Coterminal Angles and Reference Angles

Name: _____	Date: _____	Score: _____ / 29

Quick Review

This section ties the two ideas together: **coterminal angles** share a terminal side, and **reference angles** measure how far that terminal side sits from the nearest x-axis.

Coterminal recipe. To find a positive angle in $[0, 360°)$ coterminal with a given angle, add or subtract $360°$ as many times as needed. Same for radians, but with 2π.

Reference angle recipe (after reducing to $[0, 360°)$).

Q1: reference $= \theta$.

Q2: reference $= 180° - \theta$.

Q3: reference $= \theta - 180°$.

Q4: reference $= 360° - \theta$.

On an axis $(0, 90, 180, 270)$: reference is the distance to the nearest x-axis (0 or $90°$).

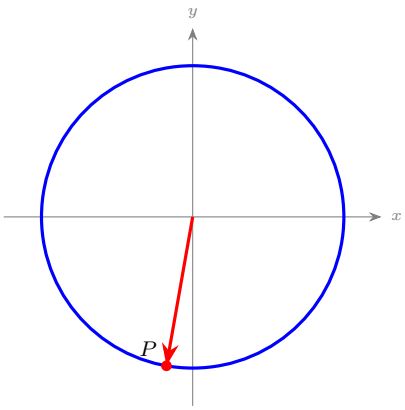
Key fact. Coterminal angles share their reference angle – and their values for all six trig functions. So if you want $\sin(-405°)$, the fastest route is: find a coterminal in $[0, 360°)$, then read off the answer there.

Common slips. Stopping after the coterminal reduction without computing the reference angle (the problem usually wants both). Picking the wrong quadrant rule – always verify which quadrant the reduced angle is in. Reporting a negative reference angle (they're always positive).

PRACTICE

For each angle, give (i) a positive coterminal angle in $[0, 360°)$ (or $[0, 2\pi)$) and (ii) the reference angle.

1. The terminal side below is for a $-100°$ angle. Give its positive coterminal angle in $[0, 360°)$. _____

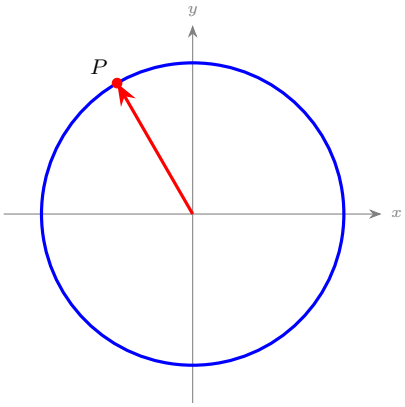


2. $200°$: a negative coterminal angle. _____

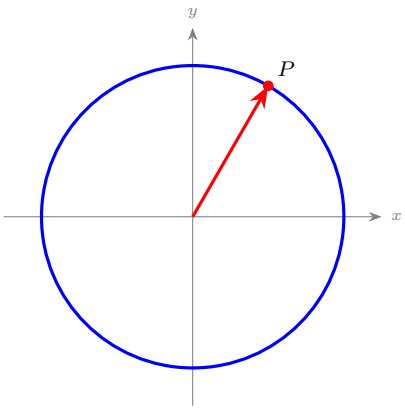
3. $-225°$: reference angle. _____

Scan Me

4. A $480°$ angle reduces to the terminal side drawn below. Find its reference angle. _____



5. The terminal side below is for a $-300°$ angle. Give its positive coterminal angle in $[0, 360°)$ and its reference angle. _____



6. $-405°$: reference angle. _____

7. $-50°$: reference angle. _____

8. $810°$: positive coterminal in $[0, 360°)$. _____

9. $\frac{7\pi}{4}$: reference angle. _____

10. $725°$: reference angle. _____

11. $-690°$: positive coterminal and reference angle. _____

12. $\frac{11\pi}{4}$: positive coterminal in $[0, 2\pi)$. _____

13. $\frac{3\pi}{4}$: reference angle. _____

14. $420°$: reference angle. _____

15. $-150°$: positive coterminal and reference angle. _____

16. $1000°$: positive coterminal in $[0, 360°)$. _____

17. $1000°$: reference angle. _____

18. $-\frac{\pi}{3}$: positive coterminal in $[0, 2\pi)$. _____

19. $-\frac{\pi}{3}$: reference angle. _____

20. True or False: coterminal angles share the same reference angle. _____

Scan Me

Word Problems

21. A satellite makes 2.25 counterclockwise rotations from a reference position. In degrees, what is its terminal angle in $[0, 360^\circ)$ and the reference angle of that terminal side? ____

22. An angle measures -585°. Find a positive coterminal angle in $[0, 360^\circ)$ and its reference angle. ____

23. A clock's second hand sweeps from the 12 position through $1{,}050^\circ$ counterclockwise (a strange clock). What's the equivalent positive angle in $[0, 360^\circ)$, and where does the hand point? ____

24. A wheel sensor logs angle $-\frac{17\pi}{6}$ in radians. Find a positive coterminal angle in $[0, 2\pi)$ and the reference angle. ____

Additional Practice

25. Find $\sin\theta$ if opposite $= 5$, hypotenuse $= 13$. _____

26. Find $\cos\theta$ if adjacent $= 12$, hypotenuse $= 13$. _____

27. Find $\tan\theta$ if opposite $= 7$, adjacent $= 4$. _____

28. Find $\sin 30^\circ$. _____

29. Find $\cos 60^\circ$. _____

Scan Me

Answer Keys

1. $260°$
2. $-160°$
3. $45°$
4. $60°$
5. $60°$; ref $60°$
6. $45°$
7. $50°$
8. $90°$
9. $\frac{\pi}{4}$
10. $5°$
11. $30°$; ref $30°$
12. $\frac{3\pi}{4}$
13. $\frac{\pi}{4}$
14. $60°$
15. $210°$; ref $30°$
16. $280°$
17. $80°$
18. $\frac{5\pi}{3}$
19. $\frac{\pi}{3}$
20. True
21. $90°$; reference $90°$
22. $135°$; reference $45°$
23. $330°$; between the 11 and 12
24. $\frac{7\pi}{6}$; reference $\frac{\pi}{6}$

Additional Practice Answers

25. $\frac{5}{13}$
26. $\frac{12}{13}$
27. $\frac{7}{4}$
28. $\frac{1}{2}$
29. $\frac{1}{2}$

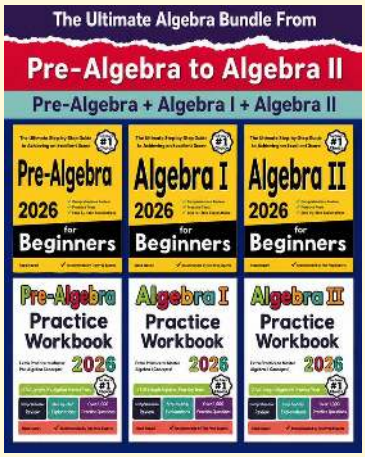
Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. A careful way to see it: $-100° + 360° = 260°$ – the same terminal side shown. This is the part to check before moving on, because it keeps the answer tied to the original question.
2. Keep the rule visible: $200° - 360° = -160°$. (Any answer that differs by a multiple of $360°$ works.) That gives a quick check on the answer.
3. One steady path is: Positive coterminal: $-225° + 360° = 135°$ (Q2). Reference: $180° - 135° = 45°$. That gives a quick check on the answer.
4. Start with the key idea: Reduce: $480° - 360° = 120°$ (Q2). Reference: $180° - 120° = 60°$. This is the part to check before moving on, because it keeps the answer tied to the original question.
5. A careful way to see it: $-300° + 360° = 60°$, which is in Q1, so reference $=$ angle $= 60°$. That gives a quick check on the answer.
6. Keep the rule visible: $-405° + 720° = 315°$ (Q4). Reference $= 360° - 315° = 45°$. This is the part to check before moving on, because it keeps the answer tied to the original question.
7. One steady path is: $-50° + 360° = 310°$ (Q4). Reference $= 360° - 310° = 50°$. (Quick rule: the reference angle of $-\theta$ equals the reference angle of $+\theta$.) That gives a quick check on the answer.
8. Start with the key idea: $810° - 720° = 90°$. This is the part to check before moving on, because it keeps the answer tied to the original question.
9. A careful way to see it: Q4. Reference $= 2\pi - \frac{7\pi}{4} = \frac{\pi}{4}$. This is the part to check before moving on, because it keeps the answer tied to the original question.
10. Keep the rule visible: Reduce: $725° - 720° = 5°$ (Q1). Reference is the angle itself: $5°$. That gives a quick check on the answer.
11. One steady path is: $-690° + 720° = 30°$ (Q1). Reference is the angle itself, $30°$. That gives a quick check on the answer.
12. Start with the key idea: $\frac{11\pi}{4} - 2\pi = \frac{11\pi}{4} - \frac{8\pi}{4} = \frac{3\pi}{4}$. This is the part to check before moving on, because it keeps the answer tied to the original question.
13. A careful way to see it: Q2. Reference $= \pi - \frac{3\pi}{4} = \frac{\pi}{4}$. This is the part to check before moving on, because it keeps the answer tied to the original question.
14. Keep the rule visible: Reduce: $420° - 360° = 60°$ (Q1). Reference $= 60°$. This is the part to check before moving on, because it keeps the answer tied to the original question.
15. One steady path is: $-150° + 360° = 210°$ (Q3). Reference $= 210° - 180° = 30°$. This is the part to check before moving on, because it keeps the answer tied to the original question.
16. Start with the key idea: $1000° - 2(360°) = 1000° - 720° = 280°$. This is the part to check before moving on, because it keeps the answer tied to the original question.
17. A careful way to see it: $280°$ is in Q4. Reference $= 360° - 280° = 80°$. This is the part to check before moving on, because it keeps the answer tied to the original question.
18. Keep the rule visible: $-\frac{\pi}{3} + 2\pi = -\frac{\pi}{3} + \frac{6\pi}{3} = \frac{5\pi}{3}$. This is the part to check before moving on, because it keeps the answer tied to the original question.
19. One steady path is: $\frac{5\pi}{3}$ is in Q4. Reference $= 2\pi - \frac{5\pi}{3} = \frac{\pi}{3}$. This is the part to check before moving on, because it keeps the answer tied to the original question.
20. They share a terminal side, so they sit in the same quadrant (or on the same axis) and have the same distance to the nearest x-axis – same reference angle.
21. A careful way to see it: 2.25 rotations $= 2.25 \cdot 360° = 810°$. Reduce: $810° - 720° = 90°$. The terminal side lies on the positive y-axis, so the reference angle is $90°$ (the distance to the nearest x-axis). That gives a quick check on the answer.
22. Keep the rule visible: $-585° + 2(360°) = -585° + 720° = 135°$. That sits in Q2, so the reference angle is $180° - 135° = 45°$. That gives a quick check on the answer.
23. One steady path is: $1050° - 2(360°) = 1050° - 720° = 330°$. That's $30°$ short of a full counterclockwise sweep – so the hand sits one hour mark before the 12, between the 11 and 12. (Each hour mark is $30°$.) That gives a quick check on the answer.
24. Add full rotations: $-\frac{17\pi}{6} + 2(2\pi) = -\frac{17\pi}{6} + \frac{24\pi}{6} = \frac{7\pi}{6}$. That's in Q3 (between $\pi = \frac{6\pi}{6}$ and $\frac{3\pi}{2} = \frac{9\pi}{6}$). Reference $= \frac{7\pi}{6} - \pi = \frac{\pi}{6}$.

Scan Me

Build Algebra Confidence From Pre-Algebra Through Algebra II



The Complete Algebra Success Bundle

Pre-Algebra, Algebra I, and Algebra II in one clear path

Friendly lessons, focused practice, and full-review support for every stage.

Scan Me

Scan for the Bundle

6 Books
3 Courses
1 Path

Bundle Value: Six coordinated books help students review missing skills, learn new algebra topics, and practice until the steps feel natural.

Complete Course Path

- ✓ Starts with Pre-Algebra foundations
- ✓ Moves smoothly into Algebra I skills
- ✓ Extends learning through Algebra II topics
- ✓ Great for review, tutoring, and summer study

One bundle, one steady path.

Step-by-Step Lessons

- ✓ Plain-English explanations students can follow
- ✓ Worked examples that show every important step
- ✓ Common mistakes called out before they stick
- ✓ Skill-building practice after each lesson
- ✓ Helpful for independent study or class support

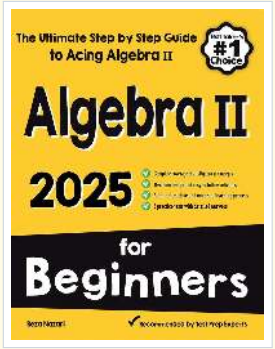
Less guessing. More understanding.

Practice That Sticks

- ✓ Matching practice workbooks for extra repetition
- ✓ Review sets to keep older skills fresh
- ✓ Answer explanations for checking thinking
- ✓ Strong support before tests and final exams
- ✓ Designed to build fluency and confidence

Practice today. Remember tomorrow.

STUDENT FAVORITE • Master Algebra II From the Ground Up



✓ 100% Guaranteed
✓ Lifetime Support
✓ Trusted by Teachers

Start Your Algebra Journey Today! →

Algebra II for Beginners

Written by a top math teacher & aligned with national and state Algebra II courses. From polynomial functions to logarithms, trigonometry, and rational expressions — explained the easy way.

- ✓ **Complete coverage** of every Algebra II concept — perfect companion to these worksheets
- ✓ **Step-by-step explanations** with worked examples on every topic
- ✓ **QR codes in every chapter** for free video lessons & bonus practice
- ✓ **2 full-length practice tests** with detailed answer keys

★ STUDENT'S #1 CHOICE ★

Teacher-recommended • 12,000+ Happy Students

↓ PDF EDITION

Scan Me

Instant download • any device

PAPERBACK

Scan Me

Paperback on Amazon

Hold it in your hands

Pair these free worksheets with *Algebra II for Beginners* and you have a complete self-paced course — concept lessons, daily practice, and full exam-style reviews, all in one path. → **EffortlessMath.com/product/algebra-ii-for-beginners**