Exponential Growth and Decay

Name: ______________	Date: ____________	Score: ______ / 32

Quick Review

Exponential growth model: $A(t) = A_0(1+r)^t$, where A_0 is the starting amount, r is the growth rate (a decimal), and t is the number of time periods. The factor $1+r$ is the per-period multiplier. A 5% growth rate gives $1+0.05 = 1.05$.

Exponential decay model: $A(t) = A_0(1-r)^t$, where $0 < r < 1$ is the decay rate. A 12% decay rate gives $1 - 0.12 = 0.88$ — you keep 88% of the previous amount each period. Don't write 0.12 as the decay factor; that would mean keeping 12%, which is a 88% loss.

Half-life form. If a quantity halves every h time units, write $A(t) = A_0\left(\frac{1}{2}\right)^{t/h}$. The exponent t/h counts how many half-lives have elapsed. At $t = h$, you get half. At $t = 2h$, a quarter. At $t = 3h$, an eighth.

Doubling-time form. Symmetric idea: if doubling time is T, write $A(t) = A_0 \cdot 2^{t/T}$.

Solving for time. To find when $A(t)$ reaches a target, set up the equation and take logarithms. Quick check: how long for $P(t) = 200(1.15)^t$ to reach 400? Solve $(1.15)^t = 2$, so $t = \log 2/\log 1.15 \approx 4.96$ years, so about 5.

Common slips. Confusing rate with factor (a 25% decay rate is factor 0.75, not 0.25). Forgetting to divide by the period length in half-life / doubling-time formulas. Assuming half-life depends on the starting amount — it doesn't.

PRACTICE

Apply growth/decay models. Find amounts, rates, or times as asked.

1. Growth model: 5% annual, $P_0 = 1000$. Write the formula. The graph below confirms the shape. ________

2. Decay model: 12% annual decay, $V_0 = 18000$. Write the formula. The graph below shows the curve. ________

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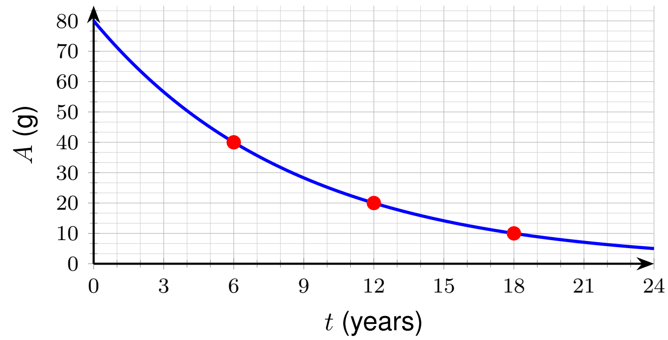
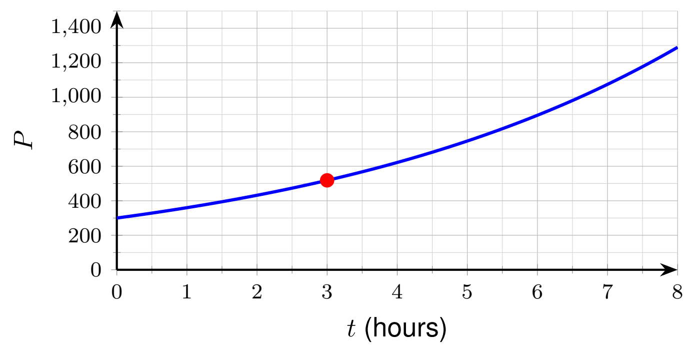
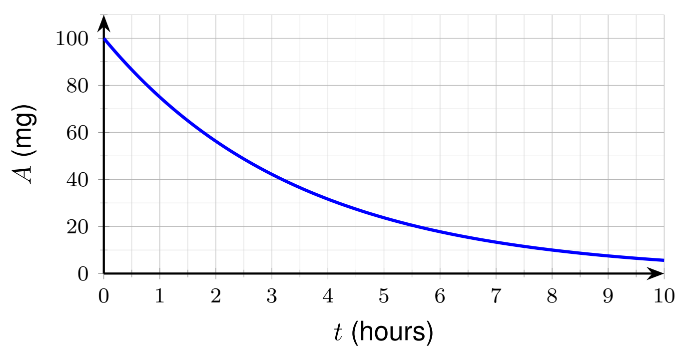
3. Half-life $h = 6$ years, $A_0 = 80$ g. How much remains after 18 years? Read the graph for confirmation. ________



4. Bacteria: 20% hourly growth, start 300. How many after 3 hours? The growth curve is shown. ________



5. Drug: 25% hourly decay, start 100 mg. Write the formula. The decay curve is shown. ________



6. Population 200 growing at 15%/year. About when does it hit 400? ________

7. Isotope half-life 12 days, start 80 g. Amount after 36 days? ________

8. Machine: $V(t) = 18000(0.88)^t$. Annual depreciation rate? ________

9. Town population $12{,}000$, growing 3%/year. Years until it exceeds $15{,}000$? ________

10. A 20% decay rate has factor ______. ________

11. Population doubles every 4 years, starting at 500. Formula? ________

12. Half-life 3 hours, starting at 80 g. Formula? ________

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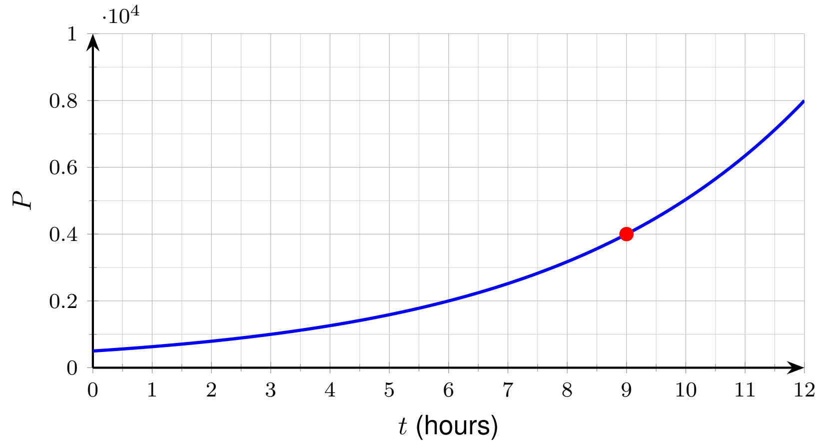
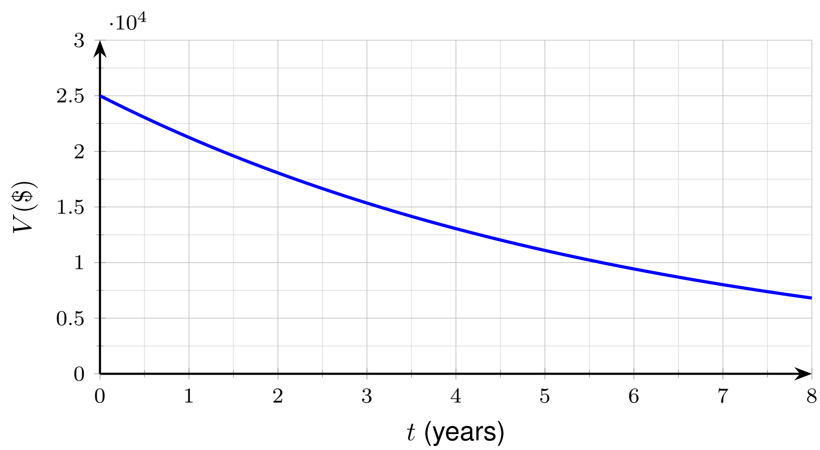
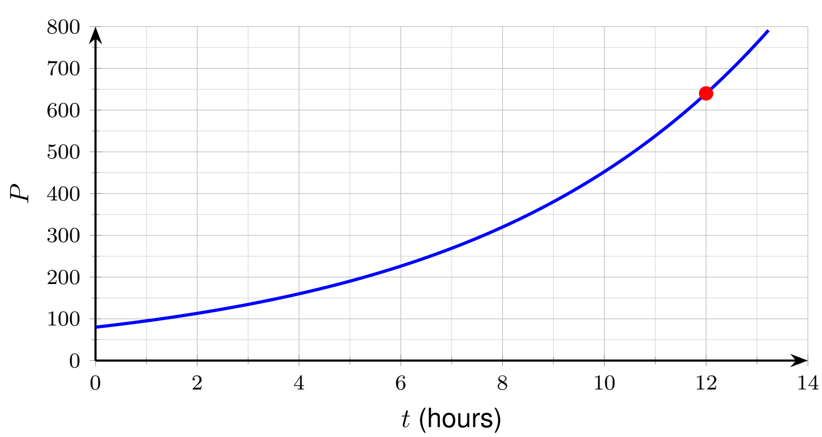
13. The graph below shows a population growth model $P(t) = 500 \cdot 2^{t/3}$. Find $P(9)$. ________



14. The graph below shows car depreciation $V(t) = 25000(0.85)^t$. What is the annual depreciation rate? ________



15. The graph below models bacteria with $P(t) = 80 \cdot 2^{t/4}$. Approximate the population at $t = 12$ hours. ________



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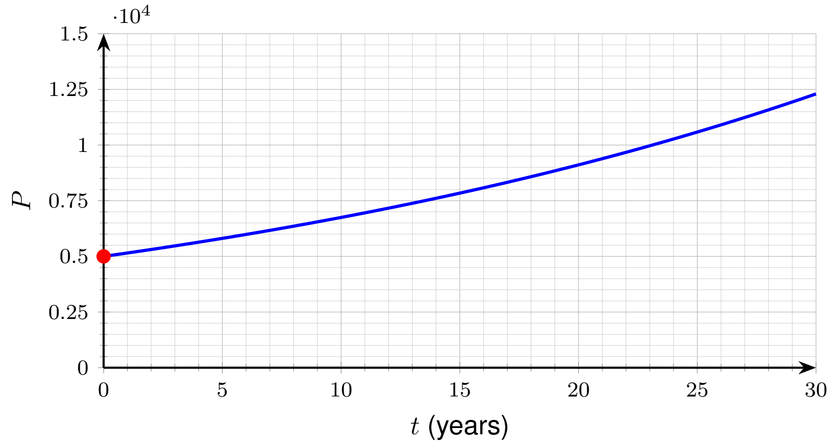
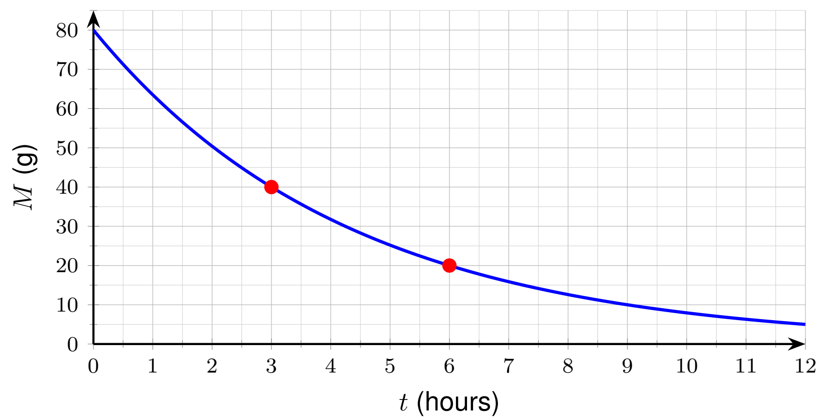
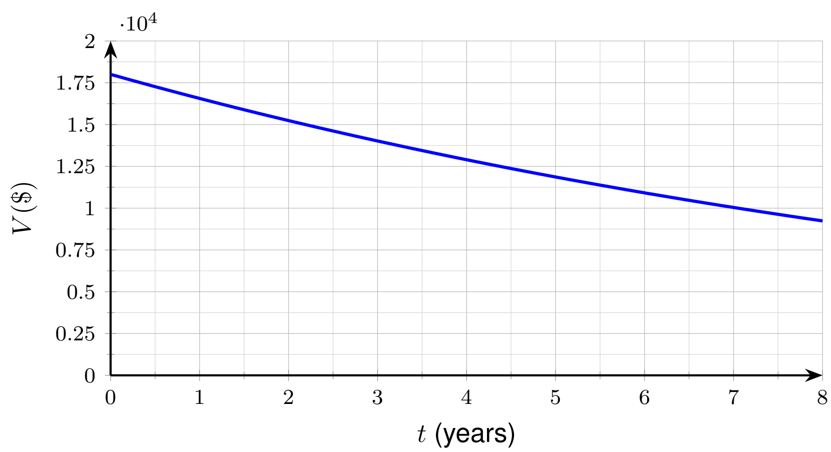
16. The graph below shows continuous growth $P(t) = 5000e^{0.03t}$. What is $P(0)$? ________



17. The graph below shows radioactive decay $M(t) = 80(0.5)^{t/3}$. What is the half-life? ________



18. The graph below shows decay $V(t) = 18000(0.92)^t$. Annual depreciation rate? ________



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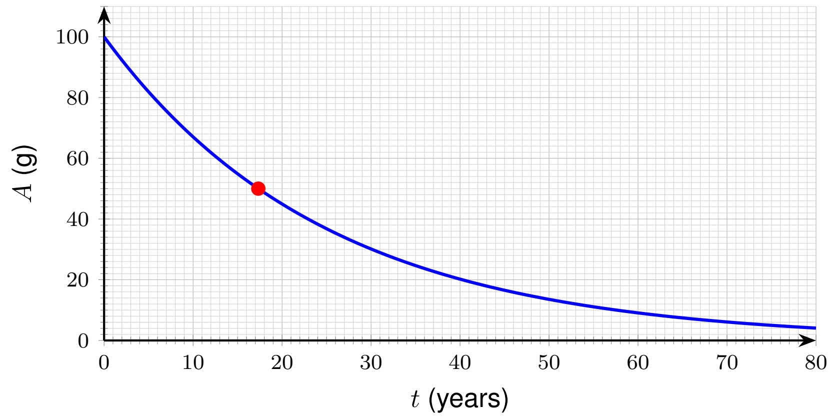
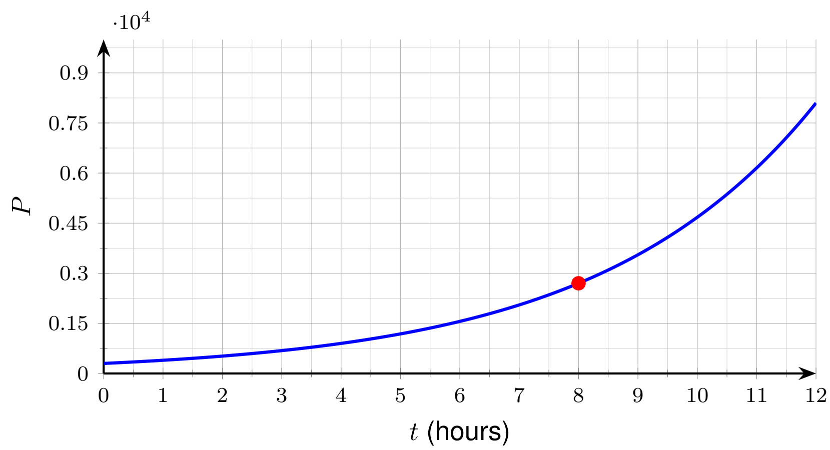
19. The graph below shows decay $A(t) = 100e^{-0.04t}$. Approximate the half-life. ________

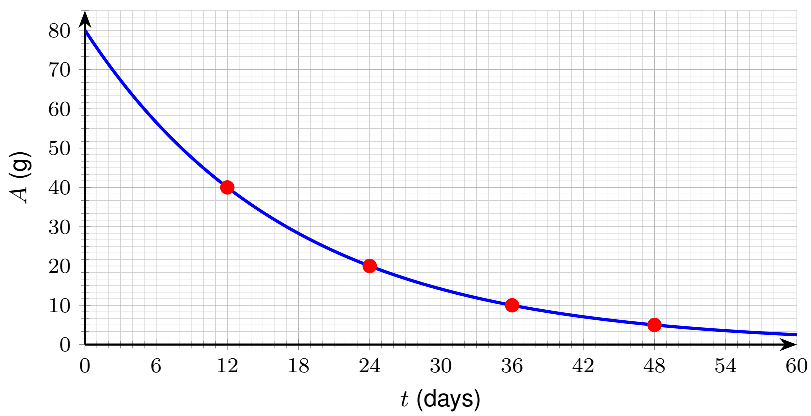


20. The graph below shows triple-every-4-hours growth $P(t) = 300 \cdot 3^{t/4}$. Find $P(8)$. ________

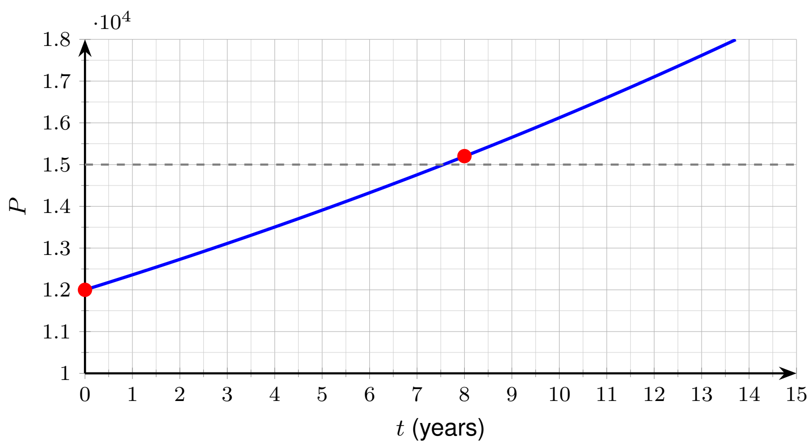


♦ Word Problems

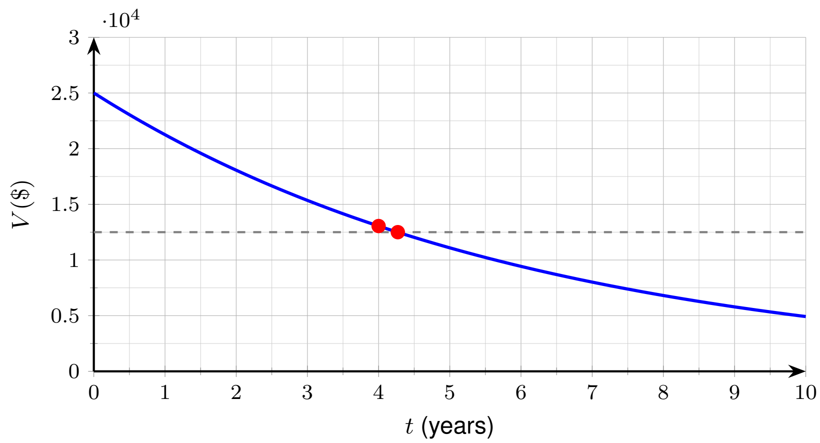
21. A radioactive isotope has a half-life of 12 days. A sample starts at 80 grams. How many grams remain after 36 days? How long until only 5 grams remain? ________



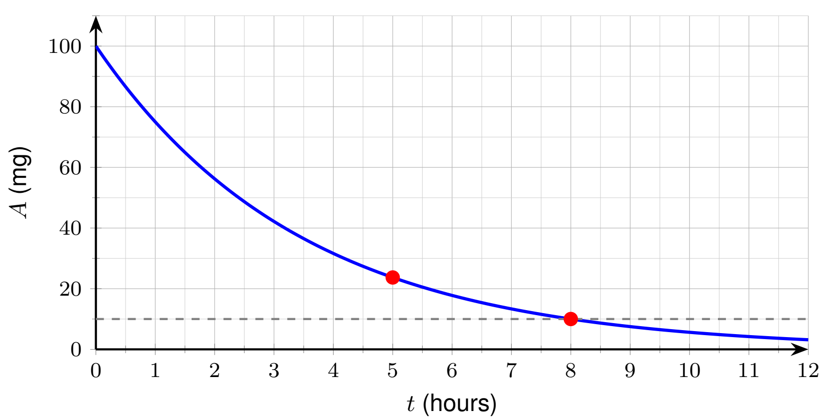
22. A town of 12,000 grows at 3% per year. Write the model $P(t)$, and find the first whole year when the population first exceeds 15,000. ________



23. A car worth $\$25{,}000$ depreciates by 15% per year. Write $V(t)$, find the value after 4 years (round to the nearest dollar), and compute the time it takes for the car to lose half its value. ________



24. A drug enters the bloodstream at 100 mg and decays at 25% per hour. Write $A(t)$, find the amount after 5 hours, and find when only 10 mg remain (round to one decimal place). ________



Additional Practice

25. Evaluate $3 \cdot 2^4$. ________

26. Find a in $y = a \cdot 3^x$ if $y(0) = 7$. ________

27. Growth or decay: $y = 12(0.8)^x$. ________

28. Growth or decay: $y = 5(1.12)^t$. ________

29. Find y when $x = 3$ for $y = 2^x + 1$. ________

30. Solve $2^x = 32$. ________

31. Solve $5^x = 125$. ________

32. Initial value of $P = 400(1.05)^t$. ________

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Answer Keys

1. $P(t) = 1000(1.05)^t$
2. $V(t) = 18000(0.88)^t$
3. 10 g
4. ≈ 518
5. $A(t) = 100(0.75)^t$
6. ≈ 5 years
7. 10 g
8. 12%
9. 8
10. 0.80
11. $P(t) = 500 \cdot 2^{t/4}$
12. $A(t) = 80(1/2)^{t/3}$
13. 4000
14. 15%
15. 640
16. 5000
17. 3 hours
18. 8%
19. ≈ 17.3 years
20. 2700
21. $A(36) = 10$ g; $t = 48$ days for 5 g
22. $P(t) = 12000(1.03)^t$; $t = 8$
23. $V(t) = 25000(0.85)^t$; $V(4) \approx \$13{,}050$; $t \approx 4.27$ years
24. $A(t) = 100(0.75)^t$; $A(5) \approx 23.7$ mg; $t \approx 8.0$ hours

Additional Practice Answers

25. 48
26. $a = 7$
27. decay
28. growth
29. 9
30. $x = 5$
31. $x = 3$
32. 400

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. For growth, the factor is $1 + r = 1 + 0.05 = 1.05$, and the starting amount 1000 is A_0. So $P(t) = 1000(1.05)^t$. The curve bends upward gently — 5% is a slow-but-steady climb.

2. For decay, the factor is $1 - r = 1 - 0.12 = 0.88$, and $A_0 = 18000$, so $V(t) = 18000(0.88)^t$. The factor 0.88 means you keep 88% each year. (The trap is writing 0.12 as the factor — that would represent keeping only 12%.)

3. One steady path is: $18 = 3$ half-lives. Halve three times: $80 \to 40 \to 20 \to 10$. Or use $80(1/2)^{18/6} = 80(1/2)^3 = 10$. That gives a quick check on the answer.

4. Growth factor $1 + 0.20 = 1.2$, so $P(3) = 300(1.2)^3$. The power is $1.2^3 = 1.728$, then $300(1.728) = 518.4$, about 518 bacteria.

5. For decay, the factor is $1 - r = 1 - 0.25 = 0.75$, and the starting dose is 100 mg. So $A(t) = 100(0.75)^t$ — the drug keeps 75% of its amount each hour.

6. Reaching 400 from 200 is a doubling, so set $(1.15)^t = 2$. Take logs: $t = \log 2 / \log 1.15 \approx 4.96$, which rounds to about 5 years.

7. Count the half-lives: $36/12 = 3$. So halve 80 three times: $80 \to 40 \to 20 \to 10$. About 10 g remain (check: $80(1/2)^3 = 80/8 = 10$).

8. Read off the factor: 0.88. Since factor $= 1 - r$, the rate is $r = 1 - 0.88 = 0.12$, so the machine loses 12% of its value each year.

9. Need $12000(1.03)^t > 15000$, i.e. $(1.03)^t > 1.25$. Take logs: $t > \log 1.25 / \log 1.03 \approx 7.55$, so the first whole year over the target is $t = 8$.

10. The decay factor is $1 - r = 1 - 0.20 = 0.80$ — you keep 80% each period. Don't use 0.20; that's the rate (the part lost), not the surviving factor.

11. Use the doubling-time form $A_0 \cdot 2^{t/T}$. Here $A_0 = 500$ and the doubling time is $T = 4$, so $P(t) = 500 \cdot 2^{t/4}$. The exponent $t/4$ counts how many doublings have happened.

12. Use the half-life form $A_0 \left(\frac{1}{2}\right)^{t/h}$ with $A_0 = 80$ and $h = 3$, giving $A(t) = 80(1/2)^{t/3}$. The exponent $t/3$ counts elapsed half-lives.

13. A careful way to see it: $P(9) = 500 \cdot 2^{9/3} = 500 \cdot 2^3 = 500 \cdot 8 = 4000$. The red dot at $(9, 4000)$ confirms. That gives a quick check on the answer.

14. Keep the rule visible: Factor 0.85 means keep 85%, so lose 15% per year. This is the part to check before moving on, because it keeps the answer tied to the original question.

15. One steady path is: $P(12) = 80 \cdot 2^{12/4} = 80 \cdot 2^3 = 80 \cdot 8 = 640$. Three doublings from 80. That gives a quick check on the answer.

16. Start with the key idea: $P(0) = 5000e^0 = 5000$. The intercept gives the initial population. That gives a quick check on the answer.

17. From 80 to 40 takes 3 hours; from 40 to 20 takes another 3. Half-life is 3 hours regardless of starting amount.

18. Keep the rule visible: $1 - 0.92 = 0.08$, or 8% depreciation per year. This is the part to check before moving on, because it keeps the answer tied to the original question.

19. Set $100e^{-0.04t} = 50$: $e^{-0.04t} = 1/2$, so $-0.04t = \ln(1/2)$, giving $t = \ln 2/0.04 \approx 17.33$ years. The red dot on the curve sits at half the starting amount.

20. Start with the key idea: $P(8) = 300 \cdot 3^{8/4} = 300 \cdot 3^2 = 300 \cdot 9 = 2700$. Two triplings from 300: $300 \to 900 \to 2700$. That gives a quick check on the answer.

21. Model: $A(t) = 80(1/2)^{t/12}$. At $t = 36$: that's 3 half-lives, so $80 \to 40 \to 20 \to 10$ grams. Plug-check: $80(1/2)^3 = 80/8 = 10$. For the second question, set $80(1/2)^{t/12} = 5$, so $(1/2)^{t/12} = 1/16 = (1/2)^4$, giving $t/12 = 4$ and $t = 48$ days. The four red dots on the curve show the sequence 40, 20, 10, 5 at 12-day intervals.

22. A 3% rate gives growth factor $1 + 0.03 = 1.03$, so $P(t) = 12000(1.03)^t$. We want $12000(1.03)^t > 15000$, which simplifies to $(1.03)^t > 1.25$. Taking logs, $t > \log 1.25 / \log 1.03 \approx 7.55$, so the first whole year is $t = 8$. Verify both sides of that: $P(7) = 12000(1.03)^7 \approx 14{,}758$ (still under) and $P(8) = 12000(1.03)^8 \approx 15{,}201$ (over). The dashed line at $y = 15{,}000$ marks the target the curve has to cross.

23. Decay factor $1 - 0.15 = 0.85$. $V(t) = 25000(0.85)^t$. At $t = 4$: $V(4) = 25000(0.85)^4 = 25000(0.52200625) \approx 13050.16$, or \$13,050. For the half-value question, set $25000(0.85)^t = 12500$. So $(0.85)^t = 0.5$. Take logs: $t = \log 0.5 / \log 0.85 \approx 4.27$ years. (After about four and a quarter years, the car has halved in value — much faster than people expect because each year's 15% loss is taken from a shrinking base.)

24. Decay factor $1 - 0.25 = 0.75$. So $A(t) = 100(0.75)^t$. At $t = 5$: $A(5) = 100(0.75)^5 = 100(0.2373) \approx 23.7$ mg. To find when $A = 10$: $100(0.75)^t = 10$, so $(0.75)^t = 0.1$. Take logs: $t = \log 0.1 / \log 0.75 = -1/(-0.1249) \approx 8.00$ hours. So the drug reaches 10 mg after about 8 hours. (Tip: when the answer is suspiciously round, plug it back to verify — $100(0.75)^8 \approx 10.01$ mg. Close to exactly 10. Confirmed.)

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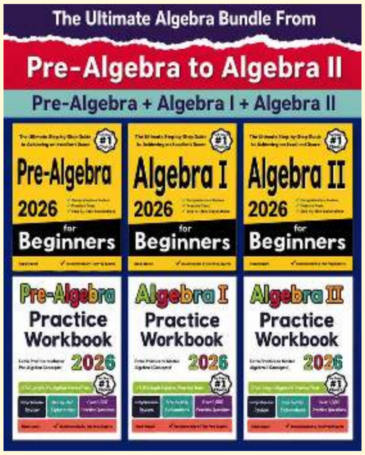

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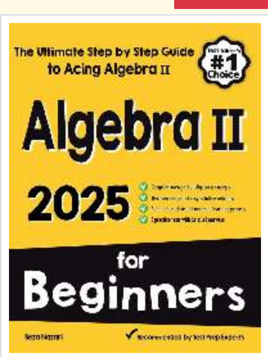

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