Double Angle Identities

Name: ____________	Date: ____________	Score: ______ / 30

Quick Review

Double-angle identities take a known sine or cosine value and give you the trig values at twice the angle, without ever computing the angle itself. Memorize all three forms of the cosine version – they're equivalent, but one of them will make each problem easier.

Sine double angle.

$\sin(2\theta) = 2\sin\theta\cos\theta$

Cosine double angle (three equivalent forms).

$\cos(2\theta) = \cos^2\theta - \sin^2\theta$

$\cos(2\theta) = 2\cos^2\theta - 1$

$\cos(2\theta) = 1 - 2\sin^2\theta$

Tangent double angle.

$\tan(2\theta) = \dfrac{2\tan\theta}{1-\tan^2\theta}$

Half-angle identities.

$\sin\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1-\cos\theta}{2}}$

$\cos\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1+\cos\theta}{2}}$

The sign in the half-angle formulas depends on the quadrant of $\dfrac{\theta}{2}$, not θ.

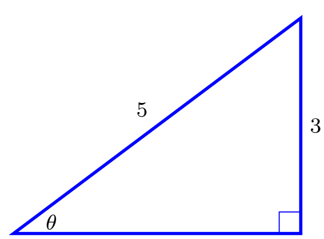
Picking the right cosine form. If the problem gives you $\sin\theta$ alone, use $\cos 2\theta = 1 - 2\sin^2\theta$ – no need to find $\cos\theta$ first. If it gives you $\cos\theta$ alone, use $2\cos^2\theta - 1$. If you have both, any form works.

Common slips. Writing $\sin 2\theta = 2\sin\theta$ (no – missing the $\cos\theta$ factor). Writing $\tan 2\theta = 2\tan\theta$ (no). Forgetting the $\pm$ in half-angle formulas, or getting the sign wrong because you checked the wrong quadrant. Confusing $\cos^2\theta$ (which is $(\cos\theta)^2$) with $\cos(\theta^2)$.

PRACTICE

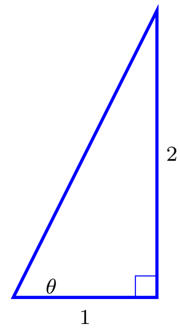
Use the double-angle (or half-angle) identity. Give exact answers where possible.

1. Write $\sin(2\theta)$ as an expression in $\sin\theta$ and $\cos\theta$. ________
2. Write $\cos(2\theta)$ in the form using $\sin\theta$ only. ________
3. If $\sin\theta = \dfrac{3}{5}$ and θ is acute, find $\sin(2\theta)$. The triangle below sets up the given ratio $\sin\theta = \dfrac{3}{5}$ (opposite over hypotenuse). ________



4. If $\sin\theta = \dfrac{3}{5}$ and θ is acute, find $\cos(2\theta)$. ________

5. If $\tan\theta = 2$, find $\tan(2\theta)$. The triangle below sets up $\tan\theta = \frac{2}{1}$ (opposite over adjacent). ________



6. If $\cos\theta = -\frac{3}{5}$ and $\theta \in$ Q III, find $\sin\theta$. ________

7. With $\cos\theta = -\frac{3}{5}$ in Q III, find $\sin(2\theta)$. ________

8. With $\cos\theta = -\frac{3}{5}$ in Q III, find $\cos(2\theta)$. ________

9. Find the exact value of $\sin(15°)$ using a half-angle identity. ________

10. Find the exact value of $\cos(22.5°)$. ________

11. Simplify $2\sin 30° \cos 30°$. ________

12. Simplify $\cos^2 75° - \sin^2 75°$. ________

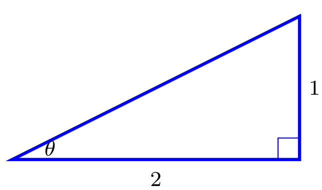
13. Simplify $1 - 2\sin^2 15°$. ________

14. If $\sin\theta = \frac{5}{13}$ and θ is in Q II, find $\sin(2\theta)$. ________

15. If $\sin\theta = \frac{5}{13}$ and θ is in Q II, find $\cos(2\theta)$. ________

16. Find the exact value of $\sin(67.5°)$. ________

17. If $\tan\theta = \frac{1}{2}$, find $\tan(2\theta)$. The triangle below shows the given ratio $\tan\theta = \frac{1}{2}$. ________



18. Simplify $2\cos^2 22.5° - 1$. ________

19. Find $\sin(2\theta)$ if $\cos\theta = \frac{1}{3}$ and θ is in Q IV. ________

20. Find the exact value of $\tan(15°)$ using the half-angle formula $\tan(\theta/2) = \frac{1 - \cos\theta}{\sin\theta}$. ________

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Word Problems

21. A projectile is launched at angle θ above level ground. The range R (over flat ground, ignoring air resistance) is $R = \dfrac{v_0^2 \sin(2\theta)}{g}$, where $v_0 = 40$ m/s and $g = 10$ m/s^2. Find R when $\theta = 30°$. ____

22. In a right triangle, $\sin\theta = \dfrac{8}{17}$ for an acute angle θ. Find $\cos(2\theta)$ exactly. ____

23. A pendulum makes angle θ with vertical, with $\cos\theta = \dfrac{7}{25}$ and θ acute. Find $\sin(2\theta)$. ____

24. An engineer needs the exact value of $\cos(75°)$ using a half-angle identity from $150°$. Compute it. ____

Additional Practice

25. Find $\sin\theta$ if opposite $= 5$, hypotenuse $= 13$. ________

26. Find $\cos\theta$ if adjacent $= 12$, hypotenuse $= 13$. ________

27. Find $\tan\theta$ if opposite $= 7$, adjacent $= 4$. ________

28. Find $\sin 30°$. ________

29. Find $\cos 60°$. ________

30. Find $\tan 45°$. ________

Answer Keys

1. $2\sin\theta\cos\theta$
2. $1-2\sin^2\theta$
3. $\frac{24}{25}$
4. $\frac{7}{25}$
5. $-\frac{4}{3}$
6. $-\frac{4}{5}$
7. $\frac{24}{25}$
8. $-\frac{7}{25}$
9. $\frac{\sqrt{2-\sqrt{3}}}{2}$
10. $\frac{\sqrt{2+\sqrt{2}}}{2}$
11. $\frac{\sqrt{3}}{2}$
12. $-\frac{\sqrt{3}}{2}$
13. $\frac{\sqrt{3}}{2}$
14. $-\frac{120}{169}$
15. $\frac{119}{169}$
16. $\frac{\sqrt{2+\sqrt{2}}}{2}$
17. $\frac{4}{3}$
18. $\frac{\sqrt{2}}{2}$
19. $-\frac{4\sqrt{2}}{9}$
20. $2-\sqrt{3}$
21. $R=80\sqrt{3}$ m ≈ 138.6 m
22. $\frac{161}{289}$
23. $\frac{336}{625}$
24. $\frac{\sqrt{2-\sqrt{3}}}{2}$

Additional Practice Answers

25. $\frac{5}{13}$
26. $\frac{12}{13}$
27. $\frac{7}{4}$
28. $\frac{1}{2}$
29. $\frac{1}{2}$
30. 1

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. A careful way to see it: This is the sine double-angle identity, memorized. This is the part to check before moving on, because it keeps the answer tied to the original question.

2. Keep the rule visible: From $\cos^2\theta-\sin^2\theta$, replace $\cos^2\theta$ with $1-\sin^2\theta$: $1-\sin^2\theta-\sin^2\theta=1-2\sin^2\theta$. This is the part to check before moving on, because it keeps the answer tied to the original question.

3. First find $\cos\theta$. From the triangle (opposite 3, hypotenuse 5) the adjacent side is $\sqrt{5^2-3^2}=4$, and since θ is acute, $\cos\theta=+\frac{4}{5}$. Now apply the double-angle identity: $\sin 2\theta=2\sin\theta\cos\theta=2\cdot\frac{3}{5}\cdot\frac{4}{5}=\frac{24}{25}$.

4. Start with the key idea: Use $1-2\sin^2\theta=1-2(9/25)=1-\frac{18}{25}=\frac{7}{25}$ – only $\sin\theta$ needed. This is the part to check before moving on, because it keeps the answer tied to the original question.

5. Use the tangent double-angle identity $\tan 2\theta=\frac{2\tan\theta}{1-\tan^2\theta}$ with $\tan\theta=2$: $\frac{2(2)}{1-2^2}=\frac{4}{1-4}=\frac{4}{-3}=-\frac{4}{3}$. The negative sign is correct – doubling an angle with tangent 2 pushes 2θ past 90°, where tangent turns negative.

6. Keep the rule visible: In Q3, sine is negative. From $\sin^2\theta+\cos^2\theta=1$: $\sin^2\theta=1-\frac{9}{25}=\frac{16}{25}$, so $\sin\theta=-\frac{4}{5}$. That gives a quick check on the answer.

7. One steady path is: $\sin\theta=-\frac{4}{5}$ (Q3). Then $\sin 2\theta=2(-4/5)(-3/5)=\frac{24}{25}$. (Two negatives multiply to positive.) That gives a quick check on the answer.

8. Start with the key idea: Use $\cos^2\theta-\sin^2\theta=\frac{9}{25}-\frac{16}{25}=-\frac{7}{25}$. Or $2\cos^2\theta-1=2(9/25)-1=\frac{18}{25}-1=-\frac{7}{25}$ ✓. That gives a quick check on the answer.

9. A careful way to see it: $15^\circ=30^\circ/2$, Q1 (so positive sign). $\sin 15^\circ=\sqrt{(1-\cos 30^\circ)/2}=\sqrt{(1-\sqrt{3}/2)/2}=\sqrt{(2-\sqrt{3})/4}=\frac{\sqrt{2-\sqrt{3}}}{2}$. That gives a quick check on the answer.

10. Keep the rule visible: $22.5^\circ=45^\circ/2$, Q1. $\cos 22.5^\circ=\sqrt{(1+\cos 45^\circ)/2}=\sqrt{(1+\sqrt{2}/2)/2}=\sqrt{(2+\sqrt{2})/4}=\frac{\sqrt{2+\sqrt{2}}}{2}$. That gives a quick check on the answer.

11. One steady path is: That's $\sin(2\cdot 30^\circ)=\sin 60^\circ=\frac{\sqrt{3}}{2}$. This is the part to check before moving on, because it keeps the answer tied to the original question.

12. Start with the key idea: That's $\cos(2\cdot 75^\circ)=\cos 150^\circ=-\frac{\sqrt{3}}{2}$. This is the part to check before moving on, because it keeps the answer tied to the original question.

13. A careful way to see it: That's $\cos(2\cdot 15^\circ)=\cos 30^\circ=\frac{\sqrt{3}}{2}$. This is the part to check before moving on, because it keeps the answer tied to the original question.

14. Q2: $\cos\theta=-\frac{12}{13}$. Then $\sin 2\theta=2(5/13)(-12/13)=-\frac{120}{169}$. (Negative makes sense: 2θ likely in Q3 or Q4.)

15. One steady path is: $\cos 2\theta=1-2\sin^2\theta=1-2(25/169)=1-\frac{50}{169}=\frac{119}{169}$. This is the part to check before moving on, because it keeps the answer tied to the original question.

16. Start with the key idea: $67.5^\circ=135^\circ/2$, Q1. $\sin 67.5^\circ=$

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$\sqrt{(1-\cos 135^\circ)/2} = \sqrt{(1+\sqrt{2}/2)/2} = \dfrac{\sqrt{2+\sqrt{2}}}{2}$. (Same value as $\cos 22.5^\circ$ – they're cofunctions.) That gives a quick check on the answer.

17. Apply $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$ with $\tan\theta = \dfrac{1}{2}$: numerator $2 \cdot \dfrac{1}{2} = 1$, denominator $1 - \left(\dfrac{1}{2}\right)^2 = 1 - \dfrac{1}{4} = \dfrac{3}{4}$. So $\tan 2\theta = \dfrac{1}{3/4} = \dfrac{4}{3}$. Positive here because the small angle doubles to something still under 90°.

18. Keep the rule visible: That's $\cos(2 \cdot 22.5^\circ) = \cos 45^\circ = \dfrac{\sqrt{2}}{2}$. This is the part to check before moving on, because it keeps the answer tied to the original question.

19. One steady path is: Q4: $\sin\theta = -\sqrt{1-1/9} = -\sqrt{8/9} = -\dfrac{2\sqrt{2}}{3}$. Then $\sin 2\theta = 2(-2\sqrt{2}/3)(1/3) = -\dfrac{4\sqrt{2}}{9}$. That gives a quick check on the answer.

20. Start with the key idea: $15^\circ = 30^\circ/2$. $\tan 15^\circ = \dfrac{1-\cos 30^\circ}{\sin 30^\circ} = \dfrac{1-\sqrt{3}/2}{1/2} = 2 - \sqrt{3}$. This is the part to check before moving on, because it keeps the answer tied to the original question.

21. A careful way to see it: $\sin(2 \cdot 30^\circ) = \sin 60^\circ = \dfrac{\sqrt{3}}{2}$. So $R = \dfrac{40^2(\sqrt{3}/2)}{10} = \dfrac{1600(\sqrt{3}/2)}{10} = 80\sqrt{3} \approx 138.56$ m. That gives a quick check on the answer.

22. Use $\cos 2\theta = 1 - 2\sin^2\theta = 1 - 2(64/289) = 1 - \dfrac{128}{289} = \dfrac{161}{289}$. (Sanity check: θ acute means small angle likely $< 45^\circ$, so $2\theta < 90^\circ$ and $\cos 2\theta > 0$ ✓.)

23. One steady path is: Acute angle: $\sin\theta = \sqrt{1-49/625} = \sqrt{576/625} = \dfrac{24}{25}$. Then $\sin 2\theta = 2(24/25)(7/25) = \dfrac{336}{625}$. That gives a quick check on the answer.

24. Start with the key idea: $75^\circ = 150^\circ/2$, and Q1 for 75° means positive sign. $\cos 75^\circ = \sqrt{(1+\cos 150^\circ)/2} = \sqrt{(1-\sqrt{3}/2)/2} = \sqrt{(2-\sqrt{3})/4} = \dfrac{\sqrt{2-\sqrt{3}}}{2}$. (Equivalent to $\dfrac{\sqrt{6}-\sqrt{2}}{4}$ – the value also reached via the difference identity $\cos(45^\circ + 30^\circ)$.) That gives a quick check on the answer.

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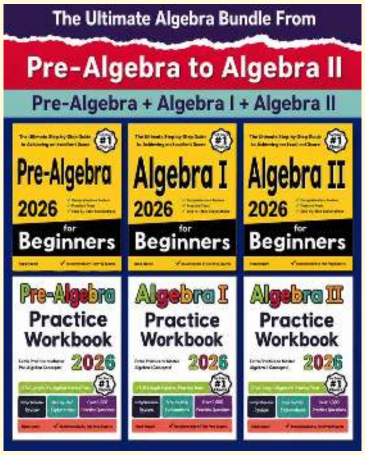

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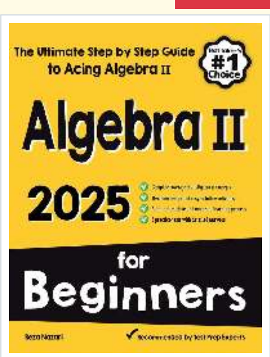

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