Factoring the Difference of Two Perfect Squares

Name: ____________	Date: ____________	Score: ______ / 24

Quick Review

The **difference of squares** pattern is the cleanest factoring move in algebra: $a^2 - b^2 = (a-b)(a+b)$. Check by expanding — the middle terms cancel and you're left with $a^2 - b^2$. The pattern works whenever both pieces are perfect squares and you have a minus sign between them. So $x^2 - 9 = x^2 - 3^2 = (x-3)(x+3)$, $4x^2 - 25 = (2x)^2 - 5^2 = (2x-5)(2x+5)$, and $25x^2 - 49y^2 = (5x-7y)(5x+7y)$.

Three reminders. (1) A **sum** of squares like x^2+9 does *not* factor over the integers — the pattern needs the minus sign. (2) A **perfect-square trinomial** like $x^2 - 2x + 1 = (x-1)^2$ has three terms, not two; it's a different pattern. (3) After one difference-of-squares step, **look again**. $x^4 - 16 = (x^2-4)(x^2+4) = (x-2)(x+2)(x^2+4)$ — the first factor is itself a difference of squares. Pull out any common factor before applying the pattern, and always check whether each result factors further.

PRACTICE

Factor completely over the integers.

1. $x^2 - 9$ ________

2. Use the difference-of-squares pattern in the table to factor $x^2 - 49$. (Identify a and b first.) ________

$a^2 - b^2$	a	b	factored
$x^2 - 4$	x	2	$(x-2)(x+2)$
$x^2 - 9$	x	3	$(x-3)(x+3)$
$x^2 - 49$	x	7	?

3. The table fills in a and b for several differences of squares. Complete the last row by factoring $4x^2 - 25$. ________

$a^2 - b^2$	a	b	factored
$9x^2 - 1$	$3x$	1	$(3x-1)(3x+1)$
$16x^2 - 9$	$4x$	3	$(4x-3)(4x+3)$
$4x^2 - 25$	$2x$	5	?

4. $9x^2 - 16$ ________

5. $x^4 - 16$ ________

6. $25x^2 - 49y^2$ ________

7. $16x^4 - 81$ ________

8. $x^2 + 9$ ________

9. $36 - y^2$ ________

10. $x^2 - 12$ ________

11. $49m^2 - 64n^2$ ________

12. $(x+6)^2 - 25$ ________

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13. Following the pattern shown, factor $x^2 - 64$. ________

expression	as $a^2 - b^2$	factored
$x^2 - 25$	$x^2 - 5^2$	$(x-5)(x+5)$
$x^2 - 36$	$x^2 - 6^2$	$(x-6)(x+6)$
$x^2 - 64$	$x^2 - 8^2$	?

14. $100x^2 - 1$ ________

15. $y^2 - 121$ ________

16. $a^4 - b^4$ ________

17. $81 - 4x^2$ ________

18. Factor $x^6 - 1$ completely (treat as $(x^3)^2 - 1^2$). ________

19. $50 - 2x^2$ ________

20. $3x^2 - 75$ ________

♦ Word Problems

21. The expression $x^2 - 49$ represents the area of a region that can be split into two rectangles sharing a side. Factor the expression to write the dimensions. ____

22. A square frame surrounds a smaller square. The outer side is $x + 6$ and the inner side is 5. The area of the frame (region between the squares) is $(x + 6)^2 - 25$. Factor this expression and interpret the factors. ____

23. Find all integer solutions to $49m^2 - 64n^2 = 0$ by factoring and using the zero-product property. ____

24. A board x inches long has a square hole of side 8 inches cut out of the middle of a square panel x inches on a side. The remaining area is $x^2 - 64$ square inches. Factor this expression, and find the side length x that makes the remaining area exactly 36 square inches. ____

Answer Keys

1.	$(x-3)(x+3)$	**13.**	$(x-8)(x+8)$
2.	$(x-7)(x+7)$	**14.**	$(10x-1)(10x+1)$
3.	$(2x-5)(2x+5)$	**15.**	$(y-11)(y+11)$
4.	$(3x-4)(3x+4)$	**16.**	$(a-b)(a+b)(a^2+b^2)$
5.	$(x-2)(x+2)(x^2+4)$	**17.**	$(9-2x)(9+2x)$
6.	$(5x-7y)(5x+7y)$	**18.**	$(x-1)(x^2+x+1)(x+1)(x^2-x+1)$
7.	$(2x-3)(2x+3)(4x^2+9)$	**19.**	$2(5-x)(5+x)$
8.	prime over the integers	**20.**	$3(x-5)(x+5)$
9.	$(6-y)(6+y)$	**21.**	$(x-7)(x+7)$
10.	not a difference of integer squares	**22.**	$(x+1)(x+11)$
11.	$(7m-8n)(7m+8n)$	**23.**	$7m=8n$ or $7m=-8n$
12.	$(x+1)(x+11)$	**24.**	$(x-8)(x+8);\ x=10$ inches

Step-by-Step Explanations

1. Both pieces are perfect squares: x^2 and $9=3^2$, with a minus between them. Apply $a^2-b^2=(a-b)(a+b)$ with $a=x, b=3$: $(x-3)(x+3)$.

2. Reading the last row: $a=x, b=7$, so $x^2-49=(x-7)(x+7)$ — same shape as the worked rows above.

3. One steady path is: $4x^2=(2x)^2$, $25=5^2$, so $a=2x, b=5$, giving $(2x-5)(2x+5)$. (The leading coefficient becomes part of a in each factor.) That gives a quick check on the answer.

4. Spot the squares: $9x^2=(3x)^2$ and $16=4^2$. So $a=3x, b=4$, giving $(3x-4)(3x+4)$. The leading coefficient folds into a.

5. First pass: $(x^2-4)(x^2+4)$. The first factor is another difference of squares: $(x-2)(x+2)$. The second, x^2+4, is a sum — doesn't factor over the integers. Complete factorization: $(x-2)(x+2)(x^2+4)$.

6. Both terms are squares: $25x^2=(5x)^2$ and $49y^2=(7y)^2$. With $a=5x, b=7y$, the pattern gives $(5x-7y)(5x+7y)$ — two variables, same rule.

7. First: $(4x^2-9)(4x^2+9)$. The first factor is $(2x-3)(2x+3)$. The second is a sum of squares — irreducible. Complete: $(2x-3)(2x+3)(4x^2+9)$.

8. Start with the key idea: Sum of squares — the pattern requires a minus sign, so this stays as is. That gives a quick check on the answer.

9. A careful way to see it: $36=6^2$, so $a=6, b=y$: $(6-y)(6+y)$. (Order matters only for the sign in the first factor.) That gives a quick check on the answer.

10. Keep the rule visible: 12 isn't a perfect square integer. Over the integers, this doesn't fit the pattern. (Over the reals, $\sqrt{12}=2\sqrt{3}$, and you could factor as $(x-2\sqrt{3})(x+2\sqrt{3})$ — but that's not asked here.) That gives a quick check on the answer.

11. One steady path is: $49m^2=(7m)^2$ and $64n^2=(8n)^2$, so $a=7m, b=8n$. Apply the pattern: $(7m-8n)(7m+8n)$. That gives a quick check on the answer.

12. Start with the key idea: $a=x+6, b=5$. So $(x+6-5)(x+6+5)=(x+1)(x+11)$. (Don't distribute the squared piece first — the pattern is faster.) That gives a quick check on the answer.

13. A careful way to see it: $64=8^2$, so $a=x, b=8$ and $x^2-64=(x-8)(x+8)$. This is the part to check before moving on, because it keeps the answer tied to the original question.

14. Keep the rule visible: $100x^2=(10x)^2$ and $1=1^2$, so $a=10x, b=1$: $(10x-1)(10x+1)$. Don't overlook 1 as a perfect square. That gives a quick check on the answer.

15. One steady path is: $121=11^2$, so $a=y, b=11$, giving $(y-11)(y+11)$. This is the part to check before moving on, because it keeps the answer tied to the original question.

16. First: $(a^2-b^2)(a^2+b^2)$. The first factor is $(a-b)(a+b)$; the second is a sum of squares — irreducible over the integers.

17. Squares again: $81=9^2$ and $4x^2=(2x)^2$, so $a=9, b=2x$. Keep the order matching the minus sign: $(9-2x)(9+2x)$.

18. Keep the rule visible: $(x^3-1)(x^3+1)$. Each factor splits as a sum/difference of cubes: $x^3-1=(x-1)(x^2+x+1)$ and $x^3+1=(x+1)(x^2-x+1)$. (The quadratic factors are irreducible over the rationals.) That gives a quick check on the answer.

19. Common factor of 2 first: $2(25-x^2)$. Then $25-x^2=(5-x)(5+x)$. Final: $2(5-x)(5+x)$.

20. Start with the key idea: Factor out 3 first: $3(x^2-25)=3(x-5)(x+5)$. This is the part to check before moving on, because it keeps the answer tied to the original question.

21. A careful way to see it: $x^2-49=x^2-7^2=(x-7)(x+7)$. The two factors are the side lengths of a rectangle with that area. (If $x>7$, both lengths are positive and the area makes sense.) That gives a quick check on the answer.

22. Difference of squares with $a=x+6, b=5$: $(x+6-5)(x+6+5)=(x+1)(x+11)$. The two factors are dimensions of an equivalent rectangle with the same area as the frame.

23. Factor: $49m^2-64n^2=(7m-8n)(7m+8n)=0$. Each factor zero gives $7m-8n=0$ or $7m+8n=0$, i.e. $7m=8n$ or $7m=-8n$. (The smallest positive integer solution is $m=8, n=7$.)

24. Start with the key idea: $x^2-64=(x-8)(x+8)$. Setting $(x-8)(x+8)=36$ means $x^2-64=36$, so $x^2=100$ and $x=10$ (positive length). Check: $100-64=36$. ✓ That gives a quick check on the answer.

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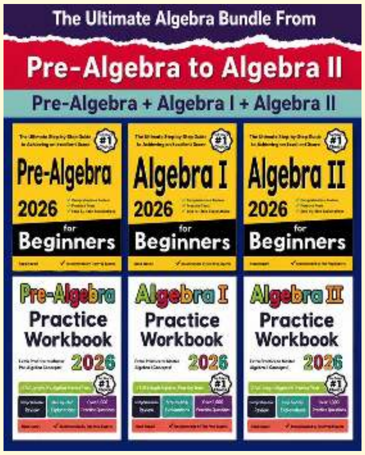

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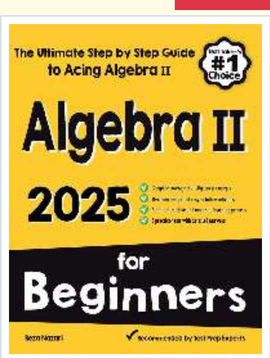

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