Direct Inverse Joint and Combined Variation

Name: _____ Date: _____ Score: _____ / 30

Quick Review

Four variation types cover almost every real-world model that uses the words "varies as." Each one is just one equation.

Direct variation: $y = kx$. y scales with x. The ratio $\frac{y}{x} = k$ stays constant.

Inverse variation: $y = \frac{k}{x}$. y and x multiply to the same constant: $xy = k$. As one grows, the other shrinks.

Joint variation: $y = kxz$ **(or** $y = kxz^2$ **etc.).** y varies directly with more than one factor. The cone volume $V = \frac{\pi}{3}r^2h$ is joint variation: V varies jointly with r^2 and h (constant $k = \frac{\pi}{3}$).

Combined variation: $y = \frac{kxz}{w}$ **(mix of direct and inverse).** Newton's gravity: $F = \frac{Gm_1m_2}{d^2}$ – joint with the masses, inverse with the square of distance.

Finding k. Use any one known data point. Plug into the model and solve for k. The same k then applies to all other data points.

Reading the words. "Varies directly" ⇒ numerator. "Varies inversely" ⇒ denominator. "With the square" ⇒ the variable is squared. "Jointly with" ⇒ multiple direct factors.

Common slips. Putting an inversely-varying variable in the numerator. Forgetting to use the data point's value *after* setting up the model. Confusing "varies directly with x^2" (so $y = kx^2$) with "varies directly with x" (so $y = kx$).

PRACTICE

Identify the variation type, set up the equation, find k from the given data, and answer the question.

1. y varies directly with x. Write the equation. _____
2. y varies inversely with x. Write the equation. _____
3. Volume V of a cone varies jointly with r^2 and h. Write the equation. _____
4. z varies directly with x and inversely with y. Write the equation. _____
5. y varies directly with x. When $x = 8$, $y = 24$. Find k. _____
6. w varies directly with x and inversely with y^2. When $x = 4$, $y = 2$, $w = 5$. Find w when $x = 9$, $y = 3$. _____
7. Gravity: $F = G\frac{m_1m_2}{d^2}$. Which type of variation describes the relationship? _____
8. y varies inversely with x. When $x = 6$, $y = 10$. Find y when $x = 15$. The table records two earlier readings – use them to pin down the constant. _____

x	y	xy
2	30	60
6	10	60

9. R varies directly with a and the square of b, and inversely with c. Given $a = 3, b = 2, c = 6, R = 10$. Find R when $a = 5, b = 3, c = 9$. _____
10. y varies directly with x. If $y = 20$ when $x = 4$, find y when $x = 10$. _____
11. Mark TRUE or FALSE: In direct variation, the constant k must be positive. _____
12. y varies inversely with $\sqrt{x}$. When $x = 9, y = 4$. Find y when $x = 16$. _____
13. V varies jointly with ℓ, w, h (rectangular box). When $\ell = 2, w = 3, h = 4, V = 24$. Find V when $\ell = 5, w = 6, h = 2$. _____
14. Mark TRUE or FALSE: "Inverse variation" means the variable is in both numerator and denominator. _____

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15. w varies directly with the cube of x. When $x = 2, w = 16$. Find w when $x = 5$. The table lists x, x^3, and w for the readings you already have. _____

x	x^3	w
1	1	2
2	8	16

16. P varies directly with T and inversely with V (ideal gas law form). At $T = 300, V = 10, P = 600$. Find P when $T = 400, V = 8$. _____

17. y varies inversely with x^2. When $x = 3, y = 4$. Find y when $x = 6$. The table shows x^2y staying constant across the known readings. _____

x	x^2	x^2y
2	4	36
3	9	36

18. z varies jointly with x and y. At $x = 2, y = 3, z = 18$. Find z when $x = 4, y = 5$. _____

19. Hooke's law: force varies directly with displacement. A spring needs 20 N to stretch it 5 cm. How much force to stretch it 12 cm? _____

20. Light intensity varies inversely with the square of distance. At 2 m, the intensity is 100 W/m^2. Find the intensity at 5 m. _____

Word Problems

21. In an ideal gas at constant temperature, pressure P varies inversely with volume V (Boyle's law). At $V = 12$ L the pressure is 2 atm. Find the pressure when the volume is compressed to 4 L. ____

22. The weight of an object on a planet varies inversely with the square of its distance from the planet's center. A satellite weighs 400 N at $6{,}400$ km from Earth's center (Earth's surface). Find its weight at $12{,}800$ km. ____

23. The resistance R of a wire varies jointly with its length ℓ and inversely with its cross-sectional area A. A 2-m wire of area 0.5 mm^2 has resistance 0.4 ohms. Find the resistance of a 5-m wire of area 0.2 mm^2 (same material). ____

24. The kinetic energy of a moving object varies jointly with its mass and the square of its speed: $KE = \frac{1}{2}mv^2$. A 1500-kg car at 20 m/s has $KE = 300{,}000$ J. If the car doubles its speed to 40 m/s (same mass), find the new KE. ____

Additional Practice

25. If y varies directly with x and $y = 18$ when $x = 6$, find k. _____

26. Direct variation with $k = 5$: write the equation. _____

27. If y varies inversely with x and $y = 4$ when $x = 3$, find k. _____

28. Inverse variation with $k = 20$: write the equation. _____

29. If $y = 7x$, find y when $x = 9$. _____

30. If $y = \frac{30}{x}$, find y when $x = 5$. _____

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Answer Keys

1. $y = kx$
2. $y = \frac{k}{x}$
3. $V = kr^2h$
4. $z = \frac{kx}{y}$
5. $k = 3$
6. $w = 5$
7. combined: joint in m_1, m_2; inverse in d^2
8. $y = 4$
9. $R = 25$
10. $y = 50$
11. FALSE
12. $y = 3$
13. $V = 60$
14. FALSE
15. $w = 250$
16. $P = 1000$
17. $y = 1$
18. $z = 60$
19. 48 N
20. 16 W/m^2
21. $P = 6$ atm
22. 100 N
23. $R = 2.5$ ohms
24. $KE = 1{,}200{,}000$ J

Additional Practice Answers

25. $k = 3$
26. $y = 5x$
27. $k = 12$
28. $y = \frac{20}{x}$
29. 63
30. 6

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. A careful way to see it: Direct variation: the variable goes in the numerator with a constant k. That gives a quick check on the answer.

2. Keep the rule visible: Inverse variation: the variable goes in the denominator. That gives a quick check on the answer.

3. Joint variation with two factors – both in the numerator. (Constant k turns out to be $\frac{\pi}{3}$ in the geometric formula.)

4. Start with the key idea: Direct $\Rightarrow$ x on top; inverse $\Rightarrow$ y on bottom. This is the part to check before moving on, because it keeps the answer tied to the original question.

5. Direct variation means $y = kx$. Substitute the data point $x = 8$, $y = 24$: $24 = k(8)$. Divide by 8 to get $k = 3$. (You can sanity-check with the ratio $\frac{y}{x} = \frac{24}{8} = 3$, which equals k in direct variation.)

6. Model: $w = \frac{kx}{y^2}$. Find k: $5 = \frac{4k}{4} = k$, so $k = 5$. Then at $x = 9, y = 3$: $w = \frac{5(9)}{9} = 5$.

7. One steady path is: F varies jointly with the two masses (numerator) and inversely with the square of distance (denominator). That gives a quick check on the answer.

8. For inverse variation $xy = k$ is constant. The table shows $k = 60$. At $x = 15$: $y = \frac{60}{15} = 4$.

9. Model: $R = \frac{kab^2}{c}$. Find k: $10 = \frac{3k(4)}{6} = 2k$, so $k = 5$. Then at $a = 5, b = 3, c = 9$: $R = \frac{5(5)(9)}{9} = 25$.

10. Direct variation $y = kx$. Find k from the first data point: $k = \frac{y}{x} = \frac{20}{4} = 5$. Then at $x = 10$, $y = 5(10) = 50$.

11. One steady path is: k can be negative (e.g. $y = -3x$ is direct variation with $k = -3$). A positive k is just the most common case in word problems. That gives a quick check on the answer.

12. Start with the key idea: Model: $y = \frac{k}{\sqrt{x}}$. Find k: $4 = \frac{k}{3}$, so $k = 12$. At $x = 16$: $y = \frac{12}{4} = 3$. That gives a quick check on the answer.

13. A careful way to see it: $V = k\ell wh$. $24 = k(2)(3)(4) = 24k$, so $k = 1$. Then $V = 1 \cdot 5 \cdot 6 \cdot 2 = 60$. (The constant $k = 1$ because rectangular-box volume *equals* the product of edges.) That gives a quick check on the answer.

14. Keep the rule visible: In simple inverse variation $y = \frac{k}{x}$, the variable is in the denominator only. That gives a quick check on the answer.

15. One steady path is: $w = kx^3$. The table shows $w/x^3 = 2$, so $k = 2$. At $x = 5$: $w = 2(125) = 250$. That gives a quick check on the answer.

16. Start with the key idea: $P = \frac{kT}{V}$. $600 = \frac{300k}{10} = 30k$, so $k = 20$. At $T = 400, V = 8$: $P = \frac{20(400)}{8} = 1000$. That gives a quick check on the answer.

17. A careful way to see it: $y = \frac{k}{x^2}$, so $x^2y = k$ is constant. The table shows $k = 36$. At $x = 6$: $y = \frac{36}{36} = 1$. (Doubling x divides y by 4, the inverse-square relation in action.) That gives a quick check on the answer.

18. Joint variation puts both factors on top: $z = kxy$. Find k from $18 = k(2)(3) = 6k$, so $k = 3$. Then at $x = 4, y = 5$: $z = 3(4)(5) = 60$.

19. Hooke's law is direct variation: $F = kx$. From $20 = k(5)$ we get $k = 4$ N/cm. Then stretching to $x = 12$ cm needs $F = 4(12) = 48$ N.

20. Start with the key idea: $I = \frac{k}{d^2}$. $100 = \frac{k}{4}$, so $k = 400$. At $d = 5$: $I = \frac{400}{25} = 16$ W/m^2. That gives a quick check on the answer.

21. Model: $P = \frac{k}{V}$. Find k from the first point: $2 = \frac{k}{12}$, so $k = 24$. At $V = 4$ L: $P = \frac{24}{4} = 6$ atm. **Sanity check:** the product PV should be constant. First state: $2 \times 12 = 24$. Second state: $6 \times 4 = 24$ ✓. (Boyle's law: compressing a gas by a factor of 3 triples the pressure – consistent with the inverse relationship.)

22. Model: $W = \frac{k}{d^2}$. Find k: $400 = \frac{k}{(6400)^2}$, so $k = 400 \cdot 6400^2$. At $d = 12{,}800$: $W = \frac{k}{(12800)^2} = \frac{400 \cdot 6400^2}{(2 \cdot 6400)^2} = \frac{400}{4} = 100$ N. Doubling the distance quarters the weight – the inverse-square law in action.

23. Model: $R = \frac{k\ell}{A}$. Find k: $0.4 = \frac{2k}{0.5} = 4k$, so $k = 0.1$. At $\ell = 5, A = 0.2$: $R = \frac{0.1(5)}{0.2} = \frac{0.5}{0.2} = 2.5$ ohms. (Longer and thinner wire both increase

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resistance – matches intuition. The new wire is $2.5\times$ longer and has 40% the area, so resistance jumps by factor $2.5/0.4 = 6.25\times$; $0.4 \times 6.25 = 2.5$ ohms ✓.)

24. From $KE = \frac{1}{2}mv^2$, the constant is $k = \frac{1}{2}$. At $v = 40$: $KE = \frac{1}{2}(1500)(40^2) = 750 \cdot 1600 = 1{,}200{,}000$ J. **Sanity by ratio:** doubling speed quadruples kinetic energy (v^2 factor). Old $KE = 300{,}000$; new should be $4 \times 300{,}000 = 1{,}200{,}000$ ✓. (This is why a 40-mph crash is *four times* as energetic as a 20-mph crash, not twice – a key insight for safety design.)

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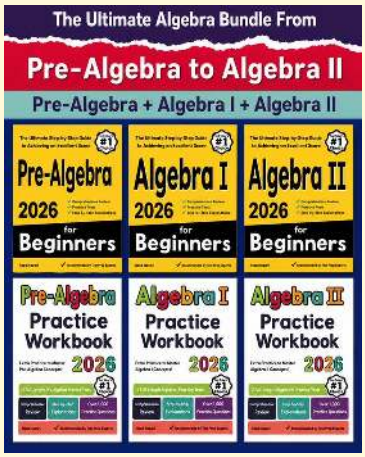

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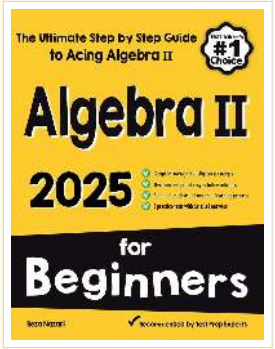

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