Even Odd and Symmetric Functions

Name: ______________	Date: ______________	Score: _____ / 37

Quick Review

Even functions satisfy $f(-x) = f(x)$ for every x in the domain. Their graphs are symmetric about the y-axis. Examples: x^2, x^4, $|x|$, $\cos x$, any polynomial with only even-degree terms (constants count as degree 0).

Odd functions satisfy $f(-x) = -f(x)$. Their graphs are symmetric about the origin (a $180°$ rotation maps the graph to itself). Examples: x, x^3, $x^5 - x$, $\sin x$, $\frac{1}{x}$, any polynomial with only odd-degree terms and *no constant term*.

Neither. Most functions are neither even nor odd. $f(x) = x^2 + x$: $f(-x) = x^2 - x \neq f(x)$ and $\neq -f(x)$. So neither.

Testing algebraically. Compute $f(-x)$ and compare with $f(x)$ and $-f(x)$. If $f(-x) = f(x)$, even. If $f(-x) = -f(x)$, odd. Otherwise neither.

Shifted graphs. A horizontal shift breaks y-axis symmetry. $f(x) = |x - 2|$ has symmetry about $x = 2$, not the y-axis — so it's neither even nor odd. The shape is still V, but the symmetry center has moved. **Common slip:** thinking "every function is even or odd." Most are neither.

PRACTICE

Classify each function as even, odd, or neither.

1. Definition: f is even iff $f(-x) =$ ________
2. Definition: f is odd iff $f(-x) =$ ________
3. Classify $f(x) = x^4$. ________
4. Classify $f(x) = x^3 - x$. ________
5. Classify $f(x) = x^3 + x^2$. ________
6. Symmetry of $|x| + 1$? ________
7. Parameter a so $f(x) = x^4 + ax^3 - 6x^2 + 9$ is even. ________
8. Classify $f(x) = 2x^5 - 3x$. ________
9. Classify $p(x) = |x - 2| + 1$. ________
10. For $f(x) = x^5 - 3x$, compute $f(1) + f(-1)$. ________
11. Classify $f(x) = \cos x$. ________
12. Classify $f(x) = \sin x$. ________
13. Classify $f(x) = x^2 + 1$. ________
14. Classify $f(x) = \dfrac{1}{x}$. ________
15. True or false: every function is either even or odd. ________

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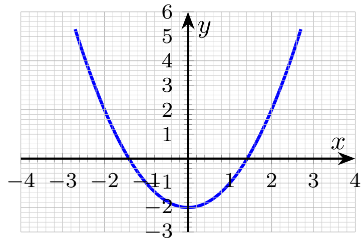
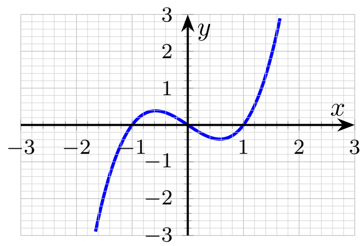
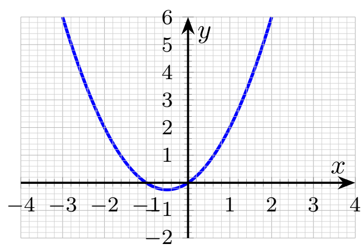
16. From the graph below, classify the function. ________



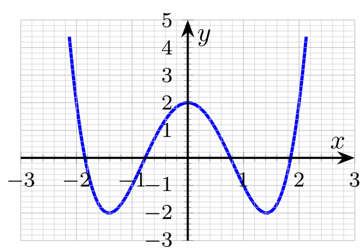
17. From the graph below, classify the function. ________



18. From the graph below, classify the function. ________



19. From the graph below, classify the function. ________



20. Decide if $f(x) = x^4 - 4x^2 + 2$ is even or odd algebraically. ________

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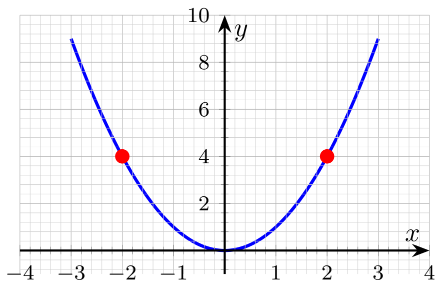
Word Problems

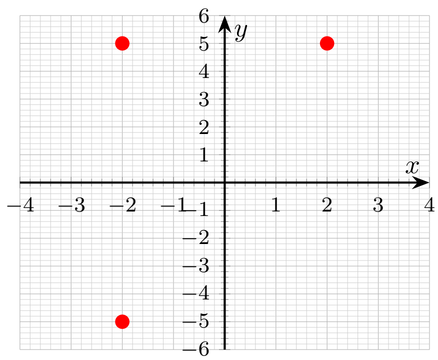
21. A potter's mug profile follows $y = x^2$ for $-3 \le x \le 3$ (a stylized cross-section). Use symmetry to find the height at $x = -2$ given that the height at $x = 2$ is 4 units, and explain. ____



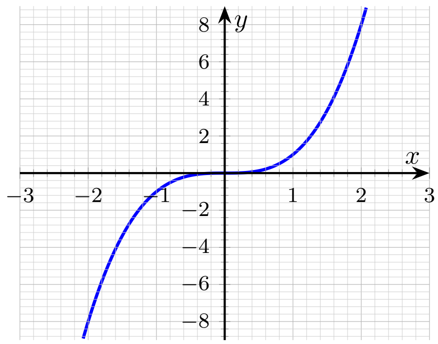
22. For each function listed in the table, classify as even (E), odd (O), or neither (N). Use the algebra test $f(-x)$ vs. $f(x)$ and $-f(x)$. ____

Function	Class
$x^2 - 3$	?
$x^3 + 2x$	?
$x + 1$	?
$\|x\| + x^2$	?

23. A graph passes through $(2, 5)$. If the function is even, what other point must be on it? If the function is odd, what other point? Sketch both cases on the same plane below. ____



24. Determine whether the function whose graph is shown below could be even, odd, or neither, and justify from symmetry alone (no formula given). ____



Additional Practice

25. If $f(x) = 2x - 5$, find $f(4)$. ________

26. If $g(x) = x^2 + 1$, find $g(-3)$. ________

27. For $f(x) = 3x + 2$, solve $f(x) = 14$. ________

28. Find $(f + g)(x)$ if $f = x + 1$, $g = 2x - 5$. ________

29. Find $(fg)(x)$ if $f = x - 2$, $g = x + 3$. ________

30. Find $f(g(x))$ if $f(x) = 2x$, $g(x) = x + 7$. ________

31. Find the inverse of $f(x) = x - 9$. ________

32. Find the inverse of $f(x) = 3x + 1$. ________

33. Domain of $f(x) = \sqrt{x - 4}$. ________

34. Domain of $f(x) = \dfrac{1}{x + 6}$. ________

35. Parent function for $y = |x| + 3$. ________

36. Shift $y = x^2$ left 4. ________

37. Average rate from $(1, 5)$ to $(4, 17)$. ________

Answer Keys

1. $f(x)$
2. $-f(x)$
3. even
4. odd
5. neither
6. y-axis (even)
7. $a = 0$
8. odd
9. neither
10. 0
11. even
12. odd
13. even
14. odd
15. false
16. even
17. odd
18. neither
19. even
20. even
21. 4 units
22. E, O, N, E
23. even $\to (-2, 5)$; odd $\to (-2, -5)$
24. odd

Additional Practice Answers

25. 3
26. 10
27. $x = 4$
28. $3x - 4$
29. $x^2 + x - 6$
30. $2x + 14$
31. $f^{-1}(x) = x + 9$
32. $f^{-1}(x) = \frac{x-1}{3}$
33. $x \geq 4$
34. $x \neq -6$
35. $y = |x|$
36. $y = (x + 4)^2$
37. 4

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. A careful way to see it: Matching outputs at opposite inputs $\Leftrightarrow$ y-axis symmetry. This is the part to check before moving on, because it keeps the answer tied to the original question.

2. Keep the rule visible: Opposite outputs $\Leftrightarrow$ origin symmetry. This is the part to check before moving on, because it keeps the answer tied to the original question.

3. Test by substituting $-x$: $f(-x) = (-x)^4 = x^4$, since an even power kills the sign. That equals $f(x)$, so f is even (y-axis symmetric).

4. Substitute $-x$: $f(-x) = (-x)^3 - (-x) = -x^3 + x$. Factor out the sign: $-x^3 + x = -(x^3 - x) = -f(x)$. Since $f(-x) = -f(x)$, the function is odd (origin symmetric).

5. Substitute $-x$: $f(-x) = (-x)^3 + (-x)^2 = -x^3 + x^2$. Compare: it isn't $f(x) = x^3 + x^2$ (so not even) and isn't $-f(x) = -x^3 - x^2$ (so not odd). Mixing odd and even powers makes it neither.

6. Keep the rule visible: $|-x| + 1 = |x| + 1$. Even. This is the part to check before moving on, because it keeps the answer tied to the original question.

7. One steady path is: Even polynomial has no odd-power term. So $a = 0$ kills the x^3 piece. That gives a quick check on the answer.

8. Substitute $-x$: $f(-x) = 2(-x)^5 - 3(-x) = -2x^5 + 3x$. Factor the sign: $-(2x^5 - 3x) = -f(x)$. All terms are odd powers with no constant, the hallmark of an odd function.

9. A careful way to see it: Shifted V is symmetric about $x = 2$, not the y-axis or origin. That gives a quick check on the answer.

10. Keep the rule visible: Odd function $\Rightarrow$ $f(1) + f(-1) = f(1) - f(1) = 0$. This is the part to check before moving on, because it keeps the answer tied to the original question.

11. One steady path is: $\cos(-x) = \cos x$. Standard fact. This is the part to check before moving on, because it keeps the answer tied to the original question.

12. Start with the key idea: $\sin(-x) = -\sin x$. Standard fact. This is the part to check before moving on, because it keeps the answer tied to the original question.

13. Substitute $-x$: $f(-x) = (-x)^2 + 1 = x^2 + 1$, which equals $f(x)$. The constant $+1$ is an even-degree term, so f stays even.

14. Keep the rule visible: $\frac{1}{-x} = -\frac{1}{x}$. This is the part to check before moving on, because it keeps the answer tied to the original question.

15. One steady path is: Most functions are neither. E.g. $f(x) = x + 1$ is neither. This is the part to check before moving on, because it keeps the answer tied to the original question.

16. Start with the key idea: Symmetric about the y-axis $\Rightarrow$ even. This is the part to check before moving on, because it keeps the answer tied to the original question.

17. A careful way to see it: Origin-symmetric S-curve $\Rightarrow$ odd. This is the part to check before moving on, because it keeps the answer tied to the original question.

18. Keep the rule visible: Vertex at $x = -\frac{1}{2}$ — not on the y-axis, no origin symmetry. Neither. That gives a quick check on the answer.

19. One steady path is: y-axis symmetric. All even powers + constant $\Rightarrow$ even. This is the part to check before moving on, because it keeps the answer tied to the original question.

20. Start with the key idea: $f(-x) = x^4 - 4x^2 + 2 = f(x)$. Even. This is the part to check before moving on, because it keeps the answer tied to the original question.

21. A careful way to see it: $f(x) = x^2$ is even, so $f(-2) = f(2) = 4$. The graph is y-axis symmetric, meaning the mirror image is the same shape. For the potter, that means the left and right walls of the mug have the same height at the same distance from center — exactly what y-axis symmetry buys you. That gives a quick check on the answer.

22. Keep the rule visible: $x^2 - 3$: even powers + constant, even. $x^3 + 2x$: odd powers, no constant, odd. $x + 1$: constant breaks the odd-function pattern (and the x term breaks evenness), neither. $|x| + x^2$: both pieces are even ($|-x| = |x|$ and $(-x)^2 = x^2$), so the sum is even. That gives a quick check on the answer.

23. Even means $f(-x) = f(x)$, so the point reflects across the y-axis: $(-2, 5)$. Odd means $f(-x) = -f(x)$, so the point rotates 180° about the origin: $(-2, -5)$. The three plotted points show all three positions; the original $(2, 5)$, the even partner $(-2, 5)$, and the odd partner $(-2, -5)$.

Answer graph

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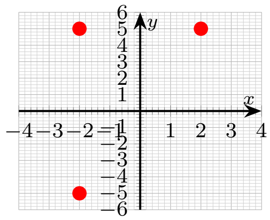

24. The graph is invariant under 180° rotation about the origin: every point (x, y) on the curve has a partner $(-x, -y)$ also on the curve. That's the geometric signature of an odd function. (The picture happens to be $y = x^3$, which is the textbook odd-function example, but the conclusion follows just from the symmetry.)

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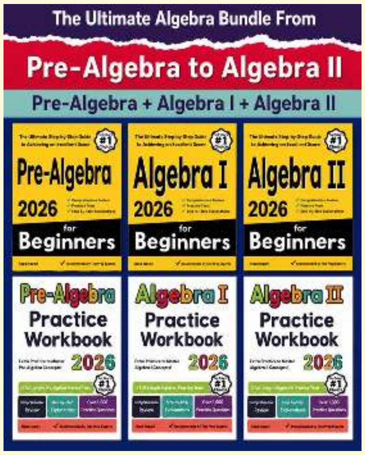

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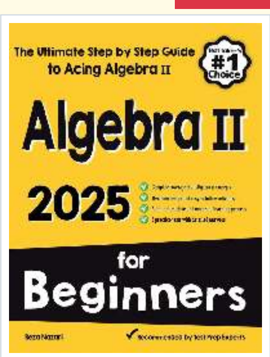

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