

# Compound and Continuously Compounded Interest

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Score: \_\_\_\_\_ / 24

## Q Quick Review

**Simple interest:**  $A = P(1 + rt)$ . Interest is computed once per year on the original principal  $P$ . Balance grows linearly.

**Compound interest** (compounded  $n$  times per year):  $A = P \left(1 + \frac{r}{n}\right)^{nt}$ . Here  $r$  is the *annual* rate (as a decimal),  $n$  is the number of compounding periods per year (yearly  $n = 1$ , semi-annually  $n = 2$ , quarterly  $n = 4$ , monthly  $n = 12$ , daily  $n = 365$ ), and  $t$  is the time in years. Each period, you multiply by  $1 + \frac{r}{n}$ , and over  $t$  years there are  $nt$  periods.

**Continuous compounding:**  $A = Pe^{rt}$ . The limit of compound interest as  $n \rightarrow \infty$ .  $e \approx 2.71828$ .

**Compounding frequency matters — a little.** For a \$1000 deposit at 8% for 5 years: annually gives \$1469.33, quarterly gives \$1485.95, monthly gives \$1489.85, daily gives \$1491.76, and continuously gives \$1491.82. More frequent compounding helps, but with diminishing returns.

**Doubling-time shortcut: rule of 72.** An investment at annual rate  $r\%$  (compounded once a year) roughly doubles in  $72/r$  years. At 6%, about 12 years; at 9%, about 8 years. Exact: solve  $(1 + r/100)^t = 2$ .

**Common slips.** Forgetting to divide  $r$  by  $n$  (using  $r$  where  $r/n$  belongs). Using  $t$  as the exponent instead of  $nt$  for compound interest. Mixing percent and decimal — 5% means  $r = 0.05$ , not 5.

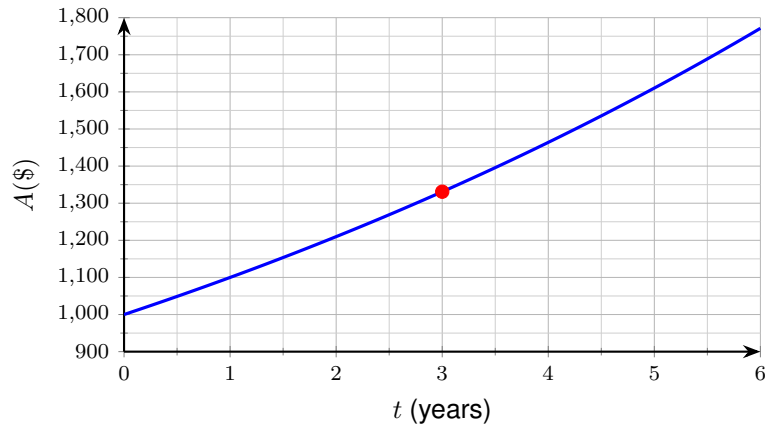
## PRACTICE

*Plug into the right formula. Round dollar amounts to the nearest cent.*

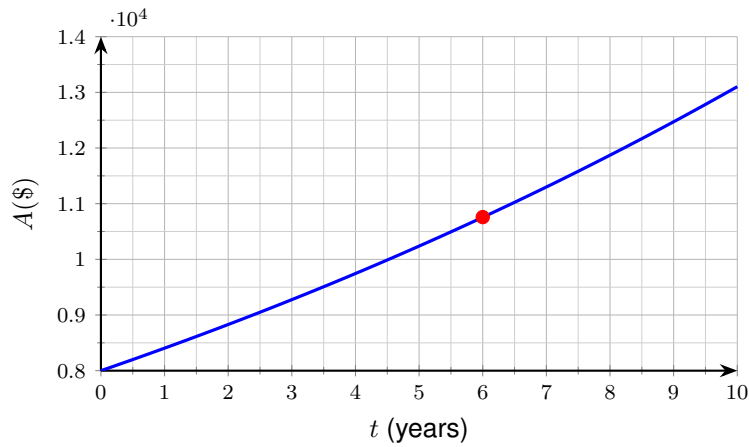
1. Compound interest formula ( $n$  periods/year)? \_\_\_\_\_
2. Continuous compounding formula? \_\_\_\_\_
3. \$1000 at 5% compounded annually for 3 years. \_\_\_\_\_
4. \$2000 at 4% compounded semi-annually for 2 years. \_\_\_\_\_
5. \$5000 at 6% compounded continuously for 4 years. \_\_\_\_\_
6. Years for an investment at 7% annual to double? \_\_\_\_\_
7. \$1000 at 8% compounded quarterly for 5 years. \_\_\_\_\_
8. \$4000 at 6% compounded monthly for 3 years; which expression? \_\_\_\_\_
9. \$2500 at 5% compounded continuously for 6 years. \_\_\_\_\_
10. Does more frequent compounding always give a larger balance? \_\_\_\_\_
11. \$1000 at 10% compounded annually for 3 years. \_\_\_\_\_
12. \$8000 at 5% compounded semi-annually for 6 years. \_\_\_\_\_
13. Is  $A = P + rt$  the compound interest formula? \_\_\_\_\_
14. If  $r = 0$  in  $A = P(1 + r/n)^{nt}$ , what is  $A$ ? \_\_\_\_\_



15. The graph below shows  $A(t) = 1000(1.1)^t$ . Find the value at  $t = 3$ . \_\_\_\_\_



16. The graph below tracks  $A(t) = 8000(1 + 0.05/2)^{2t}$ . Find the value at  $t = 6$ . \_\_\_\_\_



17. Compounding monthly gives  $r/n =$  \_\_\_\_\_ for  $r = 12\%$ . \_\_\_\_\_

18. If  $P = \$500$ ,  $r = 0.04$ ,  $n = 4$ ,  $t = 2$ , find  $A$ . \_\_\_\_\_

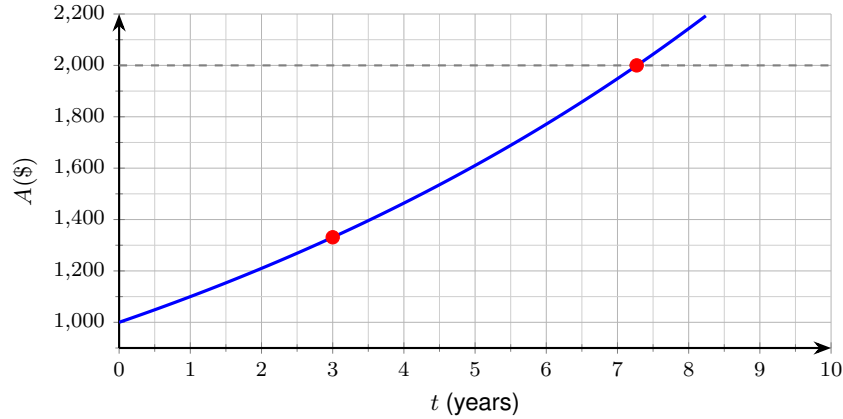
19. For continuous compounding at 3%, what factor does the balance multiply by each year? \_\_\_\_\_

20. Rule of 72: approximate doubling time at 6%. \_\_\_\_\_

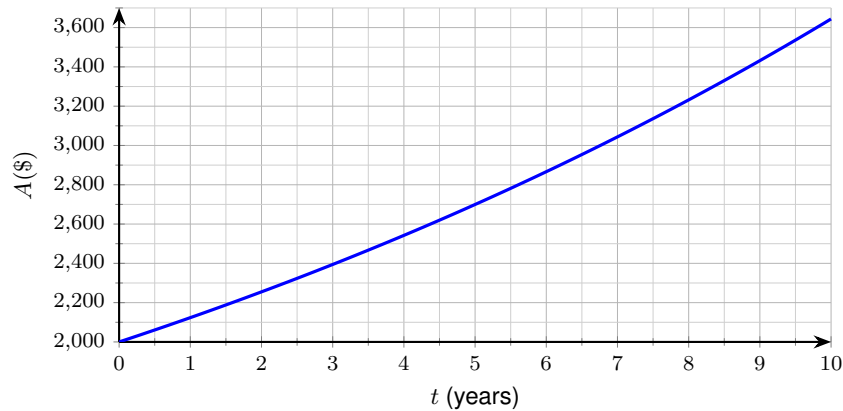


◆ Word Problems

21. An investment of \$1,000 earns 10% annual interest, compounded annually. Find the balance after 3 years, and find how long it takes the investment to double.



22. Compare three compounding strategies for \$2,000 at 6% for 10 years: annual, quarterly, and continuous. How much does the more-frequent compounding add?



23. How long does it take an investment to double at 7% compounded annually? Compare your exact answer with the rule-of-72 estimate.

24. A family puts \$10,000 into a college fund at 5% compounded continuously for a newborn. How much will be in the account on the child's 18th birthday?



## Answer Keys

1.  $A = P \left(1 + \frac{r}{n}\right)^{nt}$
2.  $A = Pe^{rt}$
3.  $\approx \$1157.63$
4.  $\approx \$2164.86$
5.  $\approx \$6356.25$
6.  $\approx 10.24$
7.  $\approx \$1485.95$
8.  $4000 \left(1 + \frac{0.06}{12}\right)^{36}$
9.  $\approx \$3374.65$
10. yes (up to the continuous limit)
11. \$1331
12.  $\approx \$10759.10$
13. no (that's wrong)
14.  $P$
15. \$1331
16.  $\approx \$10759.10$
17. 0.01
18.  $\approx \$541.43$
19.  $e^{0.03} \approx 1.0305$
20. 12 years
21.  $A(3) = \$1,331$ ;  $t \approx 7.27$  years to double
22.
 

annual: \$3,581.70
quarterly: \$3,628.04
continuous: \$3,644.24
23.  $t \approx 10.24$  years (rule of 72 estimate: 10.29)
24.  $\approx \$24,596.03$

## Step-by-Step Explanations

1. Each period multiplies the balance by  $1 + \frac{r}{n}$ , and there are  $nt$  periods in  $t$  years. Here  $r$  is the annual rate as a decimal,  $n$  the periods per year, and  $t$  the years.
2. This is the limit of  $A = P(1 + r/n)^{nt}$  as  $n \rightarrow \infty$  — compounding instant by instant. The constant  $e \approx 2.71828$  replaces the repeated factor.
3. Annually means  $n = 1$ , so  $A = 1000(1 + 0.05)^3 = 1000(1.05)^3$ . Compute  $1.05^3 = 1.157625$ , then  $1000(1.157625) \approx \$1157.63$ .
4. Semi-annual means  $n = 2$ , so  $r/n = 0.04/2 = 0.02$  and  $nt = 2(2) = 4$ . Then  $A = 2000(1.02)^4 = 2000(1.08243) \approx \$2164.86$ .
5. A careful way to see it: Use  $A = Pe^{rt}$  with  $rt = 0.06(4) = 0.24$ . So  $A = 5000e^{0.24} = 5000(1.27125) \approx \$6356.25$ . That gives a quick check on the answer.
6. Doubling means  $(1.07)^t = 2$ . Take logs:  $t = \log 2 / \log 1.07 \approx 10.24$  years. The rule of 72 gives  $72/7 \approx 10.3$  — a close, quick estimate.
7. Quarterly means  $n = 4$ , so  $r/n = 0.08/4 = 0.02$  and  $nt = 4(5) = 20$ . Then  $A = 1000(1.02)^{20} = 1000(1.48595) \approx \$1485.95$ .
8. Monthly gives  $n = 12$ , so  $r/n = 0.06/12 = 0.005$ , and the exponent is  $nt = 12(3) = 36$ . Substitute into  $P(1 + r/n)^{nt}$  to get  $4000 \left(1 + \frac{0.06}{12}\right)^{36}$ .
9. A careful way to see it: Use  $A = Pe^{rt}$  with  $rt = 0.05(6) = 0.30$ . So  $A = 2500e^{0.30} = 2500(1.34986) \approx \$3374.65$ . That gives a quick check on the answer.
10. More frequent compounding always helps, because interest starts earning interest sooner. But the gains shrink with each step up, and continuous compounding ( $A = Pe^{rt}$ ) is the ceiling you can never beat.
11. Annually means  $n = 1$ , so  $A = 1000(1.10)^3$ . Compute  $1.1^3 = 1.331$ , giving  $1000(1.331) = \$1331$  exactly — a clean integer.
12. Semi-annual gives  $n = 2$ , so  $r/n = 0.05/2 = 0.025$  and  $nt = 2(6) = 12$ . Then  $A = 8000(1.025)^{12} = 8000(1.34489) \approx \$10759.10$ .
13. No — that isn't even simple interest, which is  $A = P(1 + rt)$ . Compound interest uses a power,  $A = P(1 + r/n)^{nt}$ , because the balance is multiplied each period rather than having a fixed amount added.
14. With  $r = 0$ , the per-period factor is  $1 + 0/n = 1$ , and 1 raised to any power is still 1. So  $A = P \cdot 1 = P$  — no interest means the balance never changes.
15. Read the model and substitute  $t = 3$ :  $A(3) = 1000(1.1)^3$ . Since  $1.1^3 = 1.331$ , the value is  $1000(1.331) = \$1331$ , matching the red dot on

the curve.

16. The model already encodes  $n = 2$  and  $r/n = 0.025$ . At  $t = 6$ , the exponent is  $2t = 12$ , so  $A(6) = 8000(1.025)^{12} = 8000(1.34489) \approx \$10759.10$ .
17. Monthly compounding means  $n = 12$ . Convert the rate to a decimal first:  $r = 0.12$ . Then  $r/n = 0.12/12 = 0.01$  per month.
18. Find the pieces:  $r/n = 0.04/4 = 0.01$  and  $nt = 4(2) = 8$ . Then  $A = 500(1.01)^8 = 500(1.08286) \approx \$541.43$ .
19. Over one year, the balance multiplies by  $A(t+1)/A(t) = e^r = e^{0.03} \approx 1.0305$ . So a 3% continuous rate behaves like about a 3.05% effective annual rate.
20. The rule of 72 estimates doubling time as  $72/r\%$ , so  $72/6 = 12$  years. The exact value  $\log 2 / \log 1.06 \approx 11.9$  years confirms the shortcut is accurate here.
21. A careful way to see it:  $A(t) = 1000(1.1)^t$ . At  $t = 3$ :  $A(3) = 1000(1.331) = 1331$ . For doubling, set  $(1.1)^t = 2$ . Take logs:  $t = \log 2 / \log 1.1 \approx 0.301/0.0414 \approx 7.27$  years. Rule-of-72 estimate:  $72/10 = 7.2$  — agrees to two digits. The red dots on the curve mark both the 3-year value and the doubling point. That gives a quick check on the answer.
22. Annual:  $A = 2000(1.06)^{10} = 2000(1.79085) \approx 3581.70$ . Quarterly ( $n = 4$ ):  $A = 2000(1.015)^{40} = 2000(1.81402) \approx 3628.04$ . Continuous:  $A = 2000e^{0.6} = 2000(1.82212) \approx 3644.24$ . Going from annual to quarterly buys you about \$46; going from quarterly to continuous buys about \$16 more. The gains shrink because continuous is the ceiling — you can't do better.
23. Set  $P(1.07)^t = 2P$ , so  $(1.07)^t = 2$ . Take logs:  $t = \log 2 / \log 1.07 \approx 0.301/0.0294 \approx 10.24$  years. Rule of 72 estimate:  $72/7 \approx 10.29$ . The two agree to within a few hundredths — the rule of 72 is surprisingly accurate for everyday rates in the 5–10% range. (For very high rates, the rule of 72 overestimates a bit; at 20%, true doubling time is about 3.8 years vs. the rule's 3.6.)
24. Start with the key idea:  $A = Pe^{rt} = 10000e^{0.05 \cdot 18} = 10000e^{0.9} \approx 10000(2.4596) = 24596.03$ . So the college fund grows from \$10,000 to about \$24,596 in 18 years — about 2.46 times the initial deposit. (At 5%, the rule of 72 predicts  $72/5 \approx 14.4$  years to double, so 18 years gives a bit more than a single doubling. That matches.) That gives a quick check on the answer.



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