

# Exponential Functions

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Score: \_\_\_\_\_ / 35

## Q Quick Review

An **exponential function** has the form  $f(x) = a \cdot b^x$ , where  $a \neq 0$  is the *initial value*,  $b > 0$  is the *base* (also called the growth or decay factor), and  $b \neq 1$ . The variable lives *in the exponent*, which is what makes the family special: small steps in  $x$  produce *multiplicative* jumps in  $y$ .

**Growth vs decay.** With  $a > 0$ : if  $b > 1$ , the function *grows* (each step multiplies by  $b$ ); if  $0 < b < 1$ , it *decays*.  $f(x) = 3 \cdot 2^x$  grows — doubling each unit.  $g(x) = 4 \cdot (0.5)^x$  decays — halving each unit.

**Y-intercept is always  $a$ .** Because  $b^0 = 1$ ,  $f(0) = a \cdot 1 = a$ . So the constant in front is where the curve crosses the  $y$ -axis.

**Horizontal asymptote at  $y = 0$ .** The graph approaches the  $x$ -axis but never touches it. (Shift the function up by  $k$  and the asymptote moves to  $y = k$  — that's how you read a transformed exponential off a graph.)

**Finding  $a$  and  $b$  from two points.** If  $f(0) = a$  is given, you have  $a$  for free. Otherwise, use  $f(0)$ . To get  $b$ , plug a second point in and solve. Quick check:  $f(0) = 6$  and  $f(2) = 24$  gives  $a = 6$ , then  $6b^2 = 24$  so  $b^2 = 4$  and  $b = 2$  (positive branch). **Common traps.**  $f(x) = ax^b$  is a *power* function, not exponential — the variable belongs in the exponent. The range of  $f(x) = a \cdot b^x$  (no shift,  $a > 0$ ) is  $y > 0$  — *strictly* positive — because  $b^x > 0$  for every real  $x$ .

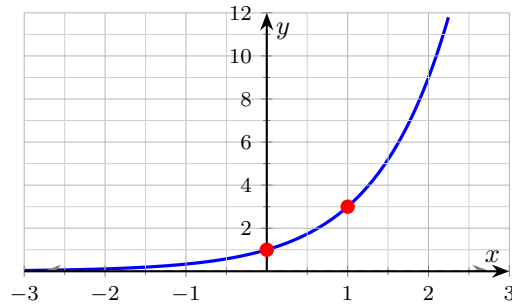
## PRACTICE

Evaluate, identify, or graph each exponential function.

1.  $f(x) = 3 \cdot 2^x$ ;  $f(4)$  \_\_\_\_\_
2.  $f(x) = 5 \cdot 3^x$ ; y-intercept \_\_\_\_\_
3.  $g(x) = 4 \cdot (0.5)^x$ ; growth or decay? \_\_\_\_\_
4.  $f(x) = 2^x$ ; domain and range \_\_\_\_\_
5.  $f(x) = a \cdot b^x$ ,  $f(0) = 6$ ,  $f(2) = 24$ ,  $b > 0$ ; find  $b$  \_\_\_\_\_
6. Is  $3 \cdot 2^x$  an exponential function? \_\_\_\_\_
7. Is  $3x^2$  an exponential function? \_\_\_\_\_
8.  $f(x) = 7 \cdot 2^x$ ;  $f(0) + f(3)$  \_\_\_\_\_
9. Horizontal asymptote of  $y = 3 \cdot 2^x$  \_\_\_\_\_
10. Horizontal asymptote of  $y = 3 \cdot 2^x + 5$  \_\_\_\_\_
11.  $f(x) = a \cdot b^x$ ,  $f(0) = 7$ ,  $f(3) = 56$ ; formula \_\_\_\_\_

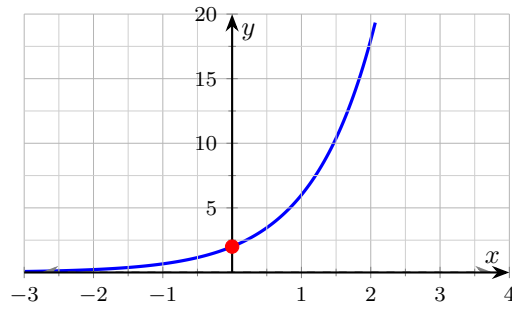


12. The graph below shows  $y = 3^x$ . Name two integer points it passes through. \_\_\_\_\_

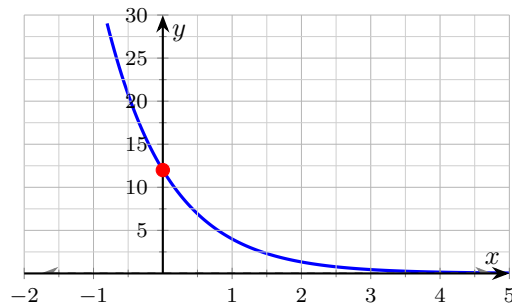


13.  $f(x) = 10 \cdot (0.8)^x$ ;  $f(2)$  \_\_\_\_\_

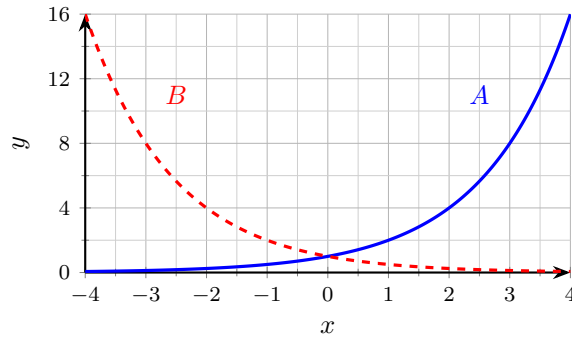
14. The graph below shows  $f(x) = 2 \cdot 3^x$ . Identify the  $y$ -intercept. \_\_\_\_\_



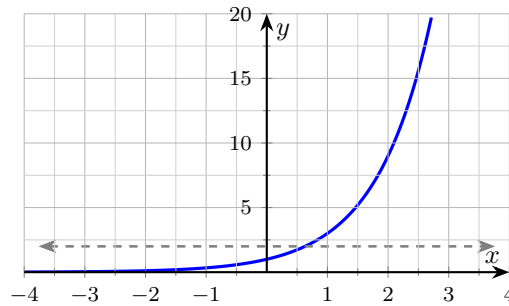
15. The graph below shows  $g(x) = 12 \cdot (1/3)^x$ . Is this growth or decay, and what is  $g(0)$ ? \_\_\_\_\_



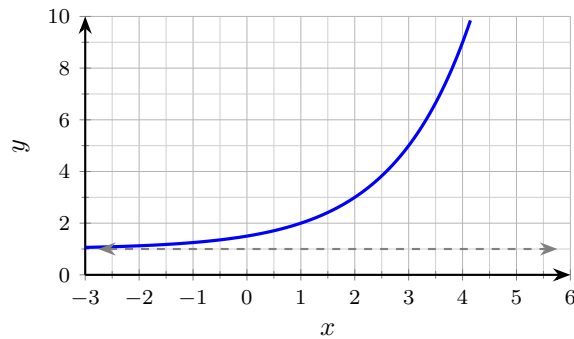
16. Which curve below is  $y = 2^x$ ? \_\_\_\_\_



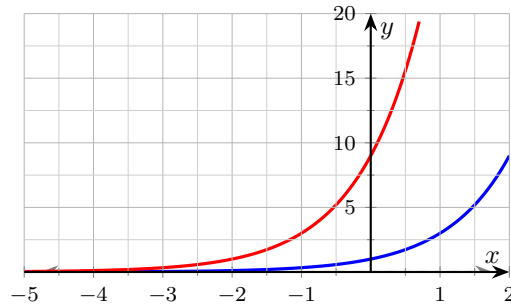
17. The graph below shows  $f(x) = 3^x + 2$ . What is the horizontal asymptote? \_\_\_\_\_



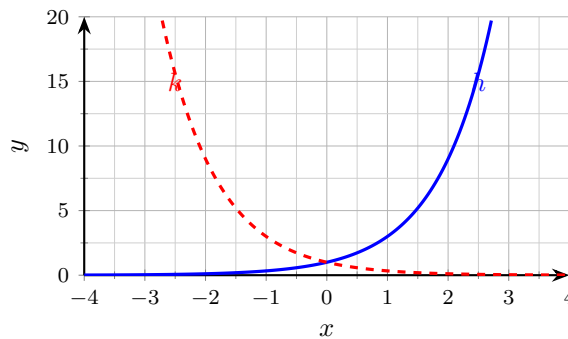
18. The graph shown is  $f(x) = 2^{x-1} + 1$ . What is the asymptote? \_\_\_\_\_



19. The graph below shows  $h(x) = 3^{x+2}$ . How does it relate to  $f(x) = 3^x$ ? \_\_\_\_\_

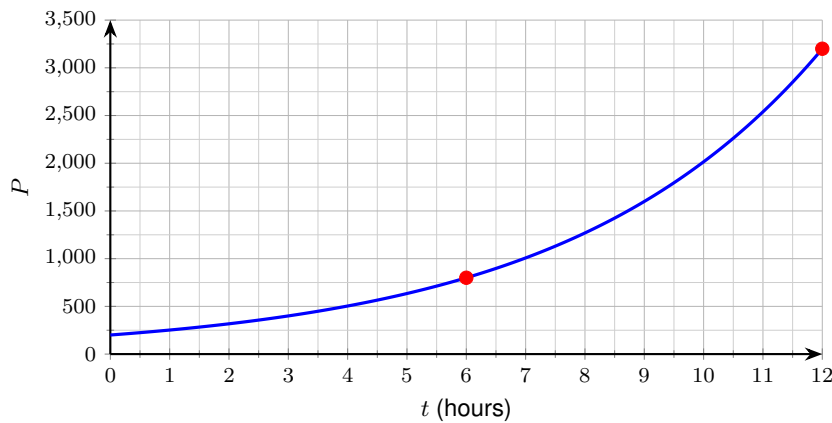


20. The graph below shows  $k(x) = 3^{-x}$ . How does it relate to  $h(x) = 3^x$ ? \_\_\_\_\_

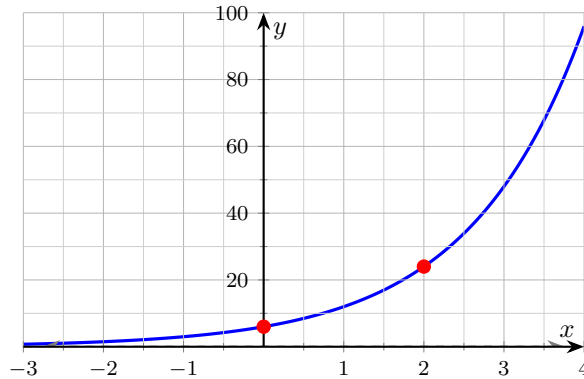


◆ Word Problems

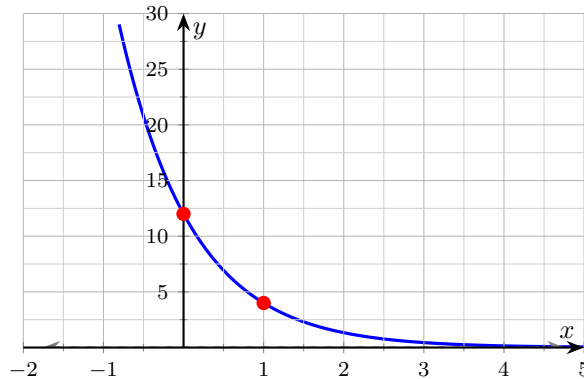
21. A bacterial population doubles every 3 hours. The colony starts at 200 bacteria. Use the model  $P(t) = 200 \cdot 2^{t/3}$  and the graph below to find the population after 6 hours and after 12 hours. \_\_\_\_\_



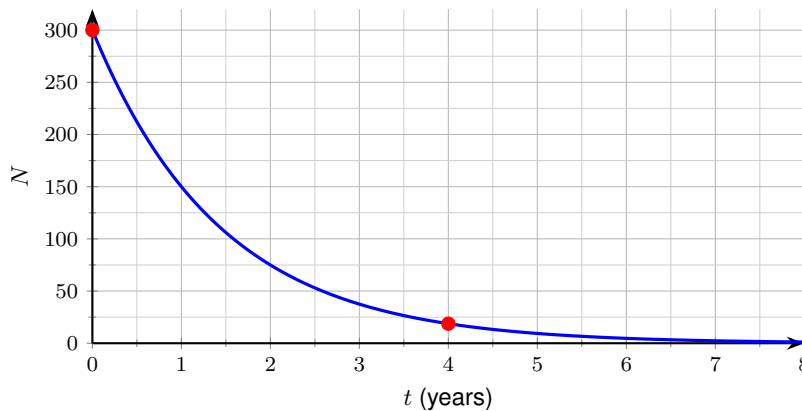
22. An exponential function has the form  $f(x) = a \cdot b^x$ . You're given  $f(0) = 6$  and  $f(2) = 24$ , with  $b > 0$ . Find  $a$ , find  $b$ , and write the formula. Sketch the resulting curve on the plane below.



23. Match the graph below to one of:  $A. y = 6(3)^x$ ,  $B. y = 12(1/3)^x$ ,  $C. y = 12(3)^x$ ,  $D. y = 6(1/3)^x$ . Justify by reading the  $y$ -intercept and shape.



24. A scientist counts 300 tagged frogs in a pond. The population shrinks by half every year due to habitat loss. Write an exponential model for the number of tagged frogs after  $t$  years, and predict the count after 4 years. The graph below shows the decay curve.



**Additional Practice**

25. Evaluate  $3 \cdot 2^4$ . \_\_\_\_\_

26. Find  $a$  in  $y = a \cdot 3^x$  if  $y(0) = 7$ . \_\_\_\_\_

27. Growth or decay:  $y = 12(0.8)^x$ . \_\_\_\_\_

28. Growth or decay:  $y = 5(1.12)^t$ . \_\_\_\_\_

29. Find  $y$  when  $x = 3$  for  $y = 2^x + 1$ . \_\_\_\_\_

30. Solve  $2^x = 32$ . \_\_\_\_\_

31. Solve  $5^x = 125$ . \_\_\_\_\_

32. Initial value of  $P = 400(1.05)^t$ . \_\_\_\_\_

33. Rate in  $P = 900(1.08)^t$ . \_\_\_\_\_

34. Rate in  $A = 1200(0.92)^t$ . \_\_\_\_\_

35. Double 60 three times. \_\_\_\_\_



## Answer Keys

1. 48
2. 5
3. decay
4.  $\mathbb{R}; y > 0$
5. 2
6. yes
7. no (it's a power/quadratic)
8. 63
9.  $y = 0$
10.  $y = 5$
11.  $f(x) = 7 \cdot 2^x$
12. (0, 1) and (1, 3)
13. 6.4
14. 2
15. decay;  $g(0) = 12$
16. A
17.  $y = 2$
18.  $y = 1$
19. shifted left 2 units
20. reflection across the  $y$ -axis
21.  $P(6) = 800, P(12) = 3200$
22.  $a = 6, b = 2, f(x) = 6 \cdot 2^x$
23. B.  $y = 12(1/3)^x$
24.  $N(t) = 300 \cdot (0.5)^t; N(4) \approx 19$

## Additional Practice Answers

25. 48
26.  $a = 7$
27. decay
28. growth
29. 9
30.  $x = 5$
31.  $x = 3$
32. 400
33. 8%
34. 8% decay
35. 480

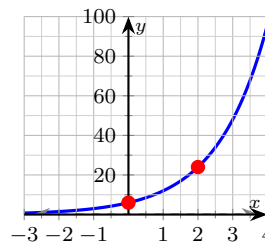
**Additional Practice:** Answers for all numbered items, including the added practice, are shown in the grid above.

## Step-by-Step Explanations

1. Exponent first:  $2^4 = 16$ . Then multiply:  $3 \cdot 16 = 48$ . (A common slip is to multiply first, getting  $6^4 = 1296$  — order of operations says the exponent binds tighter.)
2. Set  $x = 0$ :  $f(0) = 5 \cdot 3^0 = 5 \cdot 1 = 5$ . Any  $b^0 = 1$ , so the  $y$ -intercept of  $a \cdot b^x$  is always  $a$ .
3. Look at the base:  $b = 0.5$ , and since  $0 < b < 1$  the function decays. Each unit step multiplies by 0.5, so the output halves and the curve falls as  $x$  increases. (If  $b$  were greater than 1, it would grow instead.)
4. You can raise 2 to any real power, so the domain is all reals. For the range, note  $2^x$  is positive for every  $x$  and approaches 0 but never reaches it, so the range is  $y > 0$  — strictly positive, never zero or negative.
5. A careful way to see it:  $f(0) = a = 6$ . Then  $6b^2 = 24$ , so  $b^2 = 4$ . Positive branch:  $b = 2$ . (Don't forget the positivity constraint —  $b$  must be positive in the standard exponential form.) That gives a quick check on the answer.
6. Match it to the form  $a \cdot b^x$ : here  $a = 3$  and  $b = 2$ , with  $b > 0$  and  $b \neq 1$ . The variable sits in the exponent, which is exactly what makes a function exponential.
7. Here the variable  $x$  is the base and the exponent 2 is constant — the reverse of an exponential. That makes it a power (quadratic) function, not exponential.
8. Evaluate each piece separately.  $f(0) = 7 \cdot 2^0 = 7 \cdot 1 = 7$ , and  $f(3) = 7 \cdot 2^3 = 7 \cdot 8 = 56$ . Add them:  $7 + 56 = 63$ .
9. There is no vertical shift added to the function, so the asymptote stays at the parent location  $y = 0$ . As  $x \rightarrow -\infty$ ,  $2^x \rightarrow 0$ , so the curve hugs the  $x$ -axis from above without ever touching it.
10. Start from the parent  $3 \cdot 2^x$ , whose asymptote is  $y = 0$ . The  $+5$  shifts the entire graph up by 5, so the asymptote moves up with it to  $y = 5$ .
11. Use the easy point first:  $f(0) = a \cdot b^0 = a = 7$ , so  $a = 7$ . Then plug in the second point:  $7b^3 = 56$  gives  $b^3 = 8$ , so  $b = 2$ . The formula is  $f(x) = 7 \cdot 2^x$ .
12. At  $x = 0$ ,  $y = 3^0 = 1$ . At  $x = 1$ ,  $y = 3$ . Every  $a \cdot b^x$  with  $a = 1$  passes through (0, 1). The two red dots mark these anchor points.
13. Handle the exponent first:  $0.8^2 = 0.64$ . Then multiply by the initial value:  $10 \cdot 0.64 = 6.4$ . Since  $0.8 < 1$ , the output came out smaller than the starting 10, just as decay predicts.
14. Keep the rule visible: Set  $x = 0$ :  $y = 2 \cdot 3^0 = 2$ . The red dot marks (0, 2) on the curve. That gives a quick check on the answer.
15. Base  $\frac{1}{3} < 1$  means decay — the curve falls. At  $x = 0$ :  $g(0) = 12$ . The dot marks the intercept.

16. Start with the key idea:  $2^x$  grows as  $x$  increases (rising right). Curve A does that. Curve B is  $(1/2)^x$  — decay, falls right. That gives a quick check on the answer.
17. The  $+2$  shifts the parent  $3^x$  up by 2, taking the asymptote with it. (Be careful: the dashed line in the figure is at  $y = 2$ , not the  $x$ -axis.)
18. Vertical shift up by 1 moves the asymptote to  $y = 1$ . (The  $x - 1$  shifts horizontally; it doesn't touch the horizontal asymptote.)
19. One steady path is:  $3^{x+2} = f(x+2)$ . Replacing  $x$  with  $x+2$  shifts left by 2 (think: the input has to be smaller by 2 to get the same output, so the curve slides left). That gives a quick check on the answer.
20. Start with the key idea:  $3^{-x} = h(-x)$ . Replacing  $x$  with  $-x$  flips the curve across the  $y$ -axis. The growth curve becomes a decay curve. That gives a quick check on the answer.
21. Plug in directly.  $P(6) = 200 \cdot 2^{6/3} = 200 \cdot 2^2 = 800$ .  $P(12) = 200 \cdot 2^{12/3} = 200 \cdot 2^4 = 200 \cdot 16 = 3200$ . Notice the doubling rhythm:  $200 \rightarrow 400 \rightarrow 800$  over the first six hours (two doublings), then  $\rightarrow 1600 \rightarrow 3200$  over the next six. The marked points on the curve confirm both values.
22. Start with the easier point:  $f(0) = a \cdot b^0 = a$ . So  $a = 6$ . Now use the second point:  $f(2) = 6b^2 = 24$ , so  $b^2 = 4$  and (since  $b > 0$ )  $b = 2$ . The formula is  $f(x) = 6 \cdot 2^x$ . The two red dots mark the data points; the curve passes through both.

## Answer graph



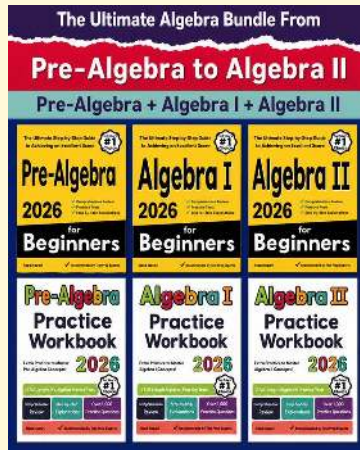
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23. Two checkpoints make this quick. The  $y$ -intercept is  $a$  (because  $b^0 = 1$ ), and the graph crosses at  $(0, 12)$ , so  $a = 12$  — rules out A and D. The curve falls as  $x$  increases, so  $0 < b < 1$  — rules out C. That leaves B:  $y = 12(1/3)^x$ . Sanity-check: at  $x = 1$ ,  $12 \cdot \frac{1}{3} = 4$ , matching the second dot.

24. Halving each year means the multiplier is 0.5, so  $N(t) = 300 \cdot (0.5)^t$ . After 4 years:  $N(4) = 300 \cdot (0.5)^4 = 300 \cdot 0.0625 = 18.75$ , or about 19 frogs. (Halving four times:  $300 \rightarrow 150 \rightarrow 75 \rightarrow 37.5 \rightarrow 18.75$ . Same answer the long way.)



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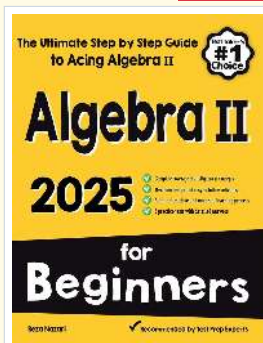
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