Compound and Continuously Compounded Interest

Name: ______________________ Date: ______________ Score: _____ / 24

Quick Review

Simple interest: $A = P(1 + rt)$. Interest is computed once per year on the original principal P. Balance grows linearly.

Compound interest (compounded n times per year): $A = P\left(1 + \frac{r}{n}\right)^{nt}$. Here r is the *annual* rate (as a decimal), n is the number of compounding periods per year (yearly $n = 1$, semi-annually $n = 2$, quarterly $n = 4$, monthly $n = 12$, daily $n = 365$), and t is the time in years. Each period, you multiply by $1 + \frac{r}{n}$, and over t years there are nt periods.

Continuous compounding: $A = Pe^{rt}$. The limit of compound interest as $n \to \infty$. $e \approx 2.71828$.

Compounding frequency matters — a little. For a $\$1000$ deposit at 8% for 5 years: annually gives $\$1469.33$, quarterly gives $\$1485.95$, monthly gives $\$1489.85$, daily gives $\$1491.76$, and continuously gives $\$1491.82$. More frequent compounding helps, but with diminishing returns.

Doubling-time shortcut: rule of 72. An investment at annual rate $r\%$ (compounded once a year) roughly doubles in $72/r$ years. At 6%, about 12 years; at 9%, about 8 years. Exact: solve $(1 + r/100)^t = 2$.

Common slips. Forgetting to divide r by n (using r where r/n belongs). Using t as the exponent instead of nt for compound interest. Mixing percent and decimal — 5% means $r = 0.05$, not 5.

PRACTICE

Plug into the right formula. Round dollar amounts to the nearest cent.

1. Compound interest formula (n periods/year)? ________
2. Continuous compounding formula? ________
3. $\$1000$ at 5% compounded annually for 3 years. ________
4. $\$2000$ at 4% compounded semi-annually for 2 years. ________
5. $\$5000$ at 6% compounded continuously for 4 years. ________
6. Years for an investment at 7% annual to double? ________
7. $\$1000$ at 8% compounded quarterly for 5 years. ________
8. $\$4000$ at 6% compounded monthly for 3 years; which expression? ________
9. $\$2500$ at 5% compounded continuously for 6 years. ________
10. Does more frequent compounding always give a larger balance? ________
11. $\$1000$ at 10% compounded annually for 3 years. ________
12. $\$8000$ at 5% compounded semi-annually for 6 years. ________
13. Is $A = P + rt$ the compound interest formula? ________
14. If $r = 0$ in $A = P(1 + r/n)^{nt}$, what is A? ________

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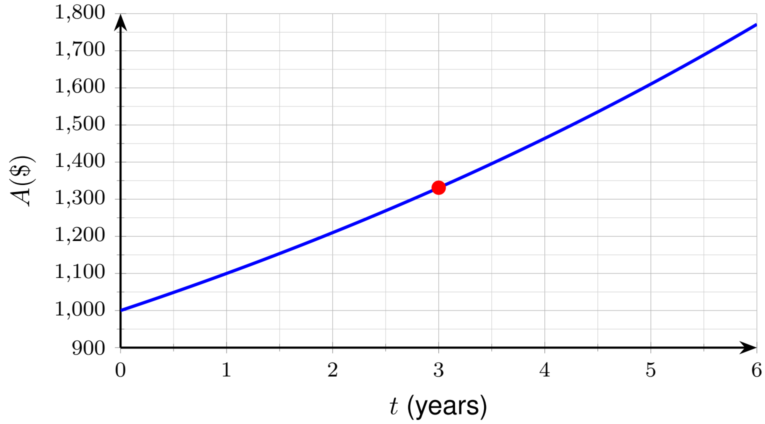
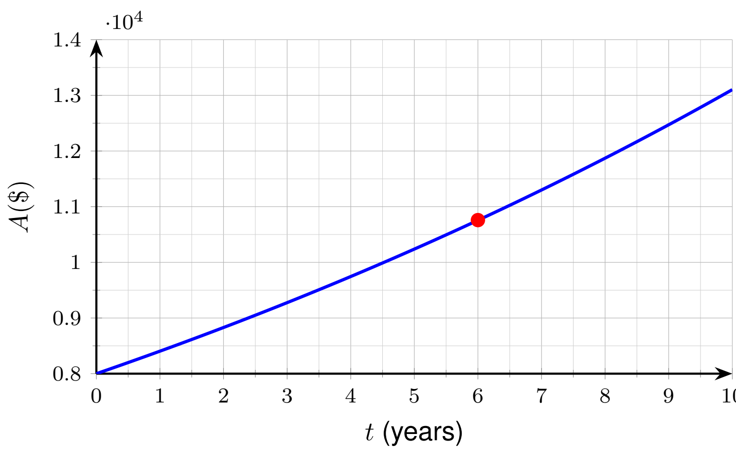
15. The graph below shows $A(t) = 1000(1.1)^t$. Find the value at $t = 3$. ________



16. The graph below tracks $A(t) = 8000(1 + 0.05/2)^{2t}$. Find the value at $t = 6$. ________



17. Compounding monthly gives $r/n =$ _____ for $r = 12\%$. ________

18. If $P = \$500$, $r = 0.04$, $n = 4$, $t = 2$, find A. ________

19. For continuous compounding at 3%, what factor does the balance multiply by each year? ________

20. Rule of 72: approximate doubling time at 6%. ________

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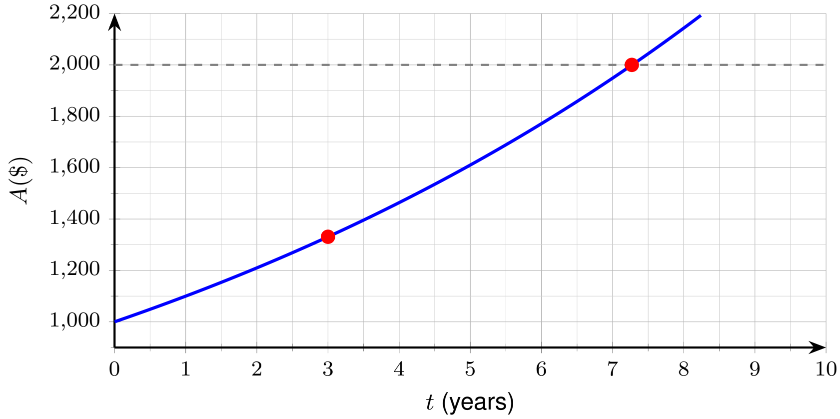
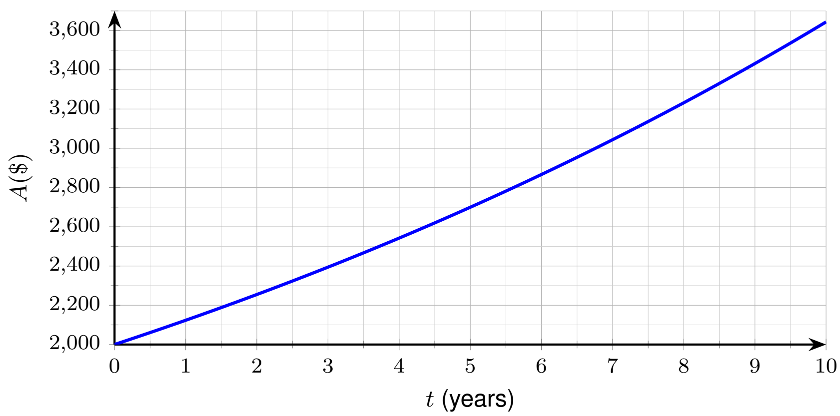
Word Problems

21. An investment of $\$1{,}000$ earns 10% annual interest, compounded annually. Find the balance after 3 years, and find how long it takes the investment to double. ________



22. Compare three compounding strategies for $\$2{,}000$ at 6% for 10 years: annual, quarterly, and continuous. How much does the more-frequent compounding add? ________



23. How long does it take an investment to double at 7% compounded annually? Compare your exact answer with the rule-of-72 estimate. ____

24. A family puts $\$10{,}000$ into a college fund at 5% compounded continuously for a newborn. How much will be in the account on the child's 18th birthday? ____

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Answer Keys

1. $A = P\left(1 + \frac{r}{n}\right)^{nt}$
2. $A = Pe^{rt}$
3. $\approx \$1157.63$
4. $\approx \$2164.86$
5. $\approx \$6356.25$
6. ≈ 10.24
7. $\approx \$1485.95$
8. $4000\left(1 + \frac{0.06}{12}\right)^{36}$
9. $\approx \$3374.65$
10. yes (up to the continuous limit)
11. $\$1331$
12. $\approx \$10759.10$
13. no (that's wrong)
14. P
15. $\$1331$
16. $\approx \$10759.10$
17. 0.01
18. $\approx \$541.43$
19. $e^{0.03} \approx 1.0305$
20. 12 years
21. $A(3) = \$1,331$; $t \approx 7.27$ years to double
22. annual: $\$3,581.70$; quarterly: $\$3,628.04$; continuous: $\$3,644.24$
23. $t \approx 10.24$ years (rule of 72 estimate: 10.29)
24. $\approx \$24,596.03$

Step-by-Step Explanations

1. Each period multiplies the balance by $1 + \frac{r}{n}$, and there are nt periods in t years. Here r is the annual rate as a decimal, n the periods per year, and t the years.
2. This is the limit of $A = P(1 + r/n)^{nt}$ as $n \to \infty$ — compounding instant by instant. The constant $e \approx 2.71828$ replaces the repeated factor.
3. Annually means $n = 1$, so $A = 1000(1 + 0.05)^3 = 1000(1.05)^3$. Compute $1.05^3 = 1.157625$, then $1000(1.157625) \approx \$1157.63$.
4. Semi-annual means $n = 2$, so $r/n = 0.04/2 = 0.02$ and $nt = 2(2) = 4$. Then $A = 2000(1.02)^4 = 2000(1.08243) \approx \$2164.86$.
5. A careful way to see it: Use $A = Pe^{rt}$ with $rt = 0.06(4) = 0.24$. So $A = 5000e^{0.24} = 5000(1.27125) \approx \$6356.25$. That gives a quick check on the answer.
6. Doubling means $(1.07)^t = 2$. Take logs: $t = \log 2/\log 1.07 \approx 10.24$ years. The rule of 72 gives $72/7 \approx 10.3$ — a close, quick estimate.
7. Quarterly means $n = 4$, so $r/n = 0.08/4 = 0.02$ and $nt = 4(5) = 20$. Then $A = 1000(1.02)^{20} = 1000(1.48595) \approx \$1485.95$.
8. Monthly gives $n = 12$, so $r/n = 0.06/12 = 0.005$, and the exponent is $nt = 12(3) = 36$. Substitute into $P(1 + r/n)^{nt}$ to get $4000\left(1 + \frac{0.06}{12}\right)^{36}$.
9. A careful way to see it: Use $A = Pe^{rt}$ with $rt = 0.05(6) = 0.30$. So $A = 2500e^{0.30} = 2500(1.34986) \approx \$3374.65$. That gives a quick check on the answer.
10. More frequent compounding always helps, because interest starts earning interest sooner. But the gains shrink with each step up, and continuous compounding ($A = Pe^{rt}$) is the ceiling you can never beat.
11. Annually means $n = 1$, so $A = 1000(1.10)^3$. Compute $1.1^3 = 1.331$, giving $1000(1.331) = \$1331$ exactly — a clean integer.
12. Semi-annual gives $n = 2$, so $r/n = 0.05/2 = 0.025$ and $nt = 2(6) = 12$. Then $A = 8000(1.025)^{12} = 8000(1.34489) \approx \$10759.10$.
13. No — that isn't even simple interest, which is $A = P(1 + rt)$. Compound interest uses a *power*, $A = P(1 + r/n)^{nt}$, because the balance is multiplied each period rather than having a fixed amount added.
14. With $r = 0$, the per-period factor is $1 + 0/n = 1$, and 1 raised to any power is still 1. So $A = P \cdot 1 = P$ — no interest means the balance never changes.
15. Read the model and substitute $t = 3$: $A(3) = 1000(1.1)^3$. Since $1.1^3 = 1.331$, the value is $1000(1.331) = \$1331$, matching the red dot on the curve.
16. The model already encodes $n = 2$ and $r/n = 0.025$. At $t = 6$, the exponent is $2t = 12$, so $A(6) = 8000(1.025)^{12} = 8000(1.34489) \approx \$10759.10$.
17. Monthly compounding means $n = 12$. Convert the rate to a decimal first: $r = 0.12$. Then $r/n = 0.12/12 = 0.01$ per month.
18. Find the pieces: $r/n = 0.04/4 = 0.01$ and $nt = 4(2) = 8$. Then $A = 500(1.01)^8 = 500(1.08286) \approx \$541.43$.
19. Over one year, the balance multiplies by $A(t+1)/A(t) = e^r = e^{0.03} \approx 1.0305$. So a 3% continuous rate behaves like about a 3.05% effective annual rate.
20. The rule of 72 estimates doubling time as $72/r\%$, so $72/6 = 12$ years. The exact value $\log 2/\log 1.06 \approx 11.9$ years confirms the shortcut is accurate here.
21. A careful way to see it: $A(t) = 1000(1.1)^t$. At $t = 3$: $A(3) = 1000(1.331) = 1331$. For doubling, set $(1.1)^t = 2$. Take logs: $t = \log 2/\log 1.1 \approx 0.301/0.0414 \approx 7.27$ years. Rule-of-72 estimate: $72/10 = 7.2$ — agrees to two digits. The red dots on the curve mark both the 3-year value and the doubling point. That gives a quick check on the answer.
22. Annual: $A = 2000(1.06)^{10} = 2000(1.79085) \approx 3581.70$. Quarterly ($n = 4$): $A = 2000(1.015)^{40} = 2000(1.81402) \approx 3628.04$. Continuous: $A = 2000e^{0.6} = 2000(1.82212) \approx 3644.24$. Going from annual to quarterly buys you about $\$46$; going from quarterly to continuous buys about $\$16$ more. The gains shrink because continuous is the ceiling — you can't do better.
23. Set $P(1.07)^t = 2P$, so $(1.07)^t = 2$. Take logs: $t = \log 2/\log 1.07 \approx 0.301/0.0294 \approx 10.24$ years. Rule of 72 estimate: $72/7 \approx 10.29$. The two agree to within a few hundredths — the rule of 72 is surprisingly accurate for everyday rates in the 5–10% range. (For very high rates, the rule of 72 overestimates a bit; at 20%, true doubling time is about 3.8 years vs. the rule's 3.6.)
24. Start with the key idea: $A = Pe^{rt} = 10000e^{0.05 \cdot 18} = 10000e^{0.9} \approx 10000(2.4596) = 24596.03$. So the college fund grows from $\$10,000$ to about $\$24,596$ in 18 years — about 2.46 times the initial deposit. (At 5%, the rule of 72 predicts $72/5 \approx 14.4$ years to double, so 18 years gives a bit more than a single doubling. That matches.) That gives a quick check on the answer.