Equation of Parabola

Name: ______	Date: ______	Score: ____ / 36

Quick Review

A **parabola** is the U-shaped graph of a quadratic function. Two equation forms come up over and over – learn to flip between them on demand.

Vertex form. $y = a(x-h)^2 + k$. Read off the vertex (h, k) in one glance – no formula needed. The sign of a tells you which way it opens: $a > 0$ opens up (the vertex is the minimum), $a < 0$ opens down (the vertex is the maximum). The size of $|a|$ controls width: $|a| > 1$ stretches the parabola vertically (narrower than $y = x^2$); $0 < |a| < 1$ compresses it (wider).

Standard form. $y = ax^2 + bx + c$. The vertex's x-coordinate is $x_v = -\frac{b}{2a}$; plug back into the equation to get the y-coordinate. Standard form is convenient for y-intercepts (just c, since $f(0) = c$) and for plugging into the quadratic formula.

Converting standard to vertex (complete the square). Factor a out of the x-terms, complete the square inside the parentheses, balance the constants you added on the way in. Quick check: $y = x^2 - 4x + 7 = (x^2 - 4x + 4) + 7 - 4 = (x-2)^2 + 3$. Vertex $(2, 3)$.

Sign trap. $y = (x-4)^2 + 3$ has vertex $(4, 3)$, not $(-4, 3)$. The form is $y = (x-h)^2 + k$, so h is the value that makes the parenthesis zero. "$x - 4$" becomes zero at $x = 4$. The sign you see is the opposite of h.

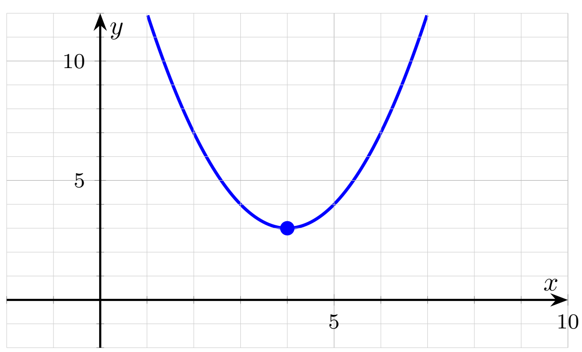
Build from a vertex and a point. Start with $y = a(x-h)^2 + k$, plug in the second point, solve for a. Quick check: vertex $(1, -2)$, point $(3, 6)$: $6 = a(2)^2 - 2 \Rightarrow 4a = 8 \Rightarrow a = 2$, so $y = 2(x-1)^2 - 2$.

Common slips. Reading $(x+5)^2$ as vertex $h = +5$ (it's -5). Forgetting to balance constants when factoring a out – $-2(x^2 - 4x) = -2[(x-2)^2 - 4]$ contributes $+8$ outside, not -8. Mixing up which sign of a opens up vs. down.

PRACTICE

For each parabola, give the vertex, opening direction, or full equation as the problem asks. Convert between standard and vertex form by completing the square.

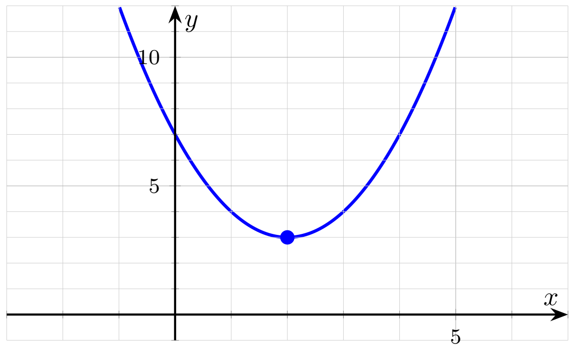
1. Find the vertex of $y = (x-4)^2 + 3$. ________



2. Find the vertex of $y = x^2 - 6x + 11$. ________

3. The parabola $y = -3(x+2)^2 + 5$ opens (up/down) and is (narrower/wider) than $y = x^2$. ________

4. Convert $y = x^2 - 4x + 7$ to vertex form. ________



5. A vertical parabola has vertex $(1, -2)$ and passes through $(3, 6)$. Find its equation in vertex form. ________

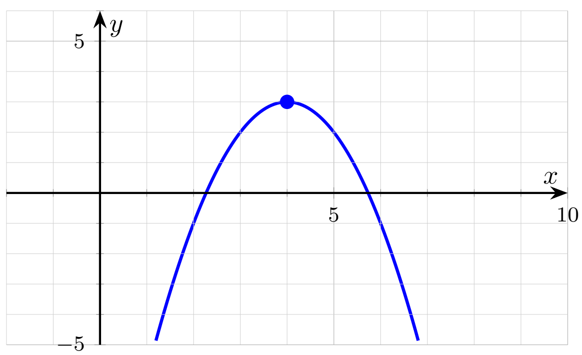
6. Convert $y = -2x^2 + 8x - 3$ to vertex form. ________

7. A vertical parabola opens upward, has vertex $(-2, 5)$, and passes through $(0, 13)$. Find its equation. ________

8. Convert $y = 3x^2 + 12x + 10$ to vertex form. ________

9. Find the vertex of $y = 2(x + 3)^2 - 7$. ________

10. Find the vertex of $y = -x^2 + 8x - 13$. ________



11. Mark TRUE or FALSE: In $y = a(x - h)^2 + k$, the parabola opens upward whenever $h > 0$. ________

12. The parabola $y = \frac{1}{4}(x - 2)^2 + 1$ is (narrower / wider) than $y = x^2$. ________

13. Find the equation of the parabola with vertex at the origin that passes through $(2, -8)$. ________

14. Convert $y = x^2 + 10x + 18$ to vertex form. ________

15. What is the axis of symmetry of $y = -2(x - 7)^2 + 9$? ________

16. Find the y-intercept of $y = (x - 3)^2 + 4$. ________

17. Find the equation in vertex form of the parabola with vertex $(0, -4)$ that passes through $(2, 0)$. ________

18. Find the vertex of $y = 4x^2 - 16x + 10$. ________

19. Mark TRUE or FALSE: The vertex of $y = (x + 5)^2 + 1$ is $(5, 1)$. ________

20. A bridge arch is modeled by $y = -\frac{1}{4}(x - 6)^2 + 9$, with x, y in meters. What's the maximum height of the arch? ________

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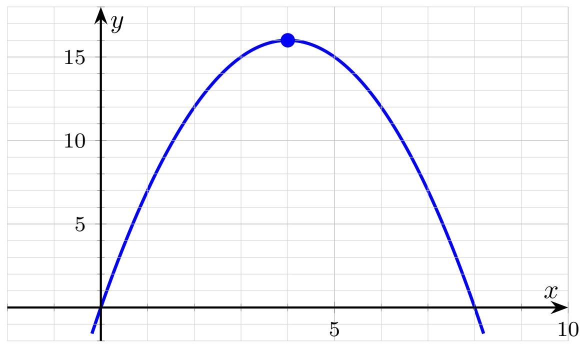
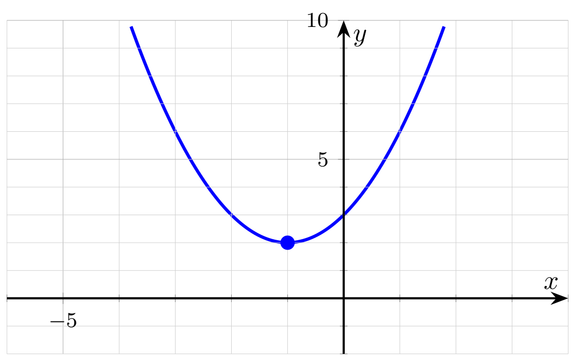
Word Problems

21. An engineering student is designing a parabolic bridge arch that passes through the origin $(0, 0)$ and has its vertex at $(4, 16)$. Find the equation of the arch in vertex form, and determine where (besides the origin) the arch meets the x-axis. ____



22. A water fountain spurts water in a parabolic arc. The water leaves the nozzle at ground level on one side, reaches a peak height of 20 feet at horizontal position $x = 5$ ft, and lands back at ground level on the other side. Find the equation of the arc, and determine where the water lands. ____

23. A graph shows a parabola with vertex $(-1, 2)$ that passes through the point $(1, 6)$. Find the parabola's equation in vertex form. ____



24. A satellite dish has a parabolic cross-section. Set up a coordinate system with the vertex at the origin so the dish opens upward. The dish is 6 feet wide at a height of 2 feet above the vertex. Write the equation of the cross-section. ____

Additional Practice

25. Center and radius of $(x-3)^2 + (y+2)^2 = 25$. ______

26. Write a circle with center $(0, 0)$ and radius 7. ______

27. Find the radius of $x^2 + y^2 = 64$. ______

28. Find the center of $(x+5)^2 + (y-1)^2 = 9$. ______

29. Vertex of $y = (x-4)^2 + 6$. ______

30. Axis of symmetry of $y = (x+2)^2 - 3$. ______

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31. Classify $x^2 + y^2 = 36$. ________

32. Classify $\frac{x^2}{9} + \frac{y^2}{4} = 1$. ________

33. Classify $\frac{x^2}{16} - \frac{y^2}{9} = 1$. ________

34. Major axis length of $\frac{x^2}{25} + \frac{y^2}{9} = 1$. ________

35. Minor axis length of $\frac{x^2}{25} + \frac{y^2}{9} = 1$. ________

36. Opening of $x = (y-1)^2 + 4$. ________

Answer Keys

1. $(4,\ 3)$
2. $(3,\ 2)$
3. down; narrower
4. $y=(x-2)^2+3$
5. $y=2(x-1)^2-2$
6. $y=-2(x-2)^2+5$
7. $y=2(x+2)^2+5$
8. $y=3(x+2)^2-2$
9. $(-3,\ -7)$
10. $(4,\ 3)$
11. FALSE
12. wider
13. $y=-2x^2$
14. $y=(x+5)^2-7$
15. $x=7$
16. $(0,\ 13)$
17. $y=x^2-4$
18. $(2,\ -6)$
19. FALSE
20. 9 m at $x=6$ m
21. $y=-(x-4)^2+16$; other x-intercept: $(8,0)$
22. $y=-\frac{4}{5}(x-5)^2+20$; lands at $(10,0)$
23. $y=(x+1)^2+2$
24. $y=\frac{2}{9}x^2$

Additional Practice Answers

25. $(3,-2),\ r=5$
26. $x^2+y^2=49$
27. 8
28. $(-5,1)$
29. $(4,6)$
30. $x=-2$
31. circle
32. ellipse
33. hyperbola
34. 10
35. 6
36. right

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. Compare to $y=(x-h)^2+k$. $h=4, k=3$. The sign inside the parentheses already tells you the answer: $(x-4)$ is zero at $x=4$, so $h=4$. Beware of swapping the sign.

2. In standard form $y=ax^2+bx+c$, the vertex's x-coordinate is $x_v=-\frac{b}{2a}=-\frac{-6}{2(1)}=3$. Substitute back to get the y-coordinate: $y=3^2-6(3)+11=9-18+11=2$. So the vertex is $(3,2)$. (Completing the square gives the same thing: $x^2-6x+11=(x-3)^2+2$.)

3. One steady path is: $a=-3$ is negative, so the parabola opens down. $|a|=3>1$, so it's stretched vertically – narrower than $y=x^2$. That gives a quick check on the answer.

4. Half of -4 is -2; square gives 4. $x^2-4x+7=(x^2-4x+4)+7-4=(x-2)^2+3$. Vertex $(2,3)$ matches the plotted dot.

5. Start with $y=a(x-1)^2-2$. Plug in $(3,6)$: $6=a(2)^2-2\Rightarrow 8=4a\Rightarrow a=2$. So $y=2(x-1)^2-2$. (Sanity check at $(3,6)$: $2(4)-2=6$ ✓.)

6. Factor -2 from the x-terms: $y=-2(x^2-4x)-3$. Inside the parentheses, $x^2-4x=(x-2)^2-4$. So $y=-2[(x-2)^2-4]-3=-2(x-2)^2+8-3=-2(x-2)^2+5$. (Watch the sign: $-2\cdot-4=+8$, not -8.)

7. The vertex $(-2,5)$ gives the shell $y=a(x+2)^2+5$ (remember $h=-2$ turns into $(x+2)$). Plug in the point $(0,13)$ to find a: $13=a(0+2)^2+5=4a+5$, so $4a=8$ and $a=2$. Since $a=2>0$, it does open upward as stated. Equation: $y=2(x+2)^2+5$.

8. Factor 3 out of the x-terms only: $y=3(x^2+4x)+10$. Inside the parentheses, half of 4 is 2 and $2^2=4$, so $x^2+4x=(x+2)^2-4$. Multiply that -4 back by the 3 outside: $y=3[(x+2)^2-4]+10=3(x+2)^2-12+10=3(x+2)^2-2$. The -12 is where students often slip by forgetting to distribute the 3.

9. Match to $y=a(x-h)^2+k$. The parenthesis $(x+3)$ is zero when $x=-3$, so $h=-3$ (the sign flips from the $+3$ you see). The constant outside is $k=-7$. Vertex $(-3,-7)$.

10. Here $a=-1, b=8, c=-13$. The vertex's x-coordinate is $x_v=-\frac{b}{2a}=-\frac{8}{2(-1)}=4$. Plug back in: $y=-(4)^2+8(4)-13=-16+32-13=3$. Vertex $(4,3)$. Since $a<0$, this is the maximum, matching the downward curve.

11. The opening direction depends on the sign of a, not on h. $a>0$ opens up; $a<0$ opens down.

12. Start with the key idea: $|a|=\frac{1}{4}<1$, so the parabola is compressed vertically – wider than $y=x^2$. That gives a quick check on the answer.

13. Vertex at the origin means $h=k=0$, so the form is just $y=ax^2$. Plug in the point $(2,-8)$: $-8=a(2)^2=4a$, so $a=-2$. Equation: $y=-2x^2$ (it opens down, as the negative y-value suggests).

14. The leading coefficient is 1, so just complete the square directly. Half of 10 is 5, and $5^2=25$. Add and subtract 25: $x^2+10x+18=(x^2+10x+25)-25+18=(x+5)^2-7$. The -25 balances the $+25$ you slipped inside.

15. The vertex of $y=-2(x-7)^2+9$ is $(7,9)$, since $(x-7)$ is zero at $x=7$. For a vertical parabola the axis of symmetry is the vertical line through the vertex's x-coordinate, so $x=7$.

16. The y-intercept is where the curve crosses the y-axis, so set $x=0$: $y=(0-3)^2+4=(-3)^2+4=9+4=13$. The intercept is $(0,13)$.

17. A careful way to see it: $y=ax^2-4$. At $(2,0)$: $0=4a-4\Rightarrow a=1$. So $y=x^2-4$. (This crosses the x-axis at ±2, matching the given point.) That gives a quick check on the answer.

18. With $a=4$, $b=-16$: $x_v=-\frac{b}{2a}=-\frac{-16}{2(4)}=\frac{16}{8}=2$. Substitute back: $y=4(2)^2-16(2)+10=16-32+10=-6$. Vertex $(2,-6)$.

19. One steady path is: $(x+5)$ is zero at $x=-5$, so the vertex is $(-5,1)$. Read the value that *makes the parenthesis zero*, not the sign you see written. That gives a quick check on the answer.

20. Vertex form: vertex $(6,9)$, $a=-\frac{1}{4}<0$ so it opens down and the vertex is the maximum. Max height $=9$ m, at horizontal position $x=6$ m.

21. Use vertex form: $y=a(x-4)^2+16$. Plug in $(0,0)$: $0=a(16)+16$, so $a=-1$. Equation: $y=-(x-4)^2+16$. For the other x-intercept, the parabola is symmetric about its axis $x=4$. The origin sits 4 units left of the axis,

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so the other intercept is 4 units right: $x = 4 + 4 = 8$, giving $(8, 0)$. **Check:** $y = -(8-4)^2 + 16 = -16 + 16 = 0$ ✓.

22. Vertex form: $y = a(x-5)^2 + 20$. The water leaves the nozzle at the origin $(0, 0)$. Plug in: $0 = a(-5)^2 + 20 = 25a + 20$, so $a = -\frac{4}{5}$. Equation: $y = -\frac{4}{5}(x-5)^2 + 20$. The water lands when $y = 0$ again: by symmetry, that's 5 units to the right of the vertex, so $x = 10$. The water lands at $(10, 0)$. **Verify:** $y = -\frac{4}{5}(25) + 20 = -20 + 20 = 0$ ✓.

23. Use vertex form: $y = a(x-(-1))^2 + 2 = a(x+1)^2 + 2$. Plug in $(1, 6)$: $6 = a(2)^2 + 2 = 4a + 2$, so $4a = 4$ and $a = 1$. Equation: $y = (x+1)^2 + 2$. **Sanity check:** at $x = -1$, $y = 2$ (vertex ✓); at $x = 1$, $y = (2)^2 + 2 = 6$ (given point ✓). The parabola opens upward because $a = 1 > 0$.

24. Vertex at origin, opens upward: $y = ax^2$ with $a > 0$. The dish is 6 ft wide at $y = 2$, so the point $(3, 2)$ lies on the parabola (half the width on each side of the axis). Plug in: $2 = a(9)$, so $a = \frac{2}{9}$. Equation: $y = \frac{2}{9}x^2$. **Sanity check:** at $x = -3$, $y = \frac{2}{9}(9) = 2$ ✓. The dish is symmetric, just as a real satellite dish should be.

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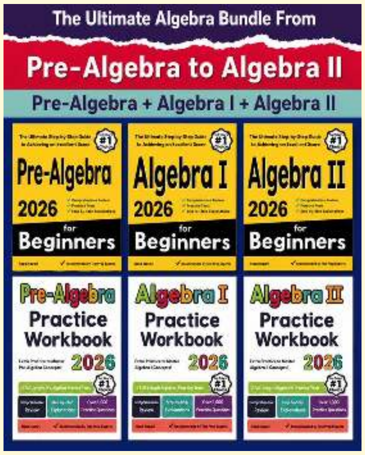

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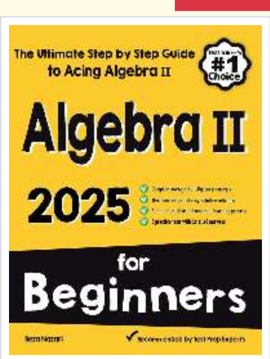

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