) - (4)(2) = -9$. The second: $(3)(1) - (4)(1) = -1$. Combine: $2(-9) - 1(-1) + 0 = -18 + 1 = -17$. (The third term drops out because of the 0 entry.)
9. A careful way to see it: $\det(M) = (2)(4) - (1)(3) = 8 - 3 = 5$. The absolute value $|\det(M)| = 5$ is the area scale factor: M takes any region and multiplies its area by 5. That gives a quick check on the answer.
10. Build the determinant with $ad - bc$: $(t)(5) - (6)(2) = 5t - 12$. Set it equal to the target: $5t - 12 = -7$, so $5t = 5$ and $t = 1$. The trick is treating t as just another entry in the $ad - bc$ formula.
11. One steady path is: $(0)(3) - (5)(-2) = 0 - (-10) = 10$. The zero in the top-left makes $ad = 0$, but the off-diagonal contribution is nonzero — the whole determinant comes from $-bc$. That gives a quick check on the answer.
12. Start with the key idea: $(7)(6) - (14)(3) = 42 - 42 = 0$. Row 2 is a scalar multiple of row 1 (in fact, $\frac{3}{7}$ times). Whenever rows are proportional, $\det = 0$. That gives a quick check on the answer.
13. A careful way to see it: $(-3)(-2) - (-1)(-5) = 6 - 5 = 1$. Four negative entries; both products come out positive. Be careful tracking the signs — $(-1)(-5) = +5$, not -5. That gives a quick check on the answer.
14. Lower triangular this time — the same product-of-diagonals rule applies: $1 \cdot 2 \cdot 7 = 14$. (Above-diagonal or below-diagonal zeros both qualify for the shortcut.)
15. Upper triangular: $3 \cdot 2 \cdot 5 = 30$. Cofactor expansion would also work but takes longer; spot the triangular structure first.
16. Two separate 2×2 determinants. First: $(1)(4) - (2)(3) = 4 - 6 = -2$. Second: $(5)(8) - (6)(7) = 40 - 42 = -2$. Sum: $-2 + (-2) = -4$. (Note that determinants don't distribute over addition in general — $\det(A + B) \neq \det(A) + \det(B)$ usually — but you can always add determinants *after* computing them separately.)
17. For an $n \times n$ matrix, $\det(kA) = k^n \det(A)$. Here $n = 2$ and $k = 2$, so $\det(2A) = 2^2 \cdot 5 = 20$. (Each row picks up one factor of k, and there are n rows.)
18. Expand along row 2 (lots of zeros there): only the middle entry survives. $\det = 3 \cdot \det[2, 1; 4, 5] \cdot (-1)^{2+2} = 3 \cdot (10 - 4) = 3 \cdot 6 = 18$. (Picking the row or column with the most zeros makes cofactor expansion much faster.)
19. Transposing a matrix doesn't change its determinant: $\det(A^T) = \det(A) = 7$. (Rows of A^T are columns of A — the formula treats them symmetrically.)
20. Cofactor along row 1: $1(45 - 48) - 2(36 - 42) + 3(32 - 35) = 1(-3) - 2(-6) + 3(-3) = -3 + 12 - 9 = 0$. With $\det = 0$, A is singular — no inverse. (Geometrically, this A collapses 3D space onto a plane.)
21. Area scale factor is $|\det(M)|$. Compute $\det(M) = (3)(5) - (1)(2) = 15 - 2 = 13$. So every region scales up by 13. Because $\det \neq 0$, M is invertible — the transformation can be undone, and the reverse map shrinks areas by a factor of $\frac{1}{13}$.
22. Keep the rule visible: $\det(C) = k(k + 1) - 12 = k^2 + k - 12$. Set this to 0: $k^2 + k - 12 = 0$, factor $(k + 4)(k - 3) = 0$, so $k = -4$ or $k = 3$. At either value, C is singular — linearly dependent rows mean no unique solution. (Two roots is normal; the quadratic in k has two zeros.) That gives a quick check on the answer.
23. The determinant of a product equals the product of the determinants: $\det(AB) = \det(A)\det(B) = (4)(-3) = -12$. (Geometrically: if A scales area by 4 and B by -3 (with a flip), the composite scales by -12.)
24. Adding multiples of one row to another doesn't change the determinant. So $\det(A)$ equals the determinant of the upper-triangular result, which is the product of diagonals: $(2)(-5)(3) = -30$. (Row swaps *would* flip the sign, so the n̈o swaps̈detail matters.)

Build Algebra Confidence From Pre-Algebra Through Algebra II



The Complete Algebra Success Bundle

Pre-Algebra, Algebra I, and Algebra II in one clear path

Friendly lessons, focused practice, and full-review support for every stage.

Scan Me

Scan for the Bundle

6 Books
3 Courses
1 Path

Bundle Value: Six coordinated books help students review missing skills, learn new algebra topics, and practice until the steps feel natural.

Complete Course Path

- ✓ Starts with Pre-Algebra foundations
- ✓ Moves smoothly into Algebra I skills
- ✓ Extends learning through Algebra II topics
- ✓ Great for review, tutoring, and summer study

One bundle, one steady path.

Step-by-Step Lessons

- ✓ Plain-English explanations students can follow
- ✓ Worked examples that show every important step
- ✓ Common mistakes called out before they stick
- ✓ Skill-building practice after each lesson
- ✓ Helpful for independent study or class support

Less guessing. More understanding.

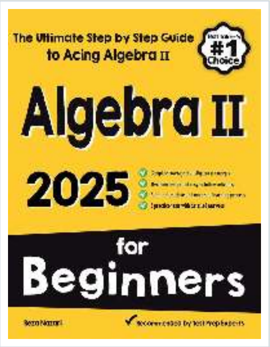
Practice That Sticks

- ✓ Matching practice workbooks for extra repetition
- ✓ Review sets to keep older skills fresh
- ✓ Answer explanations for checking thinking
- ✓ Strong support before tests and final exams
- ✓ Designed to build fluency and confidence

Practice today. Remember tomorrow.

STUDENT FAVORITE • Master Algebra II From the Ground Up

Algebra II for Beginners



Written by a top math teacher & aligned with national and state Algebra II courses. From polynomial functions to logarithms, trigonometry, and rational expressions — explained the easy way.

- ✓ **Complete coverage** of every Algebra II concept — perfect companion to these worksheets
- ✓ **Step-by-step explanations** with worked examples on every topic
- ✓ **QR codes in every chapter** for free video lessons & bonus practice
- ✓ **2 full-length practice tests** with detailed answer keys

✓ 100% Guaranteed
✓ Lifetime Support
✓ Trusted by Teachers

Start Your Algebra Journey Today! →

⋆ STUDENT'S #1 CHOICE ⋆

Teacher-recommended • 12,000+ Happy Students

↓PDF EDITION

Scan Me

Instant download • any device

PAPERBACK

Scan Me

Paperback on Amazon

Hold it in your hands

Pair these free worksheets with *Algebra II for Beginners* and you have a complete self-paced course — concept lessons, daily practice, and full exam-style reviews, all in one path. → **EffortlessMath.com/product/algebra-ii-for-beginners**