Even and Odd Functions

Name: ____________________ Date: ______________ Score: _____ / 28

Quick Review

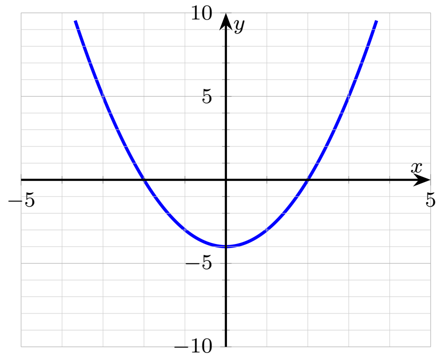
A function is **even** when $f(-x) = f(x)$ for every x in the domain. The graph is mirror-symmetric across the y-axis: whatever happens to the right also happens to the left at the same height.

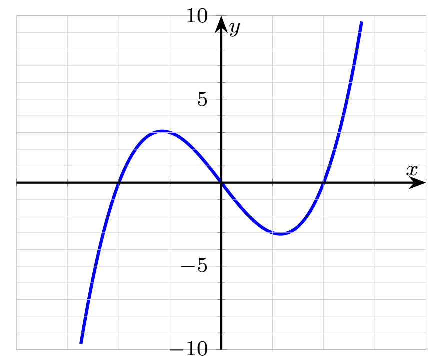
A function is **odd** when $f(-x) = -f(x)$ for every x. The graph has *origin symmetry*: rotate $180°$ around $(0, 0)$ and you get the same picture.

Polynomial shortcut:

- All terms have *even* exponents (including the constant, which is degree 0) $\Rightarrow$ *even* function. Quick check: $f(x) = x^4 - 3x^2 + 5$.
- All terms have *odd* exponents (no constant) $\Rightarrow$ *odd* function. Quick check: $g(x) = 2x^3 - 5x$.
- A *mix* of even and odd exponents $\Rightarrow$ neither. Quick check: $h(x) = x^2 + x$.

To test by computation, substitute $-x$ everywhere and simplify. If the result equals $f(x)$, it's even; if it equals $-f(x)$, it's odd; otherwise neither. Two graphs below: $f(x) = x^2 - 4$ (even, y-axis symmetry) and $g(x) = x^3 - 4x$ (odd, origin symmetry).



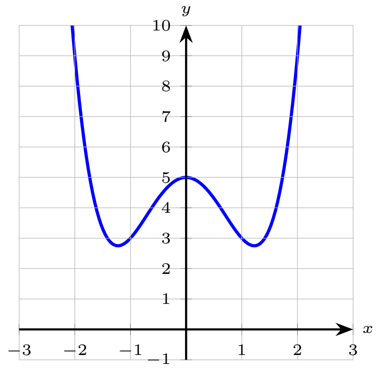


Fact: the only function that is both even and odd is $f(x) = 0$. (If $f(-x) = f(x)$ and $f(-x) = -f(x)$, then $f(x) = -f(x)$, so $2f(x) = 0$ everywhere.) Not every function falls into one of the three categories — but every *polynomial with only one parity of exponents* does.

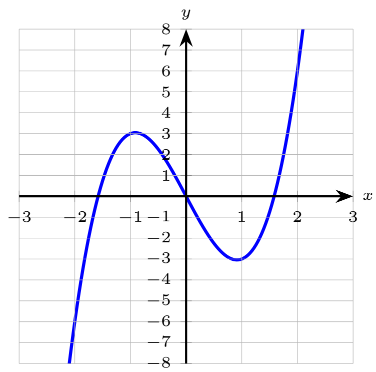
PRACTICE

Classify each polynomial as even, odd, or neither. Show the substitution test when helpful.

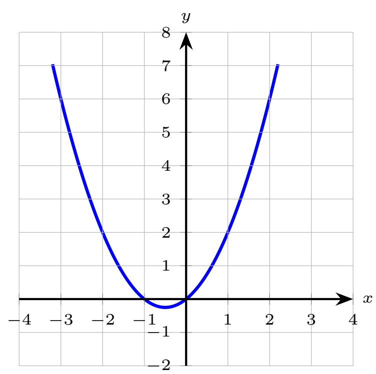
1. The graph of $f(x) = x^4 - 3x^2 + 5$ is shown. Use its symmetry to classify f as even, odd, or neither, then confirm with the substitution test. ________



2. The graph of $f(x) = 2x^3 - 5x$ is shown. Use its symmetry to classify f as even, odd, or neither, then confirm with the substitution test. ________



3. The graph of $f(x) = x^2 + x$ is shown. Use its symmetry to classify f as even, odd, or neither, then confirm with the substitution test. ________



4. $x^5 - 4x^3 + x$ ________
5. $x^4 + x^2 + 1$ ________
6. $x^3 + x^2$ ________
7. $x^2 + x + 1$ ________
8. $f(x) = 3x^6 - 2x^2 + 7$; find $f(2) - f(-2)$ ________
9. $-4x^5 + x^3$ ________
10. $-2x^4 + 3x^2 + 8$ ________

Scan Me

11. $x^3 - x$ ________
12. 6 ________
13. Which polynomial is symmetric about the origin: $x^5 - 4x^3 + x,\ x^4 + x^2 + 1,\ x^3 + x^2,\ x^2 + x + 1$? ________
14. If $p(x) = x^4 + kx^3 + 6x^2 + 9$ is even, find k ________
15. If $q(x) = ax^5 + bx^4 + 3x^3 + cx^2 + dx$ is odd, find b ________
16. True/False: every polynomial is even or odd ________
17. Test $f(x) = x^2 - 9$ for symmetry ________
18. Test $f(x) = x^3 - 4x$ for symmetry ________
19. Sum of an even function and an odd function is ________
20. Product of two odd functions is ________

♦ Word Problems

21. A function $f(x) = ax^4 + bx^2 + c$ has $f(2) = 37$ and $f(-2) = 37$. Explain how this is consistent with the function being even, and what kind of symmetry the graph has. ____
22. A signal-processing function is $g(x) = x^5 - 2x^3 + x$. Show that $g(-3) = -g(3)$ algebraically and identify the graph's symmetry. ____
23. A student claims that $h(x) = x^2 + x$ is even because both terms are positive when x is positive. Test the claim and explain the verdict. ____
24. A polynomial is given by $p(x) = 4x^6 + kx^3 + x^2 - 5$. For what value of k is p an even function? ____

Additional Practice

25. If $f(x) = 2x - 5$, find $f(4)$. ________
26. If $g(x) = x^2 + 1$, find $g(-3)$. ________
27. For $f(x) = 3x + 2$, solve $f(x) = 14$. ________
28. Find $(f + g)(x)$ if $f = x + 1,\ g = 2x - 5$. ________

Scan Me

Answer Keys

1. $\boxed{even}$
2. $\boxed{odd}$
3. $\boxed{neither}$
4. $\boxed{odd}$
5. $\boxed{even}$
6. $\boxed{neither}$
7. $\boxed{neither}$
8. $\boxed{0}$
9. $\boxed{odd}$
10. $\boxed{even}$
11. $\boxed{odd}$
12. $\boxed{even}$
13. $\boxed{x^5 - 4x^3 + x}$
14. $\boxed{k = 0}$
15. $\boxed{b = 0}$
16. $\boxed{false}$
17. $\boxed{even}$
18. $\boxed{odd}$
19. $\boxed{neither(ingeneral)}$
20. $\boxed{even}$
21. even; graph is symmetric about the y-axis
22. $g(3) = 192,\ g(-3) = -192;$ odd; origin symmetry
23. false; h is neither
24. $\boxed{k = 0}$

Additional Practice Answers

25. $\boxed{3}$
26. $\boxed{10}$
27. $\boxed{x = 4}$
28. $\boxed{3x - 4}$

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. The curve is a mirror image across the y-axis, which signals an even function. Confirm: $f(-x) = (-x)^4 - 3(-x)^2 + 5 = x^4 - 3x^2 + 5 = f(x)$. Even.

2. Rotating the graph 180° about the origin lands it back on itself — origin symmetry, the mark of an odd function. Confirm: $f(-x) = -2x^3 + 5x = -(2x^3 - 5x) = -f(x)$. Odd.

3. The parabola's axis sits at $x = -\frac{1}{2}$, not the y-axis, and there's no origin symmetry — so it's neither. Confirm: $f(-x) = x^2 - x$, which equals neither $f(x) = x^2 + x$ nor $-f(x) = -x^2 - x$.

4. All exponents odd: $5, 3, 1$. Origin symmetry. Test: $f(-x) = -x^5 + 4x^3 - x = -(x^5 - 4x^3 + x) = -f(x)$.

5. A careful way to see it: Exponents $4, 2, 0$ all even. y-axis symmetry. This is the part to check before moving on, because it keeps the answer tied to the original question.

6. Keep the rule visible: Mixed: 3 (odd) and 2 (even). Test: $f(-x) = -x^3 + x^2 \neq \pm f(x)$. That gives a quick check on the answer.

7. One steady path is: Mixed exponents $2, 1, 0$ — both parities present. This is the part to check before moving on, because it keeps the answer tied to the original question.

8. Start with the key idea: f has only even exponents, so f is even. That means $f(2) = f(-2)$, so $f(2) - f(-2) = 0$. That gives a quick check on the answer.

9. Exponents $5, 3$ both odd; no constant. Test: $f(-x) = 4x^5 - x^3 = -(-4x^5 + x^3) = -f(x)$.

10. Keep the rule visible: Exponents $4, 2, 0$ all even. This is the part to check before moving on, because it keeps the answer tied to the original question.

11. One steady path is: Exponents $3, 1$ both odd. Test: $f(-x) = -x^3 + x = -(x^3 - x) = -f(x)$. That gives a quick check on the answer.

12. Start with the key idea: $f(x) = 6 = 6x^0$ has only the (even) exponent 0. Test: $f(-x) = 6 = f(x)$. (Note: it's not odd, since $-f(x) = -6 \neq 6$ unless we're talking about the zero function.) That gives a quick check on the answer.

13. Origin symmetry = odd function = all odd exponents, no constant. Only $x^5 - 4x^3 + x$ fits.

14. For p to be even, every term must have an even exponent. The kx^3 piece has an odd exponent, so $k = 0$ is the only way to make p even (the term then disappears).

15. All even-exponent terms must vanish. The bx^4 piece has an even exponent, so $b = 0$. (Similarly $c = 0$; a and d can be anything.)

16. Polynomials with mixed-parity exponents (like $x^2 + x$) are neither even nor odd.

17. A careful way to see it: $f(-x) = (-x)^2 - 9 = x^2 - 9 = f(x)$. The function is even; the graph is a parabola symmetric about the y-axis. That gives a quick check on the answer.

18. Keep the rule visible: $f(-x) = (-x)^3 - 4(-x) = -x^3 + 4x = -(x^3 - 4x) = -f(x)$. Odd. This is the part to check before moving on, because it keeps the answer tied to the original question.

19. If f is even and g is odd (both nonzero), then $f + g$ has both even and odd pieces, so it's neither.

20. If f and g are both odd, then $(fg)(-x) = f(-x)g(-x) = (-f(x))(-g(x)) = f(x)g(x) = (fg)(x)$. So fg is even.

21. Every exponent in f is even $(4, 2, 0)$, so f is automatically even. Then $f(-x) = f(x)$ for every x, including $x = 2$: $f(2) = f(-2)$. The graph mirrors across the y-axis. (The specific constants a, b, c don't change the symmetry — only the exponent parities do.)

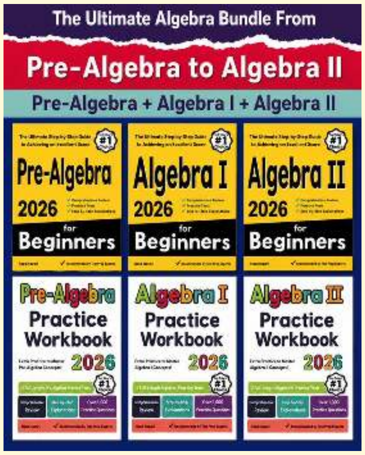
22. Keep the rule visible: $g(3) = 243 - 54 + 3 = 192$. $g(-3) = -243 + 54 - 3 = -192$. So $g(-3) = -g(3)$ ✓. Since all exponents are odd, g is odd everywhere, not just at $x = 3$. The graph has origin symmetry: rotate 180° around $(0, 0)$ and the picture matches. That gives a quick check on the answer.

23. Sign of terms doesn't decide evenness — symmetry does. Compute $h(-x) = x^2 - x$. Compare with $h(x) = x^2 + x$: not equal. Compare with $-h(x) = -x^2 - x$: not equal. So h is neither even nor odd. The student's reasoning confused *positive* with *symmetric*.

24. Start with the key idea: p is even iff every term has an even exponent. Exponents $6, 2, 0$ are already even; the troublemaker is kx^3 (exponent 3 is odd). The only way to make that term vanish is $k = 0$. Then $p(x) = 4x^6 + x^2 - 5$ — purely even. That gives a quick check on the answer.

Scan Me

Build Algebra Confidence From Pre-Algebra Through Algebra II



The Complete Algebra Success Bundle

Pre-Algebra, Algebra I, and Algebra II in one clear path

Friendly lessons, focused practice, and full-review support for every stage.

Scan Me

Scan for the Bundle

6 Books
3 Courses
1 Path

Bundle Value: Six coordinated books help students review missing skills, learn new algebra topics, and practice until the steps feel natural.

Complete Course Path

- ✓ Starts with Pre-Algebra foundations
- ✓ Moves smoothly into Algebra I skills
- ✓ Extends learning through Algebra II topics
- ✓ Great for review, tutoring, and summer study

One bundle, one steady path.

Step-by-Step Lessons

- ✓ Plain-English explanations students can follow
- ✓ Worked examples that show every important step
- ✓ Common mistakes called out before they stick
- ✓ Skill-building practice after each lesson
- ✓ Helpful for independent study or class support

Less guessing. More understanding.

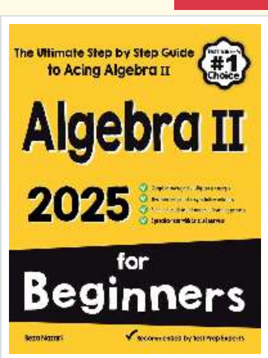
Practice That Sticks

- ✓ Matching practice workbooks for extra repetition
- ✓ Review sets to keep older skills fresh
- ✓ Answer explanations for checking thinking
- ✓ Strong support before tests and final exams
- ✓ Designed to build fluency and confidence

Practice today. Remember tomorrow.

STUDENT FAVORITE • Master Algebra II From the Ground Up

Algebra II for Beginners



✓ 100% Guaranteed
✓ Lifetime Support
✓ Trusted by Teachers

Start Your Algebra Journey Today! →

Written by a top math teacher & aligned with national and state Algebra II courses. From polynomial functions to logarithms, trigonometry, and rational expressions — explained the easy way.

- ✓ **Complete coverage** of every Algebra II concept — perfect companion to these worksheets
- ✓ **Step-by-step explanations** with worked examples on every topic
- ✓ **QR codes in every chapter** for free video lessons & bonus practice
- ✓ **2 full-length practice tests** with detailed answer keys

⋆ STUDENT'S #1 CHOICE ⋆

Teacher-recommended • 12,000+ Happy Students

↓ PDF EDITION

Scan Me

Instant download • any device

PAPERBACK

Scan Me

Paperback on Amazon

Hold it in your hands

Pair these free worksheets with *Algebra II for Beginners* and you have a complete self-paced course — concept lessons, daily practice, and full exam-style reviews, all in one path. → **EffortlessMath.com/product/algebra-ii-for-beginners**