Factors and Greatest Common Factors

Name: ______________________ Date: ______________ Score: _____ / 37

Quick Review

A **factor** of an expression is anything that divides it evenly — no remainder, no negative exponents, no fractions left over. So $6xy$ is a factor of $24x^3y^2$ because $24x^3y^2/(6xy) = 4x^2y$, a clean polynomial. But $6x^4y$ is *not* a factor because dividing leaves $4x^{-1}y$, with a negative exponent.

The **greatest common factor (GCF)** is the biggest factor shared by all the expressions in a list. For numbers, take the prime factorization and pick the lowest power of each shared prime: $24 = 2^3 \cdot 3$, $36 = 2^2 \cdot 3^2$, $60 = 2^2 \cdot 3 \cdot 5$ share 2^2 and 3, so $\gcd = 12$.

For monomials, the GCF combines coefficient-GCF with the smallest exponent of each shared variable. If a variable doesn't appear in every monomial, it doesn't appear in the GCF. So x^3 and y^4 share no variable, GCF $= 1$.

Useful facts:

- 1 is a factor of every integer (and every polynomial).
- Every nonzero integer divides itself, so $\gcd(n, n) = n$.
- The GCF of two positives is at most the smaller number.
- **Factor Theorem**: $(x - c)$ is a factor of $p(x)$ if and only if $p(c) = 0$. This tells you whether a linear factor divides the polynomial without doing any long division.

Trap: don't confuse GCF (smallest powers, smaller number) with LCM (largest powers, larger number). They're opposites.

PRACTICE

Find the factor or GCF as requested.

1. Is $6xy$ a factor of $24x^3y^2$? ________
2. Common factor of $18x^4$ and $24x^2$ ________
3. GCF of $24, 36, 60$ ________
4. The table breaks two monomials into coefficient and exponents. Find the GCF of $12x^3y^2$ and $18x^2y^4$. ________

Monomial	Coef.	Power x	Power y
$12x^3y^2$	12	3	2
$18x^2y^4$	18	2	4

5. The table shows a few values of $f(x) = x^2 - 5x + 6$. Use the factor theorem to decide whether $(x - 2)$ is a factor (compute the value the table doesn't give you). ________

x	-1	0	1
$f(x)$	12	6	2

6. GCF of $6x^4y$ and $9x^2y^3$ ________
7. GCF of $12x^5,\ 18x^3,\ 30x^2$ ________
8. Does $p(-4) = 0$ prove $(x + 4)$ is a factor of $p(x)$? ________
9. GCF of $20x^4y^3$ and $30x^2y^5$ ________
10. All positive factors of 12 ________
11. GCF of $14a^2b$ and $35ab^3$ ________
12. GCF of x^3 and y^4 ________

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13. The table shows a few values of $f(x) = x^3 - 2x^2 + 5x - 4$. Use the factor theorem to decide whether $(x - 1)$ is a factor. ________

x	-1	0	2	3
$f(x)$	-12	-4	6	20

14. GCF of $8x^3,\ 12x^2y,\ 20xy^2$ ________

15. GCF of 9 and 16 ________

16. Is $5x$ a factor of $10x^2 + 15x$? ________

17. GCF of $21, 28, 42$ ________

18. Is x^2 a factor of $6x^3 + 9x$? ________

19. GCF of $4x^2 - 8$ written as factor extraction ________

20. Is $(x - 3)$ a factor of $x^2 - 7x + 12$? ________

♦ Word Problems

21. A worker arranges three batches of 12, 30, and 42 tiles into equal rows with no leftovers, using the largest possible row size that works for all three batches. How many tiles per row? ____

22. A bakery's daily output is $20x^4y^3$ small pastries and $30x^2y^5$ large pastries, where x and y track ingredient sourcing. Find the GCF of the two output expressions. ____

23. Use the Factor Theorem to decide whether $(x + 3)$ is a factor of $p(x) = x^3 - 3x + 2$, and explain. ____

24. The polynomial $p(x) = x^3 + 4x^2 - x - 4$ has $(x - 1)$ as a factor. Verify this with the Factor Theorem. ____

Additional Practice

25. Write $3x - 5 + x^3$ in standard form. ________

26. Find the degree of $7x^4 - 2x^2 + 9$. ________

27. Add $(2x^2 + 3x - 1) + (x^2 - 5x + 4)$. ________

28. Subtract $(5x^2 - x + 6) - (2x^2 + 3x - 1)$. ________

29. Multiply $(x + 4)(x - 3)$. ________

30. Factor $x^2 + 9x + 20$. ________

31. Factor $6x^2 + 9x$. ________

32. Find the GCF of $12x^3$ and $18x^2$. ________

33. Divide $(x^2 + 5x + 6)$ by $(x + 2)$. ________

34. Find the remainder when $x^2 - 1$ is divided by $x - 3$. ________

35. Zeros of $(x - 5)(x + 1)$. ________

36. Is $x = 2$ a zero of $x^2 - 4$? ________

37. Expand $(x - 2)^2$. ________

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Answer Keys

1. yes, with quotient $4x^2y$
2. $6x^2$
3. 12
4. $6x^2y^2$
5. yes
6. $3x^2y$
7. $6x^2$
8. yes (Factor Theorem)
9. $10x^2y^3$
10. $1, 2, 3, 4, 6, 12$
11. $7ab$
12. 1
13. yes
14. $4x$
15. 1
16. yes
17. 7
18. no
19. $4(x^2 - 2)$
20. yes
21. 6 tiles per row
22. $10x^2y^3$
23. no, since $p(-3) = -16 \neq 0$
24. $p(1) = 0$ confirms it

Additional Practice Answers

25. $x^3 + 3x - 5$
26. 4
27. $3x^2 - 2x + 3$
28. $3x^2 - 4x + 7$
29. $x^2 + x - 12$
30. $(x + 4)(x + 5)$
31. $3x(2x + 3)$
32. $6x^2$
33. $x + 3$
34. 8
35. $x = 5, \ -1$
36. yes
37. $x^2 - 4x + 4$

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. Try the division: $24x^3y^2/(6xy) = 4x^2y$. Clean polynomial, no negative exponents — so yes.
2. Keep the rule visible: $\gcd(18, 24) = 6$. Smallest x-exponent: $\min(4, 2) = 2$. So $6x^2$. This is the part to check before moving on, because it keeps the answer tied to the original question.
3. Factor: $24 = 2^3 \cdot 3$, $36 = 2^2 \cdot 3^2$, $60 = 2^2 \cdot 3 \cdot 5$. Lowest powers of shared primes: 2^2 and 3, so $\gcd = 12$.
4. Start with the key idea: $\gcd(12, 18) = 6$ from the coefficient column. Take the smaller exponent for each variable: $\min(3, 2) = 2$ for x, $\min(2, 4) = 2$ for y. GCF $= 6x^2y^2$. That gives a quick check on the answer.
5. The factor theorem says $(x - 2)$ is a factor exactly when $f(2) = 0$. The table doesn't list $x = 2$, so compute it: $f(2) = 4 - 10 + 6 = 0$. Since it's zero, yes, $(x - 2)$ is a factor.
6. Coefficients: $\gcd(6, 9) = 3$. For each shared variable take the smaller exponent: $\min(4, 2) = 2$ for x and $\min(1, 3) = 1$ for y. GCF $= 3x^2y$.
7. Coefficient GCF is $\gcd(12, 18, 30) = 6$. The smallest x-power among the three is $\min(5, 3, 2) = 2$. So GCF $= 6x^2$.
8. Start with the key idea: $(x + 4) = (x - (-4))$, so we test $c = -4$. $p(-4) = 0$ exactly matches the factor theorem condition. So yes. That gives a quick check on the answer.
9. Coefficients: $\gcd(20, 30) = 10$. Smaller exponent on each variable: $\min(4, 2) = 2$ for x, $\min(3, 5) = 3$ for y. So GCF $= 10x^2y^3$.
10. Find the factor pairs that multiply to 12: $1 \cdot 12$, $2 \cdot 6$, $3 \cdot 4$. Listing both numbers from each pair gives all six factors: $1, 2, 3, 4, 6, 12$.
11. Coefficients: $\gcd(14, 35) = 7$. Smaller exponent on each: $\min(2, 1) = 1$ for a, $\min(1, 3) = 1$ for b. So GCF $= 7ab$.
12. No shared variable: x doesn't appear in y^4, and y doesn't appear in x^3. The only common factor is 1.
13. A careful way to see it: $(x - 1)$ is a factor exactly when $f(1) = 0$. The table skips $x = 1$, so compute it: $f(1) = 1 - 2 + 5 - 4 = 0$. It's zero, so yes, $(x - 1)$ is a factor. That gives a quick check on the answer.
14. Keep the rule visible: $\gcd(8, 12, 20) = 4$. x appears in all three, smallest exponent 1. y does *not* appear in $8x^3$, so it's not in the GCF. GCF $= 4x$. That gives a quick check on the answer.
15. One steady path is: $9 = 3^2$ and $16 = 2^4$ share no primes, so they're coprime — $\gcd = 1$. That gives a quick check on the answer.
16. Start with the key idea: $(10x^2 + 15x)/(5x) = 2x + 3$. Clean quotient, so yes. (You can also see the GCF $5x$ directly.) That gives a quick check on the answer.
17. A careful way to see it: $21 = 3 \cdot 7$, $28 = 2^2 \cdot 7$, $42 = 2 \cdot 3 \cdot 7$. Only 7 is shared by all three. That gives a quick check on the answer.
18. Keep the rule visible: $(6x^3 + 9x)/x^2 = 6x + 9/x$. The second piece is $9x^{-1}$, which means x^2 doesn't divide every term. So x^2 is not a factor of the whole polynomial. That gives a quick check on the answer.
19. One steady path is: $\gcd(4, 8) = 4$. No common variable. Pull out 4: $4(x^2 - 2)$. This is the part to check before moving on, because it keeps the answer tied to the original question.
20. Start with the key idea: $f(3) = 9 - 21 + 12 = 0$, so by the factor theorem, yes. This is the part to check before moving on, because it keeps the answer tied to the original question.
21. The largest row size that divides all three is $\gcd(12, 30, 42)$. Factor: $12 = 2^2 \cdot 3$, $30 = 2 \cdot 3 \cdot 5$, $42 = 2 \cdot 3 \cdot 7$. Shared primes: 2 and 3, lowest powers 2^1 and 3^1. So $\gcd = 6$. Each batch splits as $12 = 2$ rows, $30 = 5$ rows, $42 = 7$ rows. All integer counts ✓.
22. Keep the rule visible: $\gcd(20, 30) = 10$. Smallest x-exponent: $\min(4, 2) = 2$. Smallest y-exponent: $\min(3, 5) = 3$. GCF $= 10x^2y^3$. (At $x = y = 1$, GCF is 10, and 20 and 30 both divide by 10.) That gives a quick check on the answer.
23. One steady path is: $(x + 3) = (x - (-3))$, so test $c = -3$: $p(-3) = (-3)^3 - 3(-3) + 2 = -27 + 9 + 2 = -16$. Since the remainder isn't 0, $(x + 3)$ is *not* a factor of $p(x)$. That gives a quick check on the answer.
24. By the Factor Theorem, $(x - 1)$ is a factor iff $p(1) = 0$. Compute: $p(1) = 1 + 4 - 1 - 4 = 0$ ✓. So yes, $(x - 1)$ is a factor. (Bonus: dividing gives $p(x) = (x - 1)(x + 1)(x + 4)$.)

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