Dividing Rational Expressions

Name: _____ Date: _____ Score: _____ / 35

Quick Review

Division of rational expressions follows the same rule as numeric fractions: *multiply by the reciprocal of the divisor.* $\frac{A}{B} \div \frac{C}{D} = \frac{A}{B} \cdot \frac{D}{C}$, valid when B, C, D are all nonzero.

The recipe. (1) Flip the second fraction (swap numerator and denominator). (2) Multiply – and now it's the same workflow as the multiplication worksheet: factor, cancel, state restrictions.

Restrictions come from three places. (a) Original denominators. (b) The divisor's numerator (since the divisor can't be zero). (c) Any factor that ends up in the new denominator after flipping. Track all three.

Polynomial divided by a fraction. Treat the polynomial as $\frac{\text{poly}}{1}$. So $(x+5) \div \frac{x+5}{x-2} = \frac{x+5}{1} \cdot \frac{x-2}{x+5} = x-2$.

Why the reciprocal rule works. Multiplying by $\frac{D}{C}$ undoes the division by $\frac{C}{D}$: $\frac{C}{D} \cdot \frac{D}{C} = 1$. The flip is just the multiplicative inverse.

Common slips. Flipping the wrong fraction (flip the *divisor*, the one after the ÷ sign). Forgetting to add restrictions from the original divisor's numerator. Multiplying first and dividing later (gets you a multiplication, not a quotient).

PRACTICE

Multiply by the reciprocal of the divisor, then factor and cancel. State every restriction.

1. Divide $\frac{x}{3} \div \frac{2}{x}$. The table evaluates the quotient at a few inputs. _____

x	3	6	9
quotient	$\frac{3}{2}$	6	$\frac{27}{2}$

2. Divide $\frac{x^2-1}{x+2} \div \frac{x-1}{x+2}$. The table samples the quotient away from the excluded values. _____

x	0	2	3
quotient	1	3	4

3. $\frac{2x^2}{3y} \div \frac{4x}{9y^2}$ _____

4. $(x+5) \div \frac{x+5}{x-2}$ _____

5. $\frac{x^2-9}{x^2+6x+9} \div \frac{x-3}{x+3}$ _____

6. Divide $\frac{x^2+4x+3}{x^2-4} \div \frac{x+3}{x-2}$. The table evaluates the quotient at several inputs. _____

x	0	1	3
quotient	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{4}{5}$

7. $\frac{x^2-16}{x^2+2x-8} \div \frac{x+4}{x-2}$ _____

8. $\frac{x^2-25}{x^2-3x-10} \div \frac{x-5}{x+2}$ _____

9. $\frac{x^2-9}{x+2} \div \frac{x-3}{x}$ _____

10. $\frac{1}{x} \div \frac{1}{x^2}$ _____

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11. $\frac{3x+6}{x^2} \div \frac{x+2}{x}$ _____

12. $\frac{x^2-x-6}{x^2-9} \div \frac{x+2}{x+3}$ _____

13. Mark TRUE or FALSE: The reciprocal of $\frac{0}{x}$ is $\frac{x}{0}$. _____

14. $\frac{x^2-1}{6} \div \frac{x+1}{3}$ _____

15. $\frac{2x}{x-1} \div \frac{4x^2}{x^2-1}$ _____

16. $\frac{x^2+5x+4}{x^2-1} \div \frac{x+4}{x-1}$ _____

17. $\frac{x+2}{x-3} \div (x+2)$ _____

18. $\frac{4x^2-1}{2x+1} \div \frac{2x-1}{1}$ _____

19. $\frac{x^2+3x}{x^2-4} \div \frac{x}{x-2}$ _____

20. $\frac{a^2-b^2}{a+b} \div \frac{a-b}{1}$ _____

Word Problems

21. A rectangle has area $\frac{x^2-9}{x^2-1}$ and width $\frac{x-3}{x+1}$. Find the length, in simplest form, and state the restrictions. _____

22. A scale model has length $\frac{x+5}{x-2}$ feet on the page that represents $\frac{x^2+5x}{x^2-4}$ feet in real life. Find the page-to-real scale ratio (page ÷ real), in simplest form. _____

23. In a chemistry rate problem, the ratio $\frac{2x^2-8}{x+1} \div \frac{x-2}{x^2-1}$ comes up. Simplify it and explain why the answer is a polynomial. _____

24. Solve for the missing value: $\frac{x^2+x-6}{x^2-1} \div \frac{?}{x+1} = \frac{x+3}{x-1}$. Find the rational expression that goes in the box. _____

Additional Practice

25. Simplify $\frac{x^2-9}{x-3}$. _____

26. Excluded value of $\frac{1}{x+4}$. _____

27. Domain of $f(x) = \frac{x}{x-5}$. _____

28. Multiply $\frac{x}{3} \cdot \frac{6}{x}$. _____

29. Divide $\frac{x^2}{5} \div \frac{x}{10}$. _____

30. Add $\frac{3}{x} + \frac{5}{x}$. _____

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31. Subtract $\frac{7}{x-1} - \frac{2}{x-1}$. _____

32. Solve $\frac{1}{x} = 4$. _____

33. Solve $\frac{x+2}{x-1} = 3$. _____

34. Vertical asymptote of $y = \frac{4}{x+8}$. _____

35. Horizontal asymptote of $y = \frac{3x+1}{x-2}$. _____

Answer Keys

1. $\frac{x^2}{6},\ x \neq 0$
2. $x+1,\ x \neq -2,\ 1$
3. $\frac{3xy}{2},\ x \neq 0,\ y \neq 0$
4. $x-2,\ x \neq -5,\ 2$
5. $1,\ x \neq \pm 3$
6. $\frac{x+1}{x+2},\ x \neq -3,\ \pm 2$
7. $\frac{x-4}{x+4},\ x \neq -4,\ 2$
8. $\frac{x+5}{x-5},\ x \neq -2,\ 5$
9. $\frac{x(x+3)}{x+2},\ x \neq -2,\ 0,\ 3$
10. $x,\ x \neq 0$
11. $\frac{3}{x},\ x \neq 0,\ -2$
12. $1,\ x \neq -3,\ -2,\ 3$
13. FALSE
14. $\frac{x-1}{2},\ x \neq -1$
15. $\frac{x+1}{2x},\ x \neq 0,\ \pm 1$
16. $1,\ x \neq \pm 1,\ -4$
17. $\frac{1}{x-3},\ x \neq 3,\ -2$
18. $1,\ x \neq -\frac{1}{2},\ \frac{1}{2}$
19. $\frac{x+3}{x+2},\ x \neq 0,\ \pm 2$
20. $1,\ a \neq -b,\ b$
21. length $= \frac{x+3}{x-1},\ x \neq \pm 1,\ 3$
22. $\frac{x+2}{x},\ x \neq 0,\ \pm 2,\ -5$
23. $2(x+2)(x-1),\ x \neq \pm 1,\ 2$
24. $? = x-2$

Additional Practice Answers

25. $x+3,\ x \neq 3$
26. $x = -4$
27. $x \neq 5$
28. 2
29. $2x$
30. $\frac{8}{x}$
31. $\frac{5}{x-1}$
32. $x = \frac{1}{4}$
33. $x = \frac{5}{2}$
34. $x = -8$
35. $y = 3$

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. Flip: $\frac{x}{3} \cdot \frac{x}{2} = \frac{x^2}{6}$. Restriction from the divisor's numerator: $x \neq 0$. (The table values match $\frac{x^2}{6}$ at $x = 3, 6, 9$.)

2. Flip and multiply: $\frac{(x-1)(x+1)}{x+2} \cdot \frac{x+2}{x-1} = x+1$. Restrictions: $x \neq -2$ (denominator) and $x \neq 1$ (divisor's numerator). (The table agrees with $x+1$ at $x = 0, 2, 3$.)

3. Flip the divisor and multiply: $\frac{2x^2}{3y} \cdot \frac{9y^2}{4x} = \frac{18x^2y^2}{12xy}$. Reduce the coefficient $\frac{18}{12} = \frac{3}{2}$, the x powers $\frac{x^2}{x} = x$, and the y powers $\frac{y^2}{y} = y$, giving $\frac{3xy}{2}$. Keep $x, y \neq 0$.

4. Treat $(x+5)$ as $\frac{x+5}{1}$. Flip the divisor: $\frac{x+5}{1} \cdot \frac{x-2}{x+5} = x-2$. Restrictions: $x \neq 2$ (denominator) and $x \neq -5$ (divisor's numerator).

5. Factor: $x^2-9 = (x-3)(x+3)$; $x^2+6x+9 = (x+3)^2$. Flip and multiply: $\frac{(x-3)(x+3)}{(x+3)^2} \cdot \frac{x+3}{x-3} = 1$. Restrictions: $x \neq -3$ (denominator) and $x \neq 3$ (divisor's numerator).

6. Factor: $x^2+4x+3 = (x+1)(x+3)$; $x^2-4 = (x-2)(x+2)$. Flip: $\frac{(x+1)(x+3)}{(x-2)(x+2)} \cdot \frac{x-2}{x+3} = \frac{x+1}{x+2}$. (The table matches $\frac{x+1}{x+2}$ at $x = 0, 1, 3$.)

7. Factor: $x^2-16 = (x-4)(x+4)$; $x^2+2x-8 = (x+4)(x-2)$. Flip: $\frac{(x-4)(x+4)}{(x+4)(x-2)} \cdot \frac{x-2}{x+4} = \frac{x-4}{x+4}$.

8. Factor: $x^2-25 = (x-5)(x+5)$; $x^2-3x-10 = (x-5)(x+2)$. Flip: $\frac{(x-5)(x+5)}{(x-5)(x+2)} \cdot \frac{x+2}{x-5} = \frac{x+5}{x-5}$.

9. Factor $x^2-9 = (x-3)(x+3)$. Flip the divisor $\frac{x-3}{x}$ to $\frac{x}{x-3}$ and multiply: $\frac{(x-3)(x+3)}{x+2} \cdot \frac{x}{x-3}$. The $(x-3)$ factors cancel, leaving $\frac{x(x+3)}{x+2}$. Restrictions come from three places: $x \neq -2$ (original denom), $x \neq 0$ (divisor's numerator), $x \neq 3$ (divisor's denom).

10. Keep the rule visible: Flip: $\frac{1}{x} \cdot \frac{x^2}{1} = x$. (Dividing by $\frac{1}{x^2}$ is the same as multiplying by x^2.) That gives a quick check on the answer.

11. Factor $3x+6 = 3(x+2)$. Flip the divisor and multiply: $\frac{3(x+2)}{x^2} \cdot \frac{x}{x+2}$. The $(x+2)$ factors cancel and one x cancels, leaving $\frac{3}{x}$. Keep $x \neq 0$ (denom) and $x \neq -2$ (divisor's numerator).

12. Factor: $x^2-x-6 = (x-3)(x+2)$ and $x^2-9 = (x-3)(x+3)$. Flip the divisor and multiply: $\frac{(x-3)(x+2)}{(x-3)(x+3)} \cdot \frac{x+3}{x+2}$. Every factor pairs off, so the quotient is 1. Keep $x \neq -3, -2, 3$ from the three restriction sources.

13. A careful way to see it: $\frac{0}{x} = 0$ (for $x \neq 0$), and zero has no reciprocal. The expression $\frac{x}{0}$ is undefined. That gives a quick check on the answer.

14. Keep the rule visible: $x^2-1 = (x-1)(x+1)$. Flip: $\frac{(x-1)(x+1)}{6} \cdot \frac{3}{x+1} = \frac{3(x-1)}{6} = \frac{x-1}{2}$. Restriction from the divisor's numerator: $x \neq -1$. That gives a quick check on the answer.

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15. Factor $x^2 - 1 = (x-1)(x+1)$. Flip: $\frac{2x}{x-1} \cdot \frac{(x-1)(x+1)}{4x^2} = \frac{2x(x+1)}{4x^2} = \frac{x+1}{2x}$.

16. Factor: $x^2 + 5x + 4 = (x+1)(x+4)$; $x^2 - 1 = (x-1)(x+1)$. Flip: $\frac{(x+1)(x+4)}{(x-1)(x+1)} \cdot \frac{x-1}{x+4} = 1$.

17. Treat $(x+2)$ as $\frac{x+2}{1}$. Flip: $\frac{x+2}{x-3} \cdot \frac{1}{x+2} = \frac{1}{x-3}$. Restriction: $x \neq -2$ (the divisor can't be zero).

18. Keep the rule visible: $4x^2 - 1 = (2x-1)(2x+1)$. Flip: $\frac{(2x-1)(2x+1)}{2x+1} \cdot \frac{1}{2x-1} = 1$. That gives a quick check on the answer.

19. Factor: $x^2 + 3x = x(x+3)$; $x^2 - 4 = (x-2)(x+2)$. Flip: $\frac{x(x+3)}{(x-2)(x+2)} \cdot \frac{x-2}{x} = \frac{x+3}{x+2}$.

20. Start with the key idea: $a^2 - b^2 = (a-b)(a+b)$. Flip: $\frac{(a-b)(a+b)}{a+b} \cdot \frac{1}{a-b} = 1$. This is the part to check before moving on, because it keeps the answer tied to the original question.

21. Length = area ÷ width = $\frac{x^2-9}{x^2-1} \div \frac{x-3}{x+1}$. Flip: $\frac{(x-3)(x+3)}{(x-1)(x+1)} \cdot \frac{x+1}{x-3} = \frac{x+3}{x-1}$. Restrictions: $x \neq \pm 1$ (original denominator) and $x \neq 3$ (divisor's numerator). At $x = 5$: length $= \frac{8}{4} = 2$. Original area $= \frac{16}{24} = \frac{2}{3}$, width $= \frac{2}{6} = \frac{1}{3}$. Ratio $\frac{2/3}{1/3} = 2$. Match.

22. Ratio $= \frac{x+5}{x-2} \div \frac{x^2+5x}{x^2-4}$. Flip: $\frac{x+5}{x-2} \cdot \frac{x^2-4}{x^2+5x}$. Factor: $x^2 - 4 = (x-2)(x+2)$; $x^2 + 5x = x(x+5)$. $\frac{x+5}{x-2} \cdot \frac{(x-2)(x+2)}{x(x+5)} = \frac{x+2}{x}$. Restrictions: $x \neq 0, \pm 2, -5$ (from each original denominator and from the divisor's numerator factors). Sanity check at $x = 3$: $\frac{8}{1} \div \frac{24}{5} = \frac{8}{1} \cdot \frac{5}{24} = \frac{40}{24} = \frac{5}{3}$. Simplified at $x = 3$: $\frac{5}{3}$. Match.

23. Factor: $2x^2 - 8 = 2(x-2)(x+2)$ and $x^2 - 1 = (x-1)(x+1)$. Flip: $\frac{2(x-2)(x+2)}{x+1} \cdot \frac{(x-1)(x+1)}{x-2} = 2(x+2)(x-1)$. Every denominator factor cancels with a numerator factor, leaving a polynomial. That happens when the divisor's denominator contains every factor the original denominator contributed – the inversion cancels everything out below the bar. Restrictions: $x \neq -1$ (original denom), $x \neq 2$ (divisor's numerator), $x \neq \pm 1$ (divisor's denominator factors).

24. Let the missing expression be E. Then $\frac{x^2+x-6}{x^2-1} \cdot \frac{x+1}{E} = \frac{x+3}{x-1}$. Factor: $x^2 + x - 6 = (x-2)(x+3)$, $x^2 - 1 = (x-1)(x+1)$. The left side becomes $\frac{(x-2)(x+3)}{(x-1)(x+1)} \cdot \frac{x+1}{E} = \frac{(x-2)(x+3)}{(x-1)E}$. Setting that equal to $\frac{x+3}{x-1}$ forces $\frac{x-2}{E} = 1$, so $E = x - 2$. (Working backwards from the desired answer is a standard reverse-engineering technique whenever you're given a target.)

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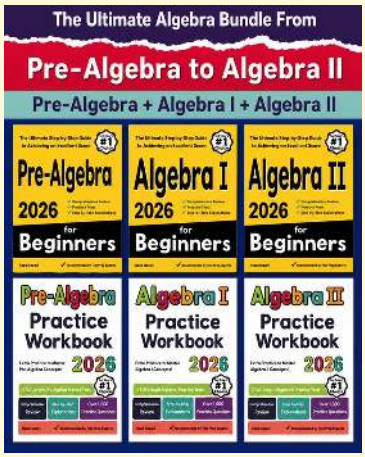

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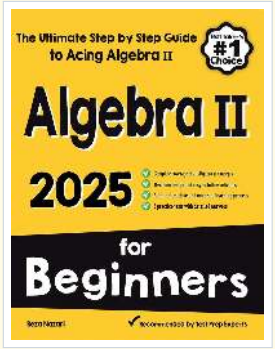

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