Equation of an Ellipse

Name: ____________	Date: ____________	Score: ______ / 30

Quick Review

An **ellipse** is the set of points whose distances to two fixed points (the **foci**) sum to a constant. Stretch a loop of string between two pins and trace with a pencil keeping the string taut – that's an ellipse.

Standard form, horizontal major axis ($a > b$). $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$. Center (h, k). *Vertices* (ends of the major axis) at $(h \pm a, k)$. *Co-vertices* (ends of the minor axis) at $(h, k \pm b)$. *Foci* at $(h \pm c, k)$ where $c^2 = a^2 - b^2$.

Vertical major axis ($b > a$). Same standard form, but now the larger denominator sits under the y-term. Vertices at $(h, k \pm b)$, co-vertices at $(h \pm a, k)$, foci at $(h, k \pm c)$ with $c^2 = b^2 - a^2$. (Whatever the orientation, c^2 is the difference of the squared semi-axes – always positive.)

Identifying the major axis. Look at the two denominators. The larger one's variable indicates the direction. $\frac{x^2}{25} + \frac{y^2}{9} = 1$: $25 > 9$, so the major axis is horizontal, $a = 5$, $b = 3$.

Ellipse vs. hyperbola: the operator. An ellipse uses a **plus** between the squared terms; a hyperbola uses a **minus**. Different shape, different formula.

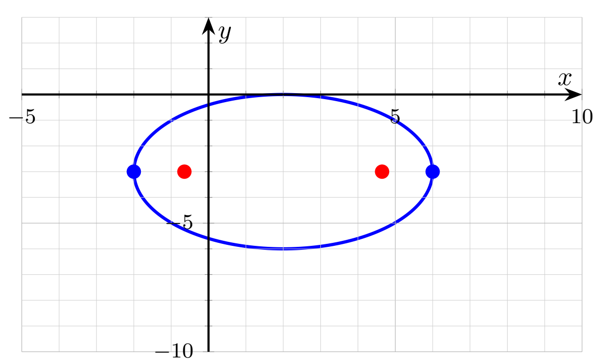
Building from features. Given the center, the vertices, and the foci, a is the center-to-vertex distance and c is the center-to-focus distance. Then $b^2 = a^2 - c^2$ (so $b < a$, as expected for the minor axis).

Common slips. Confusing a and b when the major axis is vertical. Using the hyperbola formula $c^2 = a^2 + b^2$ instead of the ellipse formula $c^2 = a^2 - b^2$. Forgetting that the larger denominator always sits under the variable whose axis is the major axis. Reading $(y + 3)$ as $k = +3$ (it's -3). The defining property: sum of focal distances $= 2a$ for every point on the ellipse – *not* a, *not* b, *not* c.

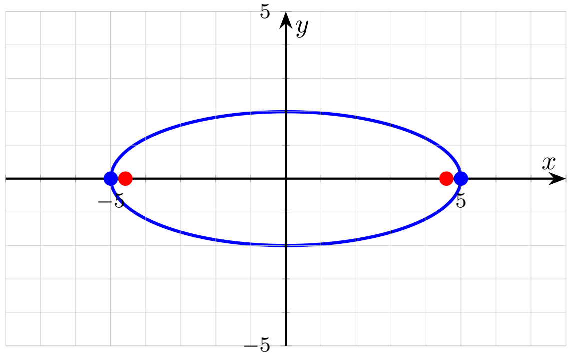
PRACTICE

For each ellipse, identify center, vertices, co-vertices, and foci as the problem requests. Always check which axis is the major axis by comparing denominators.

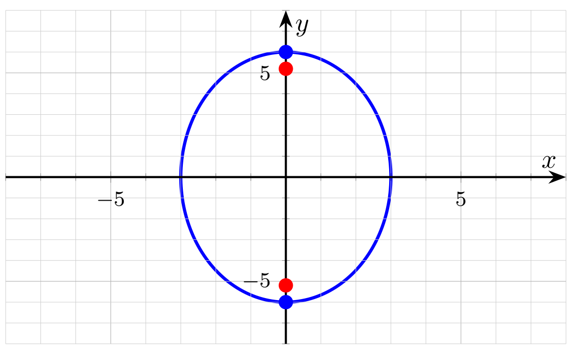
1. What is the center of $\frac{(x-2)^2}{16} + \frac{(y+3)^2}{9} = 1$? ________



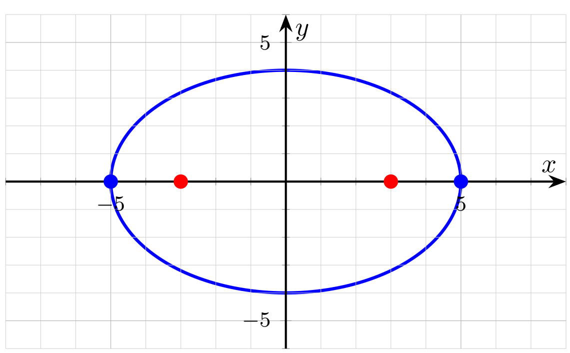
2. For $\frac{x^2}{25} + \frac{y^2}{4} = 1$, find the vertices. ________



3. For $\frac{x^2}{9} + \frac{y^2}{36} = 1$, identify the major axis and its vertices. ________



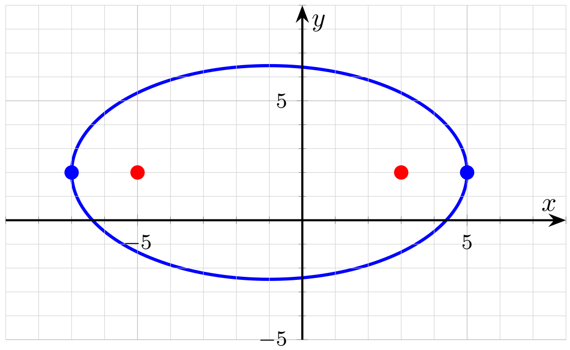
4. For $\frac{x^2}{25} + \frac{y^2}{16} = 1$, find the foci. ________



5. An ellipse has center $(0, 0)$, vertices $(\pm 5, 0)$, and foci $(\pm 4, 0)$. Find its equation. ________

6. For $\frac{x^2}{49} + \frac{y^2}{24} = 1$, find the foci. ________

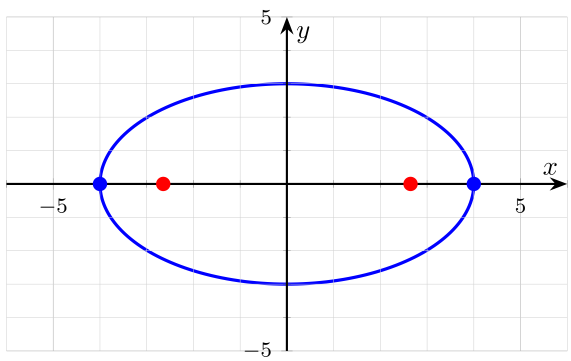
7. For $\frac{(x+1)^2}{36}+\frac{(y-2)^2}{20}=1$, identify the center and the vertices. ________



8. An ellipse has center $(3,-2)$, vertices $(3,6)$ and $(3,-10)$, and co-vertices $(-2,-2)$ and $(8,-2)$. Find its equation. ________

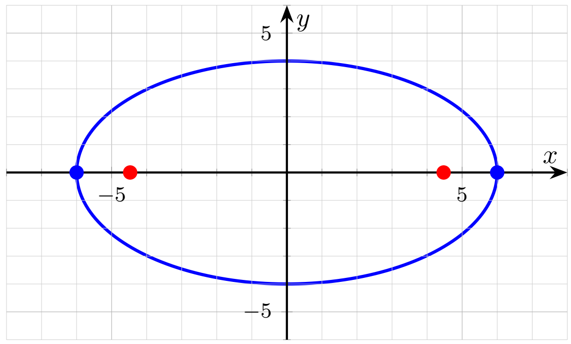
9. Mark TRUE or FALSE: For ellipses, $c^2=a^2+b^2$. ________

10. Find the equation of the ellipse with center at the origin, semi-major axis 4 horizontal, semi-minor axis 3. ________

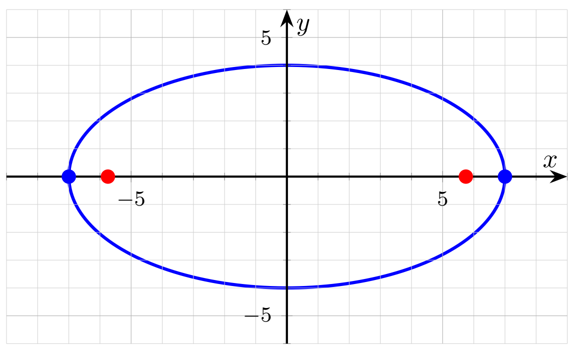


11. For $\frac{x^2}{16}+\frac{y^2}{9}=1$, find the co-vertices. ________

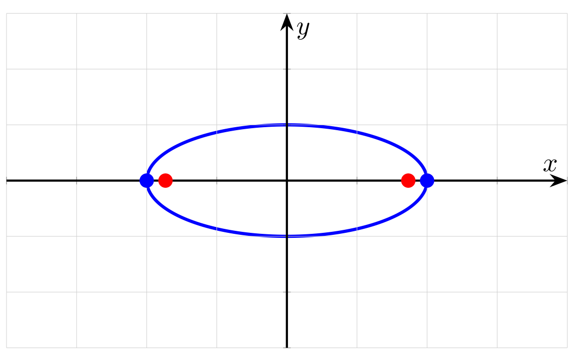
12. Find the foci of $\frac{x^2}{36} + \frac{y^2}{16} = 1$. ________



13. Find the equation of an ellipse with center at the origin, semi-axes 7 (horizontal) and 4 (vertical). ________

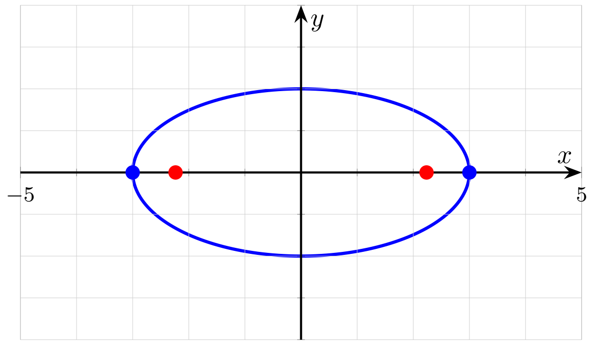


14. For $\frac{x^2}{4} + \frac{y^2}{1} = 1$, find the vertices. ________

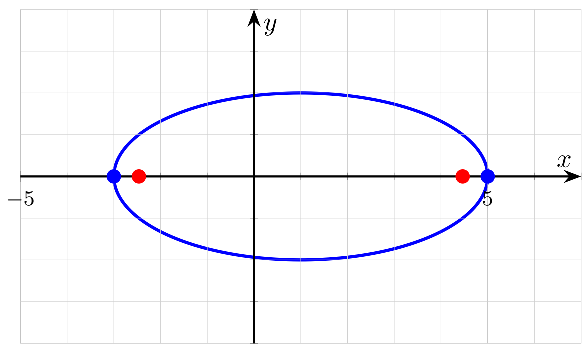


15. A parametric curve has $x = 3\cos t$, $y = 2\sin t$ for $0 \le t \le 2\pi$. Eliminate t to find its equation. ________

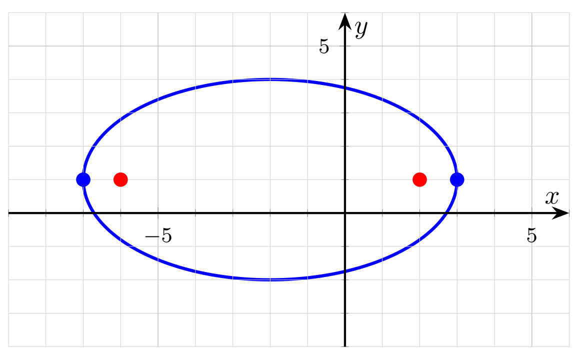
16. For $\frac{x^2}{9} + \frac{y^2}{4} = 1$, find the foci. ________



17. Find the equation of an ellipse centered at $(1, 0)$ with horizontal radius 4 and vertical radius 2. ________



18. Find the equation of an ellipse centered at $(-2, 1)$ with horizontal radius 5 and vertical radius 3. ________



19. Mark TRUE or FALSE: When the two semi-axes of an ellipse are equal, the ellipse becomes a circle. ________

20. For the ellipse $\frac{x^2}{49} + \frac{y^2}{16} = 1$, find the length of the minor axis. ________

Word Problems

21. A planet's orbit around the Sun is approximately elliptical. The Sun sits at one focus. Suppose an orbit has the Sun at $(3, 0)$ (in astronomical units), the other focus at $(-3, 0)$, and a semi-major axis of 5 AU. Find the equation of the orbit and the orbit's perihelion (closest approach to the Sun) distance. ____

22. A whispering gallery is shaped like the upper half of an ellipse. The room is 50 feet wide and 20 feet tall at the center. Two people standing at the foci can hear each other clearly. Find the distance between the two foci. ____

23. An ellipse-shaped patio has a major-axis length of 24 ft and a minor-axis length of 10 ft, centered at the origin with the major axis horizontal. Write its equation, and find the location of its foci. ____

24. An athletic track has a straight portion bordered by two semicircles, but a designer proposes replacing it with a perfect ellipse for a new infield. The infield is 100 m long (major axis) and 60 m wide (minor axis), centered at the origin. Find the equation of the elliptical boundary and the distance from the center to each focus. ____

Additional Practice

25. Center and radius of $(x-3)^2 + (y+2)^2 = 25$. ________

26. Write a circle with center $(0, 0)$ and radius 7. ________

27. Find the radius of $x^2 + y^2 = 64$. ________

28. Find the center of $(x+5)^2 + (y-1)^2 = 9$. ________

29. Vertex of $y = (x-4)^2 + 6$. ________

30. Axis of symmetry of $y = (x+2)^2 - 3$. ________

Answer Keys

1. $(2, -3)$
2. $(\pm 5,\ 0)$
3. vertical; $(0, \pm 6)$
4. $(\pm 3,\ 0)$
5. $\frac{x^2}{25} + \frac{y^2}{9} = 1$
6. $(\pm 5,\ 0)$
7. center $(-1, 2)$; vertices $(-7, 2), (5, 2)$
8. $\frac{(x-3)^2}{25} + \frac{(y+2)^2}{64} = 1$
9. FALSE
10. $\frac{x^2}{16} + \frac{y^2}{9} = 1$
11. $(0, \pm 3)$
12. $(\pm 2\sqrt{5},\ 0)$
13. $\frac{x^2}{49} + \frac{y^2}{16} = 1$
14. $(\pm 2,\ 0)$
15. $\frac{x^2}{9} + \frac{y^2}{4} = 1$
16. $(\pm\sqrt{5},\ 0)$
17. $\frac{(x-1)^2}{16} + \frac{y^2}{4} = 1$
18. $\frac{(x+2)^2}{25} + \frac{(y-1)^2}{9} = 1$
19. TRUE
20. 8
21. $\frac{x^2}{25} + \frac{y^2}{16} = 1$; perihelion: 2 AU
22. $2c = 30$ ft
23. $\frac{x^2}{144} + \frac{y^2}{25} = 1$; foci $(\pm\sqrt{119}, 0)$
24. $\frac{x^2}{2500} + \frac{y^2}{900} = 1$; $c = 40$ m

Additional Practice Answers

25. $(3, -2),\ r = 5$
26. $x^2 + y^2 = 49$
27. 8
28. $(-5, 1)$
29. $(4, 6)$
30. $x = -2$

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. A careful way to see it: $h = 2$ (from $(x-2)$), $k = -3$ (from $(y+3) = y-(-3)$). The sign-flip is the classic trap. That gives a quick check on the answer.

2. Larger denominator is 25 (under x^2), so the major axis is horizontal and $a = 5$. The vertices sit on the major axis at $(\pm 5, 0)$, with co-vertices on the minor axis at $(0, \pm 2)$.

3. Larger denominator 36 is under y^2: vertical major axis. $b^2 = 36$, so $b = 6$. Vertices at $(0, \pm 6)$. (Co-vertices at $(\pm 3, 0)$.)

4. Start with the key idea: $a^2 = 25,\ b^2 = 16.\ c^2 = a^2 - b^2 = 9$, so $c = 3$. Horizontal major axis: foci at $(\pm 3, 0)$. That gives a quick check on the answer.

5. Vertices and foci lie on the x-axis, so the major axis is horizontal. Center-to-vertex gives $a = 5$ ($a^2 = 25$); center-to-focus gives $c = 4$. For an ellipse, $b^2 = a^2 - c^2 = 25 - 16 = 9$. Equation: $\frac{x^2}{25} + \frac{y^2}{9} = 1$.

6. Larger denominator 49 sits under x^2, so the major axis is horizontal with $a^2 = 49$ and $b^2 = 24$. For an ellipse, $c^2 = a^2 - b^2 = 49 - 24 = 25$, so $c = 5$. Foci lie on the major axis: $(\pm 5, 0)$. (Don't use $a^2 + b^2$ – that's the hyperbola formula.)

7. Center $(-1, 2)$. $a^2 = 36$ (larger), under x, so horizontal major axis, $a = 6$. Vertices: $(-1 \pm 6, 2) = (-7, 2)$ and $(5, 2)$.

8. The vertices share $x = 3$, so the major axis is vertical. Center-to-vertex distance is $|6 - (-2)| = 8$, so the semi-major axis is 8 (denominator $= 64$, under the y-term). The co-vertices give the semi-minor axis: $|8 - 3| = 5$ (denominator 25, under the x-term). Because the major axis is vertical, the larger denominator goes under y: $\frac{(x-3)^2}{25} + \frac{(y+2)^2}{64} = 1$.

9. That's the hyperbola formula. For ellipses, $c^2 = a^2 - b^2$ (or $b^2 - a^2$ for a vertical major axis). The difference of squares, not the sum.

10. Center at the origin, semi-major axis $a = 4$ horizontal (squared: 16, under x^2) and semi-minor axis $b = 3$ (squared: 9, under y^2): $\frac{x^2}{16} + \frac{y^2}{9} = 1$. The larger denominator under x^2 confirms the horizontal major axis.

11. Larger denominator 16 under x^2 means the major axis is horizontal, so the smaller denominator gives $b^2 = 9$, $b = 3$. Co-vertices are the ends of the minor (vertical) axis, on the y-axis at $(0, \pm 3)$.

12. Start with the key idea: $c^2 = 36 - 16 = 20$, so $c = 2\sqrt{5} \approx 4.47$. Horizontal major axis: foci $(\pm 2\sqrt{5}, 0)$. That gives a quick check on the answer.

13. The horizontal semi-axis 7 sits under x^2 (squared: 49) and the vertical semi-axis 4 under y^2 (squared: 16). Equation: $\frac{x^2}{49} + \frac{y^2}{16} = 1$. (Square each semi-axis before placing it.)

14. Keep the rule visible: $a^2 = 4 > 1$, horizontal major axis, $a = 2$. Vertices $(\pm 2, 0)$. That gives a quick check on the answer.

15. From $x = 3\cos t$: $\cos t = \frac{x}{3}$. From $y = 2\sin t$: $\sin t = \frac{y}{2}$. Pythagorean identity: $\cos^2 t + \sin^2 t = 1$ gives $\frac{x^2}{9} + \frac{y^2}{4} = 1$. An ellipse with $a = 3,\ b = 2$.

16. Start with the key idea: $c^2 = 9 - 4 = 5$, so $c = \sqrt{5} \approx 2.24$. Horizontal major axis. This is the part to check before moving on, because it keeps the answer tied to the original question.

17. A careful way to see it: $h = 1,\ k = 0,\ a = 4,\ b = 2$: $\frac{(x-1)^2}{16} + \frac{(y-0)^2}{4} = 1$ simplifies to $\frac{(x-1)^2}{16} + \frac{y^2}{4} = 1$. That gives a quick check on the answer.

18. Center $(-2, 1)$ gives $(x + 2)$ and $(y - 1)$ (signs flip). The horizontal radius 5 squares to 25 under the x-term; the vertical radius 3 squares to 9 under the y-term: $\frac{(x+2)^2}{25} + \frac{(y-1)^2}{9} = 1$.

19. If $a = b$, then $c = 0$ (the foci collapse to the center) and the ellipse equation reduces to $\frac{(x-h)^2 + (y-k)^2}{a^2} = 1$, i.e., $(x-h)^2 + (y-k)^2 = a^2$ – a circle of radius a.

20. The larger denominator 49 is under x^2, so the minor axis is vertical with $b^2 = 16$, giving $b = 4$. The full minor-axis length spans from $-b$ to $+b$, so it is $2b = 8$.

21. Foci at $(\pm 3, 0)$ with center at the origin – so $c = 3$. Semi-major axis $a = 5$. Find b: $b^2 = a^2 - c^2 = 25 - 9 = 16$, so $b = 4$. Horizontal major axis: equation is $\frac{x^2}{25} + \frac{y^2}{16} = 1$. **Perihelion** is the closest distance from the planet to the Sun

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(the closer focus). The nearest vertex to focus $(3, 0)$ is the right vertex at $(5, 0)$, distance $5 - 3 = 2$ AU. (In general, perihelion $= a - c$ and aphelion $= a + c$. Here aphelion would be $5 + 3 = 8$ AU.)

22. Set up coordinates with the floor along the x-axis, centered at the room's midpoint. The horizontal semi-major axis is $a = 25$ (half the 50-ft width); the vertical semi-minor axis is $b = 20$ (the full height to the ceiling, since the room is the upper half of the ellipse). Find c: $c^2 = a^2 - b^2 = 625 - 400 = 225$, so $c = 15$ ft. Distance between the two foci: $2c = 30$ ft. (Whispering galleries work because of the reflective property: any sound emitted at one focus reflects off the elliptical wall and converges at the other focus – the U.S. Capitol's Statuary Hall is a famous example.)

23. Major-axis length $2a = 24$, so $a = 12$, $a^2 = 144$. Minor-axis length $2b = 10$, so $b = 5$, $b^2 = 25$. Equation: $\dfrac{x^2}{144} + \dfrac{y^2}{25} = 1$. For the foci: $c^2 = 144 - 25 = 119$, so $c = \sqrt{119} \approx 10.91$. Foci at $(\pm\sqrt{119}, 0)$, about ± 10.91 ft from the center along the major axis. (Not a clean integer for c, but a clean set-up otherwise.)

24. Half-lengths: $a = 50$ m (half the 100-m major axis), $b = 30$ m (half the 60-m minor axis). $a^2 = 2500$, $b^2 = 900$. Equation: $\dfrac{x^2}{2500} + \dfrac{y^2}{900} = 1$. Focus distance: $c^2 = 2500 - 900 = 1600$, so $c = 40$ m. The foci are at $(\pm 40, 0)$. (Sanity check via the defining property: the right vertex $(50, 0)$ has focal distances $|50 - 40| = 10$ and $|50 - (-40)| = 90$, summing to $100 = 2a$ ✓.)

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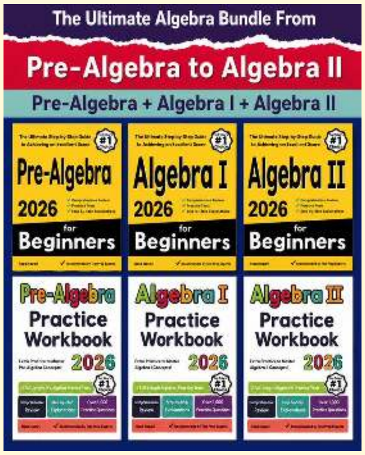

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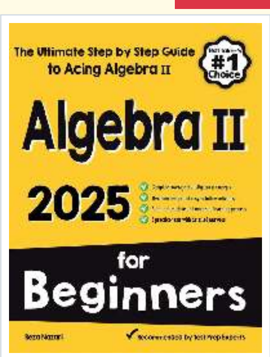

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