

Conditional and Binomial Probabilities

Name: _____ Date: _____ Score: _____ / 35

Quick Review

Conditional probability updates a probability when you learn that something has already happened. The formula: $P(A | B) = \frac{P(A \cap B)}{P(B)}$, valid whenever $P(B) > 0$. Read the bar as *given*: condition first, ratio second.

New sample space. Conditioning shrinks the universe to just the part where B holds. “Given the student studies math, what’s the chance they also study physics?” makes the math students the new 100%.

Direction matters. $P(A | B)$ and $P(B | A)$ are usually different. “Given it rains, the ground is wet” is near 1. “Given the ground is wet, it just rained” is much smaller – maybe someone watered the lawn. Don’t flip the bar without thinking.

Binomial probability. A binomial situation has n independent trials, each with two outcomes (success/failure), and a fixed success probability p . The chance of exactly k successes is $P(X = k) = \binom{n}{k} p^k (1 - p)^{n - k}$.

Why the three pieces? The binomial coefficient counts how many ways to arrange k successes among n trials. Then p^k is the chance those k chosen trials succeed and $(1 - p)^{n - k}$ is the chance the others fail.

Binomial mean. $\mu = np$ (the expected number of successes). Intuitively: n trials at probability p each give about np hits on average.

Common slips. Forgetting the binomial coefficient (giving only one specific sequence, not all arrangements). Confusing $P(A | B)$ with $P(B | A)$. Putting $P(A \cap B)$ in the denominator instead of $P(B)$.

PRACTICE

Apply the conditional formula or the binomial formula as needed.

- $P(A) = 0.8, P(B) = 0.5, P(A \cap B) = 0.4. P(A | B)$ _____
- $P(A \cap B) = 0.2, P(B) = 0.6. P(A | B)$ _____
- The distribution of heads in 4 fair flips is shown. Find $P(X = 2)$. _____

X	0	1	2	3	4
$P(X)$	$\frac{1}{16}$	$\frac{4}{16}$	$\frac{6}{16}$	$\frac{4}{16}$	$\frac{1}{16}$

- Binomial mean μ for $n = 20, p = 0.3$ _____
- Deck: pull ace, no replacement. $P(\text{2nd ace} | \text{1st ace})$ _____
- $\binom{5}{3} \left(\frac{1}{4}\right)^3 \left(\frac{3}{4}\right)^2$ _____
- From the class enrollment below (25 students), find $P(\text{physics} | \text{math})$. _____

Group	Students
Math	15
Physics	10
Both	6
Total	25

- $P(A | B) = 0.6, P(B) = 0.5. P(A \cap B)$ _____
- Binomial: $n = 10, p = 0.5, k = 10. P(X = 10)$ _____
- Binomial: $n = 3, p = 0.4, k = 1. P(X = 1)$ _____
- $P(A | B) \neq P(A)$ for $A, B \dots$ _____
- Binomial: $n = 6, p = \frac{1}{2}, k = 0. P(X = 0)$ _____
- Is $P(A | B) = P(B | A)$ always true? _____
- Binomial: $n = 4, p = 0.25, k = 2. P(X = 2)$ _____



15. Binomial mean for $n = 50, p = 0.10$ _____
16. From the face-card breakdown below, find $P(\text{red}|\text{face card})$. _____

	Red	Black	Total
Face cards	6	6	12

17. Two dice. $P(\text{sum} = 8 | \text{first die} = 3)$ _____
18. Binomial: $n = 2, p = 0.7, k = 2. P(X = 2)$ _____
19. Required for binomial: trials must be ... _____
20. $\binom{5}{0}p^0(1 - p)^5$ with $p = 0.2$ _____

◆ Word Problems

21. A class of 25 students includes 15 who study math, 10 who study physics, and 6 who study both. A student is chosen at random. Given that the student studies math, what is the probability they also study physics? _____
22. A student guesses on 5 multiple-choice questions, each with 4 options. What's the probability of getting exactly 3 correct? (Each guess is independent and the probability of a correct guess is $\frac{1}{4}$.) _____
23. A deck of cards is shuffled. The first card drawn is an ace and is set aside (not replaced). What's the probability that the second card drawn is also an ace? _____
24. A free-throw shooter makes 80% of her shots. If she takes 10 independent shots in a game, what's the expected number she'll make, and what's the probability she makes exactly 8? _____

Additional Practice

25. Probability of rolling an even number on a fair die. _____
26. Probability of drawing a heart from a standard deck. _____
27. Complement of $P(A) = 0.37$. _____
28. If events are independent, $P(A) = 0.4, P(B) = 0.5$, find $P(A \cap B)$. _____
29. Find $P(A \cup B)$ if $P(A) = 0.6, P(B) = 0.3, P(A \cap B) = 0.1$. _____
30. Choose 3 from 8. _____
31. Arrange 4 distinct books. _____
32. Find 7P_2 . _____
33. Find 7C_2 . _____
34. Probability of two heads in two coin flips. _____
35. Expected wins in 80 trials with $p = 0.25$. _____



Answer Keys

1. 0.8	13. Not in general
2. $\frac{1}{3}$	14. $\frac{27}{128}$
3. $\frac{6}{16} = \frac{3}{8}$	15. 5
4. 6	16. $\frac{1}{2}$
5. $\frac{3}{51} = \frac{1}{17}$	17. $\frac{1}{6}$
6. $\frac{45}{512}$	18. 0.49
7. $\frac{2}{5}$	19. independent, fixed n , constant p
8. 0.3	20. 0.32768
9. $\frac{1}{1024}$	21. $\frac{2}{5}$
10. 0.432	22. $\frac{45}{512}$
11. dependent	23. $\frac{3}{51} = \frac{1}{17}$
12. $\frac{1}{64}$	24. $\mu = 8; P(X = 8) = \binom{10}{8}(0.8)^8(0.2)^2 \approx 0.302$
Additional Practice Answers	
25. $\frac{1}{2}$	30. 56
26. $\frac{1}{4}$	31. 24
27. 0.63	32. 42
28. 0.20	33. 21
29. 0.8	34. $\frac{1}{4}$
	35. 20

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

- A careful way to see it: $P(A | B) = \frac{0.4}{0.5} = 0.8$. Conditioning on B divides by $P(B)$, not $P(A \cap B)$. That gives a quick check on the answer.
- Conditional probability is joint over the condition: $P(A | B) = \frac{P(A \cap B)}{P(B)} = \frac{0.2}{0.6} = \frac{1}{3}$. Divide by $P(B)$, never by $P(A \cap B)$.
- This is $P(2 \text{ heads in } 4 \text{ flips})$. $\binom{4}{2} = 6$, and $(0.5)^4 = \frac{1}{16}$, so the $X = 2$ entry is $\frac{6}{16} = \frac{3}{8}$.
- The mean of a binomial is $\mu = np$ – the number of trials times the per-trial success probability. So $\mu = 20 \times 0.3 = 6$, meaning about six successes on average over many repeats.
- Conditioning on “first card was an ace” updates the deck: one ace is gone, leaving 3 aces among 51 cards. So $P(2\text{nd ace} | 1\text{st ace}) = \frac{3}{51} = \frac{1}{17}$. Without replacement always shrinks both the favorable count and the total.
- Keep the rule visible: $\binom{5}{3} = 10$. Then $10 \cdot \frac{1}{64} \cdot \frac{9}{16} = \frac{90}{1024} = \frac{45}{512}$. That’s $P(3 \text{ correct out of } 5 \text{ four-choice guesses})$. That gives a quick check on the answer.
- One steady path is: Restrict to the 15 math students. Of those, 6 also do physics. $\frac{6}{15} = \frac{2}{5}$. That gives a quick check on the answer.
- Rearrange the conditional formula $P(A | B) = \frac{P(A \cap B)}{P(B)}$ by multiplying both sides by $P(B)$: $P(A \cap B) = P(A | B) \cdot P(B) = 0.6 \times 0.5 = 0.3$.
- A careful way to see it: $\binom{10}{0} = 1$, then $(0.5)^{10} \cdot (0.5)^0 = \frac{1}{1024}$. Just one way to get all ten – one specific sequence. That gives a quick check on the answer.
- Binomial formula $\binom{n}{k}p^k(1-p)^{n-k}$ with $n = 3, k = 1$. The coefficient

- $\binom{3}{1} = 3$ counts which trial succeeds; one success and two failures give $0.4^1 \cdot 0.6^2 = 0.4 \times 0.36 = 0.144$. Multiply: $3 \times 0.144 = 0.432$.
- If conditioning changes the probability, the events are dependent. Independence is the exact case $P(A | B) = P(A)$.
- Start with the key idea: $\binom{6}{0} = 1, (0.5)^0 = 1, (0.5)^6 = \frac{1}{64}$. Zero successes means all six trials failed – one specific sequence. That gives a quick check on the answer.
- They’re only equal in special cases (e.g., $P(A) = P(B)$). $P(\text{rain} | \text{cloudy})$ is much smaller than $P(\text{cloudy} | \text{rain})$.
- Keep the rule visible: $\binom{4}{2} = 6$. $6 \cdot (0.25)^2 \cdot (0.75)^2 = 6 \cdot \frac{1}{16} \cdot \frac{9}{16} = \frac{54}{256} = \frac{27}{128}$. That gives a quick check on the answer.
- Binomial mean is $\mu = np: 50 \times 0.10 = 5$ successes expected on average. The mean doesn’t need the binomial coefficient – it’s just trials times per-trial probability.
- Start with the key idea: Restrict to face cards: 12 total, 6 are red. $\frac{6}{12} = \frac{1}{2}$. That gives a quick check on the answer.
- Conditioning on “first die = 3” shrinks the sample space to the 6 possible values of the second die. Exactly one of them, a 5, makes the sum 8, so $P = \frac{1}{6}$. The first die is already fixed, so only the second roll varies.
- All trials must succeed, so there’s only one arrangement: $\binom{2}{2} = 1$. Both successes give $(0.7)^2 = 0.49$, with no failure factor since $(1-p)^0 = 1$.
- The four ingredients: fixed n , two outcomes per trial, independence, and constant p . Lose any one and the binomial formula stops applying.
- That’s $P(0 \text{ successes})$ with $n = 5, p = 0.2$. $\binom{5}{0} = 1, p^0 = 1, (0.8)^5 = 0.32768$. So about 32.8% chance of zero successes.
- The condition “given they study math” shrinks the universe to the 15 math



students. Of those, 6 also study physics. $P(\text{phys} \mid \text{math}) = \frac{6}{15} = \frac{2}{5}$. Notice the total of 25 never appears – it's the new sample space (15) that drives the denominator.

22. Binomial setup: $n = 5$, $p = \frac{1}{4}$, $k = 3$. $\binom{5}{3} = 10$. Then $P(X = 3) = 10 \cdot \left(\frac{1}{4}\right)^3 \cdot \left(\frac{3}{4}\right)^2 = 10 \cdot \frac{1}{64} \cdot \frac{9}{16} = \frac{90}{1024} = \frac{45}{512} \approx 0.088$. So about 8.8% chance of pulling off three out of five on guesses alone – not great odds.

23. After one ace is gone, 3 aces remain in a deck of 51 cards. The conditional

probability is $P(\text{2nd ace} \mid \text{1st ace}) = \frac{3}{51} = \frac{1}{17} \approx 0.059$. (If the first card had been replaced, the answer would jump back to $\frac{4}{52} = \frac{1}{13}$ – replacement restores independence.)

24. Binomial with $n = 10$, $p = 0.8$. Mean: $\mu = np = 10 \times 0.8 = 8$ shots made on average. For exactly 8: $\binom{10}{8} = 45$. $(0.8)^8 \approx 0.1678$ and $(0.2)^2 = 0.04$. Multiply: $45 \times 0.1678 \times 0.04 \approx 0.302$. So about a 30% chance of landing exactly 8. (That's smaller than you might expect – “exactly 8” is just one specific count, and 7 and 9 are also realistic.)



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