

Field Properties of Real and Complex Numbers

Name: _____ Date: _____ Score: _____ / 24

Quick Review

The real numbers form a **field**: a system where addition and multiplication both have identities (0 for +, 1 for ×), inverses (every real has $-a$; every *nonzero* real has $\frac{1}{a}$), and the usual commutative, associative, and distributive laws. **Zero has no multiplicative inverse** — that’s the one missing piece, because nothing times 0 equals 1. **Subtraction and division are not commutative**: $5 - 3 \neq 3 - 5$. The same field axioms hold for the complex numbers \mathbb{C} , but *not* for matrices — matrix multiplication isn’t commutative in general, and not every nonzero matrix has an inverse. **Order properties** (less-than, greater-than) work on \mathbb{R} but *not* on \mathbb{C} — complex numbers don’t have a natural ordering. When solving rational equations or inequalities, watch for values that make a denominator zero — those have to be excluded from the solution set.

PRACTICE

Solve, simplify, or identify the property at work.

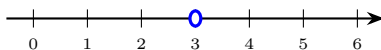
- Property: $(2x - 5)(x^2 + 1) + (7 - 2x)(x^2 + 1) = [(2x - 5) + (7 - 2x)](x^2 + 1)$ _____
- Multiplicative inverse of $\frac{x - 3}{x^2 - 9}$ _____
- Solve $\frac{2x - 5}{x + 1} \geq 3$ _____
- Is $L = \{ax + b : a, b \in \mathbb{R}\}$ closed under multiplication? _____
- Is L closed under addition? _____
- Does 0 have a multiplicative inverse in \mathbb{R} ? _____
- Commutativity asks whether order matters. Use the table to decide: is subtraction commutative on \mathbb{R} ? _____

a	b	$a - b$	$b - a$
5	3	2	-2
8	1	7	-7

- Additive identity in \mathbb{R} _____
- Multiplicative identity in \mathbb{R} _____
- The table pairs each number with its additive inverse (the value that adds to 0). Fill in the additive inverse of -7 . _____

a	additive inverse
3	-3
$\frac{1}{2}$	$-\frac{1}{2}$
-7	?

- Multiplicative inverse of $-\frac{2}{5}$ _____
- Solve $\frac{1}{x - 3} > 0$. The number line marks the critical value $x = 3$ (open, since the expression is undefined there). Decide which ray is the solution. _____



- What’s the mistake in: “solve $\frac{2x - 5}{x - 3} > 2$ by multiplying both sides by $x - 3$ ”? _____
- Is multiplication commutative on \mathbb{R} ? _____
- Solve $\frac{x + 2}{x - 1} = 3$ _____



16. Does \mathbb{C} have a natural order ($<$)? _____
17. Solve $\frac{x}{x-2} = \frac{4}{x-2}$ _____
18. Solve $\frac{3}{x} + \frac{2}{x} = 1$ _____
19. True or false: $0 \cdot a = 0$ for every $a \in \mathbb{R}$. _____
20. True or false: matrix multiplication is commutative _____

◆ Word Problems

21. A student tries to solve $\frac{2x-5}{x-3} > 2$ by multiplying both sides by $x-3$ and concludes that every real number except 3 is a solution. Explain the mistake, and find the correct solution. _____
22. Which field property justifies rewriting $5x + (3y + 2x)$ as $(5x + 2x) + 3y$? _____
23. Solve the rational equation $\frac{1}{x} + \frac{1}{x+1} = \frac{3}{4}$. State the restrictions and check your solution. _____
24. True or false (with reason): the set \mathbb{Z} of integers is closed under addition, subtraction, and multiplication, but not division. _____



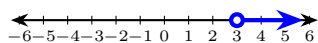
Answer Keys

- | | |
|--|---|
| <p>1. Distributive</p> <p>2. $x + 3 (x \neq \pm 3)$</p> <p>3. $[-8, -1]$</p> <p>4. no</p> <p>5. yes</p> <p>6. no</p> <p>7. no</p> <p>8. 0</p> <p>9. 1</p> <p>10. 7</p> <p>11. $-\frac{5}{2}$</p> <p>12. $x > 3$</p> | <p>13. sign of $x - 3$ unknown</p> <p>14. yes</p> <p>15. $x = \frac{5}{2}$</p> <p>16. no</p> <p>17. $x = 4$</p> <p>18. $x = 5$</p> <p>19. true</p> <p>20. false</p> <p>21. $x > 3$</p> <p>22. Commutative and associative (+)</p> <p>23. $x = \frac{5 + \sqrt{73}}{6}$ or $x = \frac{5 - \sqrt{73}}{6}$</p> <p>24. True</p> |
|--|---|

Step-by-Step Explanations

1. The factor $(x^2 + 1)$ is being pulled out of two terms. That's $ab + cb = (a + c)b$ in reverse — the distributive property, used to factor.
2. Reciprocal: $\frac{x^2 - 9}{x - 3} = \frac{(x - 3)(x + 3)}{x - 3} = x + 3$. But the original required $x \neq 3$ and $x \neq -3$, and those restrictions carry over.
3. Move everything to one side: $\frac{2x - 5}{x + 1} - 3 \geq 0 \Rightarrow \frac{-x - 8}{x + 1} \geq 0$. Critical values: -8 (numerator zero, included) and -1 (denominator zero, excluded). Sign chart shows the expression is ≥ 0 on $[-8, -1)$.
4. A counterexample is $x \cdot x = x^2$, which isn't linear. One counterexample is enough.
5. A careful way to see it: $(ax + b) + (cx + d) = (a + c)x + (b + d)$ — still linear. So yes. This is the part to check before moving on, because it keeps the answer tied to the original question.
6. No real number x satisfies $0 \cdot x = 1$. The field axiom requires every *nonzero* number to have an inverse, and zero is the lone exception.
7. Commutative would mean $a - b = b - a$ for every pair. But the table shows $5 - 3 = 2$ while $3 - 5 = -2$ — not equal. A single counterexample is enough to say no.
8. Start with the key idea: $a + 0 = a$ for every real a . Zero is the do-nothing number for addition. That gives a quick check on the answer.
9. A careful way to see it: $a \cdot 1 = a$ for every real a . One is the do-nothing number for multiplication. That gives a quick check on the answer.
10. The additive inverse of a is $-a$ — the number that brings it back to 0. For -7 that's $-(-7) = 7$. Check: $-7 + 7 = 0$. ✓
11. One steady path is: Flip and keep the sign: $-\frac{2}{5} \rightarrow -\frac{5}{2}$. Check: $-\frac{2}{5} \cdot -\frac{5}{2} = 1$. ✓ That gives a quick check on the answer.
12. The numerator 1 is always positive, so the fraction is positive exactly when the denominator is: $x - 3 > 0$, i.e. $x > 3$. That's the ray to the right of the open dot at 3.

Answer graph



13. Multiplying by an expression whose sign you don't know can flip the inequality without you noticing. Move to one side and use a sign chart instead.
14. Keep the rule visible: $ab = ba$ for all real a, b . (This breaks for matrices, but holds for real and complex numbers.) That gives a quick check on the answer.
15. Restriction: $x \neq 1$. Multiply: $x + 2 = 3(x - 1)$. Expand: $x + 2 = 3x - 3$. Subtract: $5 = 2x$, so $x = \frac{5}{2}$. (Not 1, so valid.)
16. There's no way to order \mathbb{C} that respects the field axioms. Is i greater than 0 or less? Either choice contradicts properties of multiplication.
17. Restriction: $x \neq 2$. Same denominator means the numerators must match: $x = 4$. (And $4 \neq 2$, so it's valid.)
18. Keep the rule visible: Restriction: $x \neq 0$. Combine the left: $\frac{5}{x} = 1$. Cross-multiply: $5 = x$. That gives a quick check on the answer.
19. The zero product property. Follows from the distributive axiom: $0 \cdot a = (0 + 0) \cdot a = 0 \cdot a + 0 \cdot a$, so $0 \cdot a = 0$.
20. In general $AB \neq BA$ for matrices. (Some special pairs do commute, but it's not a general rule like it is for real numbers.)
21. The error: $x - 3$ could be positive or negative, and multiplying by a negative flips the inequality direction. Safer approach — move everything to one side: $\frac{2x - 5}{x - 3} - 2 > 0 \Rightarrow \frac{1}{x - 3} > 0$. The numerator 1 is positive, so the fraction is positive only when the denominator is: $x - 3 > 0$, i.e., $x > 3$.
22. Two moves happen. Inside the parentheses, $3y + 2x = 2x + 3y$ (commutative property of addition). Then $5x + (2x + 3y) = (5x + 2x) + 3y$ (associative property of addition). Once regrouped, the like terms can combine to $7x + 3y$.
23. Restrictions: $x \neq 0$ and $x \neq -1$. Multiply both sides by $4x(x + 1)$: $4(x + 1) + 4x = 3x(x + 1)$. Expand: $4x + 4 + 4x = 3x^2 + 3x$, so $8x + 4 = 3x^2 + 3x$, then $3x^2 - 5x - 4 = 0$. Quadratic formula: $x = \frac{5 \pm \sqrt{25 + 48}}{6} = \frac{5 \pm \sqrt{73}}{6}$. Quick numeric check: $(5 + \sqrt{73})/6 \approx 2.26$, and $\frac{1}{2 \cdot 2.26} + \frac{1}{3 \cdot 2.26} \approx 0.443 + 0.307 = 0.75 = \frac{3}{4}$. ✓ Both values differ from 0 and -1 , so both are valid.
24. Adding, subtracting, or multiplying two integers always gives an integer. But dividing 1 by 2 gives $\frac{1}{2}$, which is not an integer — so \mathbb{Z} fails closure under division. (That's why \mathbb{Z} is a ring, not a field. \mathbb{Q}, \mathbb{R} , and \mathbb{C} are all fields.)



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