

Composition of Functions

Name: _____ Date: _____ Score: _____ / 29

Q Quick Review

Composition chains two functions: $(f \circ g)(x) = f(g(x))$ means “apply g first, then feed the result into f .” Read it inside-out. The little circle \circ is not multiplication — don’t confuse $(f \circ g)$ with $(f \cdot g)$.

Order matters. Composition is *not* commutative. Usually $(f \circ g)(x) \neq (g \circ f)(x)$. Example with $f(x) = 2x + 1$ and $g(x) = x^2$: $(f \circ g)(x) = 2x^2 + 1$, but $(g \circ f)(x) = (2x + 1)^2 = 4x^2 + 4x + 1$. Different polynomials.

To evaluate at a number, work inside-out. For $(f \circ g)(4)$ with $f(x) = 2x + 3$, $g(x) = x - 1$: first $g(4) = 3$, then $f(3) = 9$. So $(f \circ g)(4) = 9$.

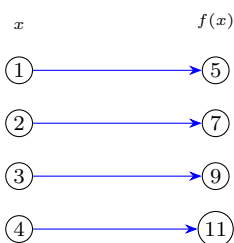
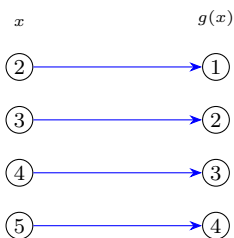
To build the formula, substitute the inner expression wherever the outer function asks for x .

Domain. $(f \circ g)$ is defined wherever x is in the domain of g and $g(x)$ is in the domain of f . The **identity** function $I(x) = x$ is the do-nothing function: $f \circ I = I \circ f = f$. Composition is also **associative**: $(f \circ g) \circ h = f \circ (g \circ h)$. Two functions are inverses of each other when $f \circ g$ and $g \circ f$ both equal the identity — that’s what an inverse *does* for a living.

PRACTICE

Compose the functions as indicated. Watch the order.

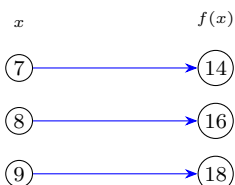
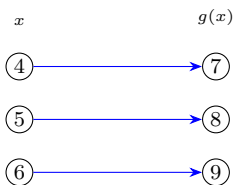
1. The mapping diagrams show g (left) and f (right). Find $(f \circ g)(4)$. _____



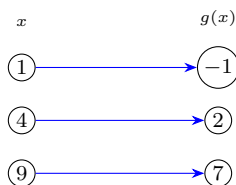
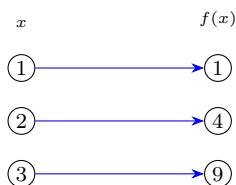
- 2. $(f \circ g)(x)$ means: _____
- 3. $f(x) = x + 5$, $g(x) = 3x$; $(f \circ g)(x)$ _____
- 4. $f(x) = x^2$, $g(x) = x + 2$; $(f \circ g)(x)$ _____
- 5. $f(x) = 2x + 1$, $g(x) = x^2$; $(g \circ f)(x)$ _____
- 6. $f(x) = 3x - 6$; $g(x)$ so that $(f \circ g)(x) = x$ _____
- 7. $f(x) = x^2 + 1$, $g(x) = x - 3$; $(f \circ g)(x)$ _____
- 8. $f(x) = \sqrt{x - 1}$, $g(x) = 2x + 5$; $(f \circ g)(x)$ _____



9. The mapping diagrams show g (left) and f (right). Find $(f \circ g)(5)$. _____



10. The mapping diagrams show f (left) and g (right). Find $(g \circ f)(3)$. _____



- 11. $f(x) = 3x + 2$, $g(x) = x$; $(f \circ g)(x)$ _____
- 12. $f(x) = \frac{1}{x}$, $g(x) = x + 1$; $(f \circ g)(x)$ _____
- 13. $f(x) = 2x - 1$, $g(x) = 3x + 4$; $(g \circ f)(x)$ _____
- 14. $f(x) = x^2$, $g(x) = x + 1$. Is $(f \circ g)(x)$ the same as $(g \circ f)(x)$? _____
- 15. $S(w) = 4w + 9$ shipping cost; $p(x) = x + 2$ packaging weight in pounds for an x -pound item. Cost in terms of x : _____
- 16. $f(x) = x^3$, $g(x) = x - 2$; $(f \circ g)(3)$ _____
- 17. $f(x) = 2x + 5$, $g(x) = \frac{x - 5}{2}$. Show $(f \circ g)(x) = x$. _____
- 18. $f(x) = x + 4$, $g(x) = 3x$; $(g \circ f)(x)$ _____
- 19. $f(x) = \sqrt{x}$, $g(x) = x - 1$. Find $(f \circ g)(x)$ and the domain. _____
- 20. $f(x) = 2x + 1$, $g(x) = \frac{x - 1}{2}$. Find $(f \circ g)(x)$ and $(g \circ f)(x)$. _____



◆ Word Problems

21. A shipping company charges $S(w) = 4w + 9$ dollars to ship a package weighing w pounds. A packing machine increases an item's weight by 2 pounds before shipping, so $p(x) = x + 2$. Write the total shipping cost as a function of the original item weight x , then find the cost for a 5-pound item. _____

22. A circle's radius after t seconds is $r(t) = 2t + 1$ centimeters. Its area in terms of radius is $A(r) = \pi r^2$. Write the area as a function of time, $A(t)$, then find $A(3)$. _____

23. Let $f(x) = 2x - 3$ and $g(x) = \frac{x+3}{2}$. Show that f and g are inverses by computing both compositions. _____

24. A clothing store applies a 20% discount and then charges 8% sales tax. Let $d(p) = 0.80p$ be the discount step (output: discounted price) and $t(p) = 1.08p$ be the tax step. Write the total-cost function $C(p) = t(d(p))$ for an item with sticker price p , then find the cost of a \$50 item. _____

Additional Practice

25. If $f(x) = 2x - 5$, find $f(4)$. _____

26. If $g(x) = x^2 + 1$, find $g(-3)$. _____

27. For $f(x) = 3x + 2$, solve $f(x) = 14$. _____

28. Find $(f + g)(x)$ if $f = x + 1$, $g = 2x - 5$. _____

29. Find $(fg)(x)$ if $f = x - 2$, $g = x + 3$. _____



Answer Keys

- | | |
|-----------------------------|---|
| 1. 9 | 13. $6x + 1$ |
| 2. $f(g(x))$ | 14. no |
| 3. $3x + 5$ | 15. $4x + 17$ |
| 4. $(x + 2)^2$ | 16. 1 |
| 5. $(2x + 1)^2$ | 17. x |
| 6. $g(x) = \frac{x + 6}{3}$ | 18. $3x + 12$ |
| 7. $x^2 - 6x + 10$ | 19. $\sqrt{x - 1}, x \geq 1$ |
| 8. $\sqrt{2x + 4}$ | 20. both equal x |
| 9. 16 | 21. $(S \circ p)(x) = 4x + 17; (S \circ p)(5) = \37 |
| 10. 7 | 22. $A(t) = \pi(2t + 1)^2, A(3) = 49\pi \text{ cm}^2$ |
| 11. $3x + 2$ | 23. $f \circ g = g \circ f = I(x) = x$ |
| 12. $\frac{1}{x + 1}$ | 24. $C(p) = 0.864p, C(50) = \$43.20$ |
- Additional Practice Answers**
- | | |
|-------------|-------------------|
| 25. 3 | 28. $3x - 4$ |
| 26. 10 | 29. $x^2 + x - 6$ |
| 27. $x = 4$ | |

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

- Work inside-out. Trace g first: the arrow from 4 lands on 3, so $g(4) = 3$. Feed that into f : the arrow from 3 lands on 9, so $f(3) = 9$. Thus $(f \circ g)(4) = 9$.
- Apply g first, feed result into f . The little circle is composition, not multiplication.
- One steady path is: $f(g(x)) = f(3x) = 3x + 5$. Plug the entire $3x$ into where x appeared in f . That gives a quick check on the answer.
- Start with the key idea: $f(g(x)) = f(x + 2) = (x + 2)^2$. Equivalently $x^2 + 4x + 4$. This is the part to check before moving on, because it keeps the answer tied to the original question.
- A careful way to see it: $g(f(x)) = g(2x + 1) = (2x + 1)^2 = 4x^2 + 4x + 1$. Different from $(f \circ g)(x) = 2x^2 + 1$. That gives a quick check on the answer.
- Keep the rule visible: g must undo f . Solve $y = 3x - 6$ for x : $x = \frac{y + 6}{3}$, so $g(x) = \frac{x + 6}{3}$. Check: $f(g(x)) = 3 \cdot \frac{x + 6}{3} - 6 = x + 6 - 6 = x$. (That's the inverse.) That gives a quick check on the answer.
- One steady path is: $f(x - 3) = (x - 3)^2 + 1 = x^2 - 6x + 9 + 1 = x^2 - 6x + 10$. This is the part to check before moving on, because it keeps the answer tied to the original question.
- Start with the key idea: $f(2x + 5) = \sqrt{(2x + 5) - 1} = \sqrt{2x + 4}$. Domain: $2x + 4 \geq 0$, so $x \geq -2$. This is the part to check before moving on, because it keeps the answer tied to the original question.
- Inside-out: g sends 5 to 8, so $g(5) = 8$. Then f sends 8 to 16. So $(f \circ g)(5) = 16$.
- Apply f first: $f(3) = 9$. Then apply g to that result: $g(9) = 7$. So $(g \circ f)(3) = 7$. Order matters — the inner function runs first.
- One steady path is: g is the identity, so $f \circ g = f$. The identity is the do-nothing function under composition. That gives a quick check on the answer.
- Start with the key idea: $f(x + 1) = \frac{1}{x + 1}$, defined for $x \neq -1$. This is the part to check before moving on, because it keeps the answer tied to the original question.
- A careful way to see it: $g(f(x)) = g(2x - 1) = 3(2x - 1) + 4 = 6x - 3 + 4 = 6x + 1$. This is the part to check before moving on, because it keeps the answer tied to the original question.
- Keep the rule visible: $(f \circ g)(x) = (x + 1)^2 = x^2 + 2x + 1$; $(g \circ f)(x) = x^2 + 1$. Different polynomials — composition is not commutative. That gives a quick check

on the answer.

- One steady path is: $S(p(x)) = 4(x + 2) + 9 = 4x + 8 + 9 = 4x + 17$. This is the part to check before moving on, because it keeps the answer tied to the original question.
- Start with the key idea: $g(3) = 1, f(1) = 1$. This is the part to check before moving on, because it keeps the answer tied to the original question.
- A careful way to see it: $f(g(x)) = 2 \cdot \frac{x - 5}{2} + 5 = (x - 5) + 5 = x$. (Hence $g = f^{-1}$.) This is the part to check before moving on, because it keeps the answer tied to the original question.
- Keep the rule visible: $g(f(x)) = g(x + 4) = 3(x + 4) = 3x + 12$. (Compare $(f \circ g)(x) = 3x + 4$.) That gives a quick check on the answer.
- One steady path is: $f(g(x)) = \sqrt{x - 1}$. The radicand must be ≥ 0 , so the domain is $x \geq 1$. That gives a quick check on the answer.
- Start with the key idea: $f(g(x)) = 2 \cdot \frac{x - 1}{2} + 1 = (x - 1) + 1 = x$.
 $g(f(x)) = \frac{(2x + 1) - 1}{2} = \frac{2x}{2} = x$. So f and g are inverses. That gives a quick check on the answer.
- The shipping function uses the packed weight, so compose: $S(p(x)) = 4(x + 2) + 9 = 4x + 8 + 9 = 4x + 17$. For a 5-pound item: $4(5) + 17 = 37$ dollars. Sanity check: packed weight is 7 pounds, and $4(7) + 9 = 37$. Same number both ways.
- Compose: $A(r(t)) = \pi(2t + 1)^2$. At $t = 3$: $r(3) = 7$, so $A = \pi(49) = 49\pi \text{ cm}^2$. Composing lets us skip the intermediate radius and write area directly as a function of time — useful when modeling a growing ripple.
- Compute $f(g(x)) = 2 \cdot \frac{x + 3}{2} - 3 = (x + 3) - 3 = x$. Compute $g(f(x)) = \frac{(2x - 3) + 3}{2} = \frac{2x}{2} = x$. Both compositions return the input untouched, which is the identity function. That's exactly the definition of inverse: each function undoes the other.
- Discount first, then tax: $t(d(p)) = 1.08(0.80p) = 0.864p$. So you pay 86.4% of the sticker price. For $p = 50$: $C(50) = 43.20$. (The two steps are simple scalings, which is why they commute — discount-then-tax and tax-then-discount give the same dollar total.)



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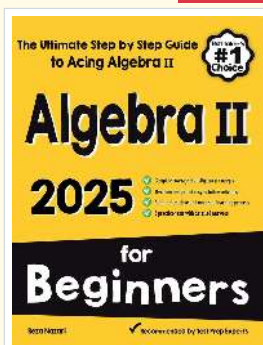
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