

# Domain and Range of Trigonometric Functions

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Score: \_\_\_\_\_ / 29

## Quick Review

Every trig function has its own personality on the unit circle, and that shows up in its domain (what inputs it accepts) and its range (what outputs it produces).

**Sine and cosine.** Both eat every real angle, so the domain of  $\sin \theta$  and  $\cos \theta$  is  $\mathbb{R}$ . Their outputs are the  $y$ - and  $x$ -coordinates of a point on the unit circle – those coordinates never leave  $[-1, 1]$ . So the range is  $[-1, 1]$  for both.

**Tangent.**  $\tan \theta = \frac{\sin \theta}{\cos \theta}$ , so tangent blows up wherever  $\cos \theta = 0$  – that's at  $\theta = \frac{\pi}{2} + n\pi$  for any integer  $n$ . Domain: all reals except those vertical-asymptote angles. Range:  $\mathbb{R}$  – between any two consecutive asymptotes, tangent slides through every real value.

**Cotangent.**  $\cot \theta = \frac{\cos \theta}{\sin \theta}$  is undefined where  $\sin \theta = 0$ , that is, at  $\theta = n\pi$ . Range:  $\mathbb{R}$ , same idea.

**Secant and cosecant.** These are the reciprocals  $\sec \theta = \frac{1}{\cos \theta}$  and  $\csc \theta = \frac{1}{\sin \theta}$ . They blow up where their partner is zero, and because  $|\cos \theta| \leq 1$  and  $|\sin \theta| \leq 1$ , their reciprocals are always  $\geq 1$  or  $\leq -1$ . Range for both:  $(-\infty, -1] \cup [1, \infty)$ .

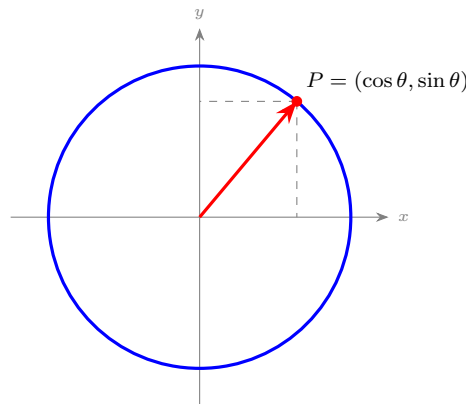
**Transformations.** If you stretch or shift, the range moves with you. For  $a \sin \theta + k$ , multiplying scales the range by  $|a|$  and the  $k$  slides it up or down. So  $3 \sin \theta - 2$  has range  $[-5, 1]$ .

**Common slips.** Saying the domain of  $\sec \theta$  excludes  $\theta = n\pi$  (that's cosecant). Mixing up the range of sine and cosine ( $[-1, 1]$ ) with the principal range of inverse sine ( $[-\pi/2, \pi/2]$ ). Writing  $(-1, 1)$  instead of  $[-1, 1]$  – sine actually hits both endpoints.

## PRACTICE

State the domain or range. Use interval or set notation as the problem requires.

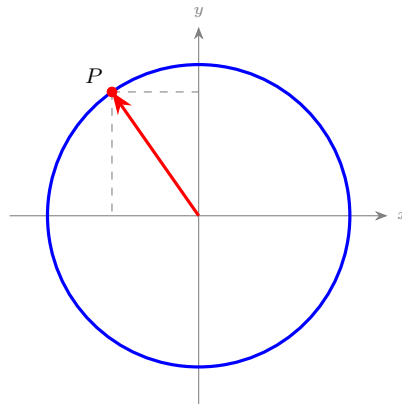
- State the domain of  $f(\theta) = \sin \theta$ . \_\_\_\_\_
- State the range of  $f(\theta) = \cos \theta$ . The point  $P$  rides the unit circle as  $\theta$  varies; cosine reads off its  $x$ -coordinate. \_\_\_\_\_



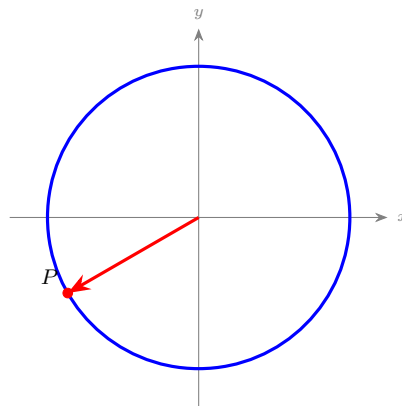
- State the domain of  $f(\theta) = \tan \theta$ . \_\_\_\_\_
- State the range of  $f(\theta) = \tan \theta$ . \_\_\_\_\_
- State the range of  $f(\theta) = \sec \theta$ . \_\_\_\_\_
- State the domain of  $f(\theta) = \csc \theta$ . \_\_\_\_\_



7. State the range of  $f(\theta) = \sin \theta$ . Use the unit circle below: sine is the  $y$ -coordinate of the point  $P$ . \_\_\_\_\_



8. State the domain of  $f(\theta) = \cos \theta$ . The ray to  $P$  can point at any angle around this circle – think about which inputs  $\theta$  are allowed. \_\_\_\_\_



- 9. State the range of  $f(\theta) = \csc \theta$ . \_\_\_\_\_
- 10. State the domain of  $f(\theta) = \cot \theta$ . \_\_\_\_\_
- 11. State the range of  $g(\theta) = 2 \sin \theta$ . \_\_\_\_\_
- 12. State the range of  $g(\theta) = \cos \theta + 5$ . \_\_\_\_\_
- 13. State the range of  $g(\theta) = 3 \sin \theta - 2$ . \_\_\_\_\_
- 14. State the range of  $g(\theta) = -\cos \theta$ . \_\_\_\_\_
- 15. State the domain of  $f(\theta) = \tan(2\theta)$ . \_\_\_\_\_
- 16. State the range of  $g(\theta) = 4 \cos \theta + 1$ . \_\_\_\_\_
- 17. State the domain of  $f(\theta) = \sec(\theta)$ . \_\_\_\_\_
- 18. Find the range of  $g(\theta) = \frac{1}{2} \sin \theta$ . \_\_\_\_\_
- 19. True or false:  $\sin \theta = 1.2$  has a real solution. \_\_\_\_\_
- 20. State the range of  $g(\theta) = 5 - 2 \cos \theta$ . \_\_\_\_\_



## ◆ Word Problems

21. A Ferris wheel cabin's height above the ground is modeled by  $h(t) = 20 \sin(t) + 25$ , where  $h$  is in feet. \_\_\_\_\_  
What is the range of possible heights? Give the minimum and maximum height the cabin reaches.
22. A sound wave's pressure variation is  $p(t) = 3 \cos(t)$ . State the range of  $p(t)$  and explain what it means in \_\_\_\_\_  
context.
23. An engineer writes the model  $v(\theta) = \tan(\theta)$  for voltage in a circuit. For what angles is  $v(\theta)$  undefined? \_\_\_\_\_  
Describe the set of forbidden inputs.
24. A tide-height model is  $H(t) = 4 \sin(t) + 10$  feet. The harbor master needs to know whether a boat that \_\_\_\_\_  
needs 15 feet of clearance can ever enter. Can it? What is the maximum height?

## Additional Practice

25. Find  $\sin \theta$  if opposite = 5, hypotenuse = 13. \_\_\_\_\_
26. Find  $\cos \theta$  if adjacent = 12, hypotenuse = 13. \_\_\_\_\_
27. Find  $\tan \theta$  if opposite = 7, adjacent = 4. \_\_\_\_\_
28. Find  $\sin 30^\circ$ . \_\_\_\_\_
29. Find  $\cos 60^\circ$ . \_\_\_\_\_



## Answer Keys

1.  $\mathbb{R}$

2.  $[-1, 1]$

3.  $\theta \neq \frac{\pi}{2} + n\pi$

4.  $\mathbb{R}$

5.  $(-\infty, -1] \cup [1, \infty)$

6.  $\theta \neq n\pi$

7.  $[-1, 1]$

8.  $\mathbb{R}$

9.  $(-\infty, -1] \cup [1, \infty)$

10.  $\theta \neq n\pi$

11.  $[-2, 2]$

12.  $[4, 6]$

13.  $[-5, 1]$

## Additional Practice Answers

25.  $\frac{5}{13}$

26.  $\frac{12}{13}$

27.  $\frac{7}{4}$

14.  $[-1, 1]$

15.  $\theta \neq \frac{\pi}{4} + \frac{n\pi}{2}$

16.  $[-3, 5]$

17.  $\theta \neq \frac{\pi}{2} + n\pi$

18.  $\left[-\frac{1}{2}, \frac{1}{2}\right]$

19. False

20.  $[3, 7]$

21.  $[5, 45]$  feet; min = 5, max = 45

22.  $[-3, 3]$

23.  $\theta = \frac{\pi}{2} + n\pi, n \in \mathbb{Z}$

24. No; max = 14 ft

28.  $\frac{1}{2}$

29.  $\frac{1}{2}$

**Additional Practice:** Answers for all numbered items, including the added practice, are shown in the grid above.

## Step-by-Step Explanations

1. Sine reads off the  $y$ -coordinate of a point spinning around the unit circle, and you can spin to any angle you like – positive, negative, or huge. Nothing makes sine undefined, so the domain is every real number,  $\mathbb{R}$ .

2. Cosine outputs the  $x$ -coordinate on the unit circle, which runs from  $-1$  to  $1$  and hits both endpoints, so the range is  $[-1, 1]$ .

3. Tangent is  $\frac{\sin \theta}{\cos \theta}$ , which dies where  $\cos \theta = 0$ . Those angles are  $\theta = \frac{\pi}{2} + n\pi$  for integer  $n$ , so exclude them.

4. Between any two consecutive asymptotes, tangent sweeps from  $-\infty$  up to  $+\infty$ , so it hits every real number. Range is all reals.

5. A careful way to see it:  $\sec \theta = \frac{1}{\cos \theta}$  and  $|\cos \theta| \leq 1$ . Reciprocating something with absolute value at most 1 produces something with absolute value at least 1. Range:  $(-\infty, -1] \cup [1, \infty)$ . That gives a quick check on the answer.

6. Keep the rule visible:  $\csc \theta = \frac{1}{\sin \theta}$ , undefined where  $\sin \theta = 0$ . That's the angles  $\theta = n\pi$ , so exclude them. That gives a quick check on the answer.

7. As  $P$  travels around the unit circle, its height (the  $y$ -coordinate that sine measures) climbs to a top of 1 at the very top of the circle and drops to a low of  $-1$  at the bottom, passing through everything between. Both endpoints are reached, so the range is the closed interval  $[-1, 1]$  – not the open  $(-1, 1)$ .

8. The ray to  $P$  can point at absolutely any angle as it sweeps around the circle, and cosine (the  $x$ -coordinate of  $P$ ) is defined for every one of them. So the domain is all reals,  $\mathbb{R}$  – cosine never blows up the way tangent or secant do.

9. Same logic as secant. Since  $|\sin \theta| \leq 1$  (and  $\sin \theta \neq 0$  where  $\csc$  lives), the reciprocal has  $|\csc \theta| \geq 1$ .

10. Keep the rule visible: Cotangent is  $\frac{\cos \theta}{\sin \theta}$ , undefined where  $\sin \theta = 0$ . Exclude  $\theta = n\pi$ . This is the part to check before moving on, because it keeps the answer tied to the original question.

11. One steady path is: Multiplying sine by 2 doubles its swing:  $[-1, 1] \rightarrow [-2, 2]$ . That gives a quick check on the answer.

12. Start with the key idea: Cosine runs  $[-1, 1]$ . Adding 5 shifts the whole range up by 5:  $[4, 6]$ . That gives a quick check on the answer.

13. Stretch first:  $3 \sin \theta$  has range  $[-3, 3]$ . Then shift down by 2:  $[-3-2, 3-2] = [-5, 1]$ .

14. Multiplying by  $-1$  flips the range upside down, but  $[-1, 1]$  flipped is still  $[-1, 1]$  – the interval is symmetric about 0.

15. Tangent dies when its input equals  $\frac{\pi}{2} + n\pi$ . Set  $2\theta = \frac{\pi}{2} + n\pi$  and divide by 2:  $\theta = \frac{\pi}{4} + \frac{n\pi}{2}$ .

16. Start with the key idea: Stretch by 4:  $[-4, 4]$ . Slide up 1:  $[-3, 5]$ . This is the part to check before moving on, because it keeps the answer tied to the original question.

17. Secant dies wherever cosine is zero – at  $\theta = \frac{\pi}{2} + n\pi$ . Don't confuse this with the cosecant rule (which excludes  $n\pi$  instead).

18. Keep the rule visible: Multiplying by  $\frac{1}{2}$  shrinks the swing:  $[-1, 1]$  becomes  $\left[-\frac{1}{2}, \frac{1}{2}\right]$ . That gives a quick check on the answer.

19. One steady path is: Sine's range tops out at 1. Anything above 1 (or below  $-1$ ) is unreachable. That gives a quick check on the answer.

20. Rewrite:  $-2 \cos \theta$  has range  $[-2, 2]$ . Add 5:  $[3, 7]$ . (The negative in front flips the cosine swing, but the interval is still  $[-2, 2]$ .)

21. Sine has range  $[-1, 1]$ . Multiplying by 20 gives  $[-20, 20]$ . Adding 25 shifts everything up:  $[-20 + 25, 20 + 25] = [5, 45]$ . The cabin's lowest point is 5 ft (loading platform), highest is 45 ft. (Sanity check: the wheel has radius 20 with center 25 ft up, which lines up.)

22. Cosine has range  $[-1, 1]$ , so  $3 \cos t$  has range  $[-3, 3]$ . The pressure swings between  $-3$  (maximum rarefaction) and  $+3$  (maximum compression), and hits every value in between.

23. Tangent dies wherever its denominator (cosine) is zero. Cosine is zero at the top and bottom of the unit circle:  $\theta = \frac{\pi}{2}, \frac{3\pi}{2}$ , and so on – which together is

$\theta = \frac{\pi}{2} + n\pi$  for any integer  $n$ . Those are the asymptotes of the tangent graph.

24. Maximum of  $\sin(t)$  is 1, so the maximum of  $H(t)$  is  $4(1) + 10 = 14$ . The boat needs 15 ft but the highest tide only reaches 14 ft. The boat can't make it through – ever.



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