

## End Behavior of Polynomial Functions

Name: \_\_\_\_\_

Date: \_\_\_\_\_

Score: \_\_\_\_\_ / 24

### Q Quick Review

The **end behavior** of a polynomial describes what happens to  $f(x)$  as  $x$  heads toward  $+\infty$  or  $-\infty$ . The *leading term* (highest-degree term) dominates everything else when  $|x|$  gets huge — lower-degree pieces become negligible.

The **four cases** depend on the degree's parity and the leading coefficient's sign:

- *Even degree, positive lead* ( $\uparrow\uparrow$ ): both ends rise to  $+\infty$ . (Quick check:  $f(x) = x^2$ ,  $f(x) = 2x^4$ .)
- *Even degree, negative lead* ( $\downarrow\downarrow$ ): both ends fall to  $-\infty$ . (Quick check:  $f(x) = -x^4$ .)
- *Odd degree, positive lead* ( $\downarrow\uparrow$ ): falls on the left, rises on the right. (Quick check:  $f(x) = x^3$ .)
- *Odd degree, negative lead* ( $\uparrow\downarrow$ ): rises on the left, falls on the right. (Quick check:  $f(x) = -x^3$ .)

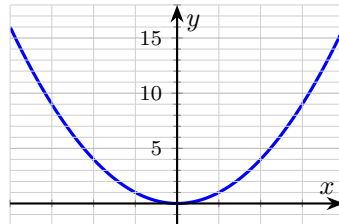
The first practice items pair each case with a graph, so you can connect the rule to the picture right away. **Rules:** lower-degree terms don't affect end behavior. Polynomials of degree  $\geq 1$  have no horizontal asymptotes (they keep growing). Even-degree means matched directions, odd-degree means opposite directions. The leading coefficient's *sign* chooses whether the matched (or right-side) end goes up or down.

### PRACTICE

Determine the end behavior of each polynomial.

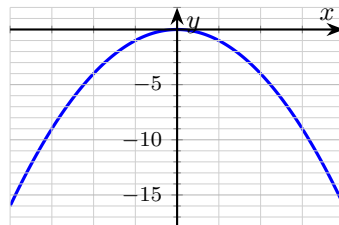
1. End behavior of  $f(x) = 3x^4 - 2x + 1$

**Case graph.** Compare the leading term's degree and sign to the arm directions.



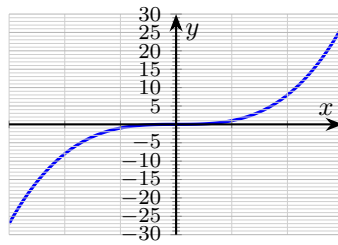
2. End behavior of  $f(x) = 2x^3 - 5x + 4$

**Case graph.** Compare the leading term's degree and sign to the arm directions.



3. End behavior of  $g(x) = -5x^4 + 3x^2 - 1$

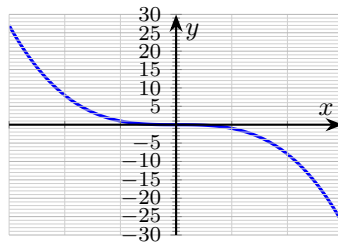
**Case graph.** Compare the leading term's degree and sign to the arm directions.



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4. End behavior of  $h(x) = -x^5 + 4x^2 - 7$

**Case graph.** Compare the leading term's degree and sign to the arm directions.



\_\_\_\_\_

5. Which form gives both ends up:  $-2x^4 + \dots$ ,  $5x^3 + \dots$ ,  $3x^6 + \dots$ ,  $-x^7 + \dots$ ?

\_\_\_\_\_

6. Degree-5 polynomial with negative leading coefficient: end behavior?

\_\_\_\_\_

7. Leading coefficient of  $f(x) = -3x^6 + 7x^4 - 2x + 5$

\_\_\_\_\_

8. For  $p(x) = ax^5 - 4x^2 + 7$  with right end down, left end up, which  $a$  works among  $-3, 0, 3, 1$ ?

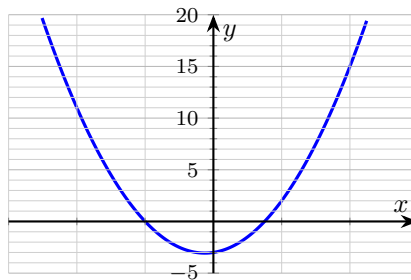
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9. End behavior with  $f(x) \rightarrow -\infty$  as  $x \rightarrow -\infty$ ,  $f(x) \rightarrow +\infty$  as  $x \rightarrow +\infty$ : one possible positive leading coefficient

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10. Determine the end behavior shown in the graph of  $f(x) = 4x^2 + x - 3$ :

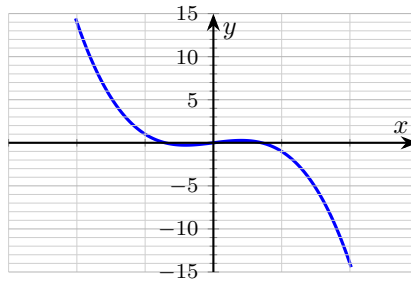
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11. Determine the end behavior shown in the graph of  $f(x) = -2x^3 + x$ : \_\_\_\_\_



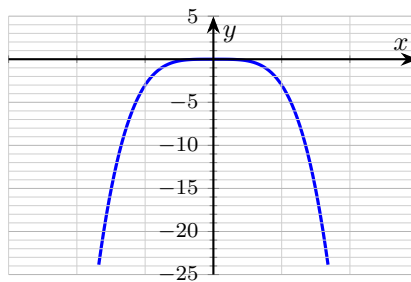
12. End behavior of  $f(x) = x^6 - 2x^2$  \_\_\_\_\_

13. End behavior of  $f(x) = -x^8 + 3x^4 + 1$  \_\_\_\_\_

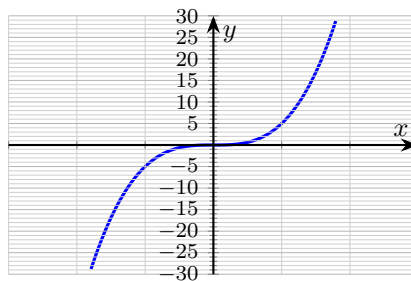
14. Are end behaviors affected by lower-degree terms? \_\_\_\_\_

15. Does a degree- 1 polynomial have a horizontal asymptote? \_\_\_\_\_

16. Determine the end behavior shown in the graph of  $f(x) = -3x^4$ : \_\_\_\_\_



17. Determine the end behavior shown in the graph of  $f(x) = 5x^3$ : \_\_\_\_\_



18. Two polynomials have leading terms  $2x^6$  and  $-2x^6$ . Do their end behaviors match? \_\_\_\_\_

19. End behavior of  $f(x) = x^7 - 1000x^2$  \_\_\_\_\_

20. End behavior of  $f(x) = -4x^2 + 10000$  \_\_\_\_\_



## ◆ Word Problems

21. A polynomial model  $p(x) = -3x^6 + 7x^4 - 2x + 5$  describes a decaying physical quantity for large  $|x|$ . \_\_\_\_\_  
Describe the end behavior and explain why the lower-degree terms don't matter.
22. For what values of  $a$  does the polynomial  $p(x) = ax^5 - 4x^2 + 7$  have end behavior *right end down, left end up*? \_\_\_\_\_
23. A graph rises to  $+\infty$  on both ends as  $|x| \rightarrow \infty$ . What can you say about its degree's parity and the leading coefficient's sign? \_\_\_\_\_
24. A polynomial  $f(x)$  has end behavior  $f(x) \rightarrow -\infty$  as  $x \rightarrow -\infty$  and  $f(x) \rightarrow +\infty$  as  $x \rightarrow +\infty$ . Give one possible polynomial of degree 3 that fits. \_\_\_\_\_



## Answer Keys

- |  |  |
|--|--|
| 1. $f(x) \rightarrow +\infty ax \rightarrow \pm\infty$   | 13. $f(x) \rightarrow -\infty ax \rightarrow \pm\infty$  |
| 2. $f(x) \rightarrow +\infty ax \rightarrow +\infty, f(x) \rightarrow -\infty ax \rightarrow -\infty$  | 14. <i>no</i>  |
| 3. $g(x) \rightarrow -\infty ax \rightarrow \pm\infty$   | 15. <i>no</i>  |
| 4. $h(x) \rightarrow -\infty ax \rightarrow +\infty, h(x) \rightarrow +\infty ax \rightarrow -\infty$  | 16. $f(x) \rightarrow -\infty ax \rightarrow \pm\infty$  |
| 5. $3x^6 + \dots$  | 17. $f(x) \rightarrow +\infty ax \rightarrow +\infty, f(x) \rightarrow -\infty ax \rightarrow -\infty$ |
| 6. $f(x) \rightarrow -\infty ax \rightarrow +\infty, f(x) \rightarrow +\infty ax \rightarrow -\infty$  | 18. <i>no</i> — opposite vertical directions   |
| 7. $-3$  | 19. $f(x) \rightarrow +\infty ax \rightarrow +\infty, f(x) \rightarrow -\infty ax \rightarrow -\infty$ |
| 8. $a = -3$  | 20. $f(x) \rightarrow -\infty ax \rightarrow \pm\infty$  |
| 9. 1 (or any positive number with an odd-degree polynomial)  | 21. $p(x) \rightarrow -\infty$ as $x \rightarrow \pm\infty$  |
| 10. $f(x) \rightarrow +\infty ax \rightarrow \pm\infty$  | 22. $a < 0$  |
| 11. $f(x) \rightarrow -\infty ax \rightarrow +\infty, f(x) \rightarrow +\infty ax \rightarrow -\infty$ | 23. <i>even degree, positive leading coefficient</i>   |
| 12. $f(x) \rightarrow +\infty ax \rightarrow \pm\infty$  | 24. $f(x) = x^3$ (or any cubic with positive leading coefficient)                                      |

## Step-by-Step Explanations

- Leading term  $3x^4$ : even degree (matched ends) and positive coefficient (rising). Both ends up.
- Leading term  $2x^3$ : odd degree (opposite ends) and positive coefficient. Right end up, left end down.
- One steady path is: Leading  $-5x^4$ : even (matched), negative (falling). Both ends down. That gives a quick check on the answer.
- Start with the key idea: Leading  $-x^5$ : odd (opposite), negative. Right end down, left end up. That gives a quick check on the answer.
- A careful way to see it: Both ends up means even degree and positive lead. Only  $3x^6$  satisfies both. That gives a quick check on the answer.
- Keep the rule visible: Odd degree + negative lead: right end falls, left end rises. That gives a quick check on the answer.
- The leading term is the highest-degree one:  $-3x^6$ . The coefficient (with its sign) is  $-3$ .
- Right down + left up = odd degree + negative lead. Need  $a < 0$ . Among the choices,  $a = -3$ .
- Opposite-direction ends  $\Rightarrow$  odd degree. Right end up  $\Rightarrow$  positive lead. So any odd-degree polynomial with positive lead works;  $f(x) = x^5$  is the simplest example.
- Only the leading term decides end behavior. Here it's  $4x^2$ : even degree means the two ends match, and a positive coefficient means they rise. So both arms go to  $+\infty$  — exactly the upward-opening parabola shown.
- The leading term is  $-2x^3$ : odd degree means the ends go opposite directions, and the negative coefficient flips the usual cubic so the right end falls and the left end rises — just as the curve shows.
- Start with the key idea: Leading  $x^6$ : even, positive. Both ends rise. This is the part to check before moving on, because it keeps the answer tied to the original question.
- A careful way to see it: Leading  $-x^8$ : even, negative. Both ends fall. This is the part to check before moving on, because it keeps the answer tied to the original

question.

- Keep the rule visible: As  $|x| \rightarrow \infty$ , the leading term dominates. Lower-degree pieces become negligible. That gives a quick check on the answer.
- Polynomials of degree  $\geq 1$  have no horizontal asymptote — they grow without bound.
- The leading (and only) term is  $-3x^4$ : even degree gives matched ends, and the negative coefficient turns them both downward. So both arms head to  $-\infty$ , matching the graph that opens downward.
- Leading term  $5x^3$ : odd degree sends the ends opposite ways, and the positive coefficient gives the standard cubic shape — right end up, left end down, as the curve shows.
- Both are even-degree, so both have matched ends. But the signs differ:  $+2x^6$  gives  $\uparrow\uparrow$ ,  $-2x^6$  gives  $\downarrow\downarrow$ .
- Even though  $-1000x^2$  is huge for moderate  $x$ , the  $x^7$  eventually dominates. Odd degree, positive lead: right up, left down.
- Leading  $-4x^2$ : even, negative. Both ends fall, no matter how large the constant 10000 is.
- The leading term is  $-3x^6$ : even-degree  $\Rightarrow$  matched ends, negative lead  $\Rightarrow$  both ends fall. As  $|x|$  grows,  $|-3x^6|$  grows like  $3x^6$ , while  $7x^4$  only grows like  $x^4$ . The ratio  $\frac{7x^4}{3x^6} = \frac{7}{3x^2} \rightarrow 0$ , so the lower term becomes negligible.
- Opposite ends  $\Rightarrow$  odd degree (already satisfied since degree is 5). Right end down + left end up  $\Rightarrow$  negative leading coefficient. So any  $a < 0$  works.
- Both ends matching direction  $\Rightarrow$  even degree. Both ends going up  $\Rightarrow$  positive leading coefficient. The specific degree (could be 2, 4, 6, ...) and the exact coefficient (any positive number) aren't pinned down by end behavior alone.
- Opposite-direction ends + right end up  $\Rightarrow$  odd degree + positive lead. With degree 3 fixed, the simplest example is  $f(x) = x^3$ . Other choices:  $2x^3$ ,  $x^3 - x$ ,  $x^3 + 5x^2 - 4$ , and so on. The leading coefficient just has to be positive.



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