

Complex Conjugates and Modulus

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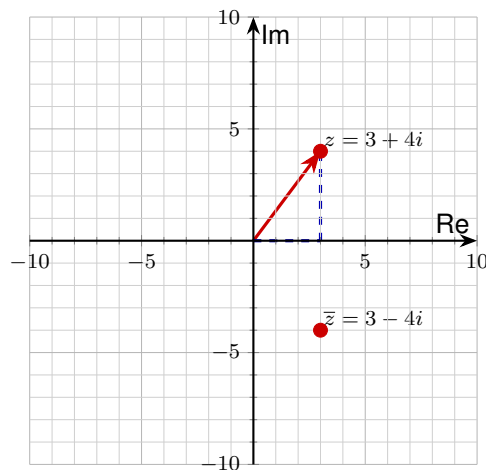
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Q Quick Review

The **conjugate** of $z = a + bi$ is $\bar{z} = a - bi$ — flip the imaginary sign, leave the real part alone. Geometrically, the conjugate is the reflection of z across the real axis. The **modulus** (or absolute value) of $z = a + bi$ is $|z| = \sqrt{a^2 + b^2}$ — the distance from z to the origin on the Argand plane. Distances are never negative, so a negative modulus is always a mistake.

Three facts you should know cold. First, $z\bar{z} = |z|^2$ for every complex z (this is why multiplying by the conjugate clears imaginary parts in division). Second, conjugation distributes over addition and multiplication: $\overline{z+w} = \bar{z} + \bar{w}$ and $\overline{zw} = \bar{z}\bar{w}$. Third, $|zw| = |z||w|$ (moduli *multiply*), but $|z+w| \leq |z| + |w|$ in general (the triangle inequality, with equality only when z and w point the same direction).

The figure shows $z = 3 + 4i$ and its conjugate $\bar{z} = 3 - 4i$ as mirror images across the real axis, with the modulus $|z| = 5$ shown as the hypotenuse of the 3-4-5 triangle.



The dashed legs are length 3 (real part) and 4 (imaginary part). The red arrow is the modulus $|z| = \sqrt{3^2 + 4^2} = 5$.

PRACTICE

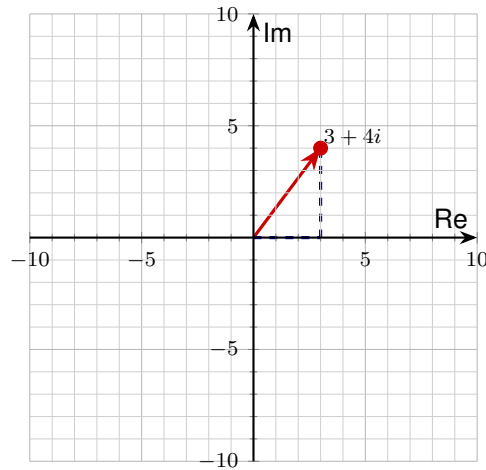
Find the conjugate or modulus as indicated. Write all answers in standard form.

1. $\overline{4 + 3i}$



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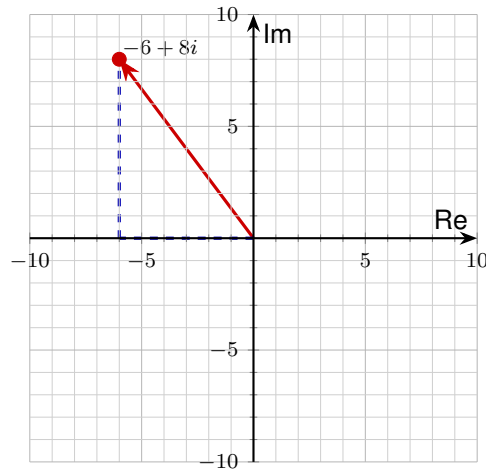
2. $|3 + 4i|$. Read the modulus as the hypotenuse of the right triangle shown below. _____



3. $\overline{-2 - 5i}$ _____

4. $z\bar{z}$ for $z = 2 + 3i$ _____

5. $|-6 + 8i|$. The point sits in the second quadrant; the dashed legs of the right triangle are drawn below. _____



6. $|5 - 12i|$ _____

7. \bar{z} for $z = \frac{1}{2 + i}$ _____

8. $\overline{5 - 12i}$ _____

9. $|z|$ for $z = -3 + 4i$ _____

10. $\overline{(2 + i) + (3 - 4i)}$ _____

11. $|2 - i| \cdot |1 + 3i|$ _____

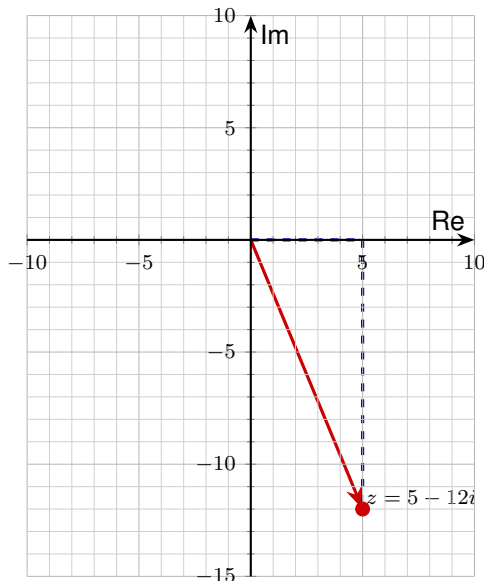
12. $|(2 - i)(1 + 3i)|$ _____

13. $\overline{(3 + 2i)(1 - i)}$ _____

14. $|1 + i|^4$ _____



- 15. \bar{z} for $z = 2 - 7i$ _____
- 16. Find z if $z\bar{z} = 25$, $\text{Re}(z) = 3$, and $\text{Im}(z) > 0$. _____
- 17. $|3 - 2i|^2$ _____
- 18. \bar{i} _____
- 19. $\left| \frac{3 + 4i}{2 - i} \right|$ _____
- 20. On the Argand plane, find the modulus and conjugate of $z = 5 - 12i$. Use the right triangle in the figure. _____



◆ Word Problems

- 21. In an AC circuit the impedance is $Z = 8 + 6i$ ohms. The magnitude of the impedance is $|Z|$. Find $|Z|$ and explain what it represents physically. _____
- 22. A point P in the complex plane satisfies $|P - (3 + i)| = 5$, meaning P is exactly 5 units from the point $3 + i$. Describe the set of all possible P geometrically. _____
- 23. Show that for $z = 1 + i$, the identity $|z|^2 = z\bar{z}$ holds by computing both sides. _____
- 24. Two complex numbers have moduli $|z| = 6$ and $|w| = 8$. What is the largest possible value of $|z + w|$, and what is the smallest? (Both z and w are non-zero.) _____

Additional Practice

- 25. Add $(3 + 2i) + (5 - i)$. _____
- 26. Subtract $(4 - i) - (1 + 6i)$. _____
- 27. Multiply $(2 + 3i)(1 - i)$. _____
- 28. Simplify i^{17} . _____
- 29. Simplify i^{22} . _____
- 30. Find the conjugate of $6 - 5i$. _____
- 31. Find $|3 + 4i|$. _____



Answer Keys

1. $4 - 3i$	13. $5 + i$
2. 5	14. 4
3. $-2 + 5i$	15. $2 - 7i$
4. 13	16. $z = 3 + 4i$
5. 10	17. 13
6. 13	18. $-i$
7. $\frac{1}{2 - i}$	19. $\sqrt{5}$
8. $5 + 12i$	20. $ z = 13, \bar{z} = 5 + 12i$
9. 5	21. $ Z = 10 \text{ ohms}$
10. $5 + 3i$	22. a circle of radius 5 centered at $3 + i$
11. $5\sqrt{2}$	23. both equal 2
12. $5\sqrt{2}$	24. largest 14, smallest 2
Additional Practice Answers	
25. $8 + i$	29. -1
26. $3 - 7i$	30. $6 + 5i$
27. $5 + i$	31. 5
28. i	

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

- Taking a conjugate flips only the imaginary sign and leaves the real part untouched: $4 + 3i \rightarrow 4 - 3i$. Geometrically this reflects the point across the real axis.
- Keep the rule visible: $\sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$. The 3-4-5 Pythagorean triple makes this come out clean — the dashed legs are 3 and 4, the arrow is the hypotenuse. That gives a quick check on the answer.
- Real part stays -2 ; imaginary part flips from -5 to $+5$. Negatives on the real side don't change.
- Start with the key idea: $(2 + 3i)(2 - 3i) = 4 + 9 = 13$. This equals $|z|^2 = (\sqrt{13})^2 = 13$, confirming $z\bar{z} = |z|^2$. That gives a quick check on the answer.
- A careful way to see it: $\sqrt{36 + 64} = \sqrt{100} = 10$. Negative real part doesn't change the modulus — you square it. The triangle has legs 6 and 8, giving hypotenuse 10. That gives a quick check on the answer.
- Square each part and add under the root: $\sqrt{5^2 + (-12)^2} = \sqrt{25 + 144} = \sqrt{169} = 13$. The negative sign disappears when you square, and the 5-12-13 triple gives a clean answer.
- Conjugation distributes over division: $\frac{1}{2 + i} = \frac{1}{2 + i} = \frac{1}{2 - i}$. (You can rationalize this further to $\frac{2 + i}{5}$, but $\frac{1}{2 - i}$ is already a valid expression for \bar{z} .)
- Flip only the imaginary sign — the $-12i$ becomes $+12i$ while the real 5 stays put: $5 + 12i$.
- A careful way to see it: $\sqrt{9 + 16} = 5$. Same 3-4-5 triple, just reflected to the second quadrant. Modulus is unchanged. That gives a quick check on the answer.
- Add first: $(2 + i) + (3 - 4i) = 5 - 3i$. Now take the conjugate: $\bar{5 - 3i} = 5 + 3i$. (Or use the identity $\bar{z + w} = \bar{z} + \bar{w}$: $2 + i + 3 - 4i = (2 - i) + (3 + 4i) = 5 + 3i$.)
- One steady path is: $|2 - i| = \sqrt{5}$ and $|1 + 3i| = \sqrt{10}$. Multiply: $\sqrt{5} \cdot \sqrt{10} = \sqrt{50} = 5\sqrt{2}$. (Or use $|zw| = |z||w|$ and compute $|(2 - i)(1 + 3i)|$ directly.) That gives a quick check on the answer.
- By $|zw| = |z||w|$ this equals $|2 - i| \cdot |1 + 3i| = \sqrt{5} \cdot \sqrt{10} = 5\sqrt{2}$. (Sanity check: FOIL $(2 - i)(1 + 3i) = 2 + 6i - i - 3i^2 = 5 + 5i$, then $|5 + 5i| = \sqrt{50} = 5\sqrt{2}$.)
- Use $\bar{zw} = \bar{z}\bar{w}$: $\bar{3 + 2i} \cdot \bar{1 - i} = (3 - 2i)(1 + i) = 3 + 3i - 2i - 2i^2 = 3 + i + 2 = 5 + i$. (Or multiply first: $(3 + 2i)(1 - i) = 3 - 3i + 2i - 2i^2 = 5 - i$;

- conjugate: $5 + i$)
- Keep the rule visible: $|1 + i| = \sqrt{2}$, so $|1 + i|^4 = (\sqrt{2})^4 = 4$. (Alternative: $|1 + i|^4 = |(1 + i)^4| = |-4| = 4$ using $(1 + i)^4 = -4$ from Section 4.2.) That gives a quick check on the answer.
 - Conjugation twice undoes itself: $\overline{\overline{2 - 7i}} = \overline{2 + 7i} = 2 - 7i = z$. The conjugate of the conjugate is always the original.
 - Start with the key idea: $z\bar{z} = |z|^2 = 25$, so $|z| = 5$. With real part 3, the imaginary part satisfies $3^2 + b^2 = 25$, giving $b^2 = 16$, so $b = \pm 4$. The constraint $\text{Im}(z) > 0$ picks $b = 4$. So $z = 3 + 4i$. That gives a quick check on the answer.
 - A careful way to see it: $|3 - 2i|^2 = 3^2 + (-2)^2 = 9 + 4 = 13$. (Or use $|z|^2 = z\bar{z} = (3 - 2i)(3 + 2i) = 13$.) That gives a quick check on the answer.
 - Write i in full form as $0 + 1i$. Flipping the imaginary sign gives $0 - 1i = -i$. So the conjugate of i is $-i$.
 - Modulus distributes over division: $\frac{|3 + 4i|}{|2 - i|} = \frac{|3 + 4i|}{|2 - i|} = \frac{5}{\sqrt{5}} = \sqrt{5}$. (Much faster than rationalizing first.)
 - Modulus: $|z| = \sqrt{5^2 + (-12)^2} = \sqrt{25 + 144} = \sqrt{169} = 13$. That's the hypotenuse of the right triangle in the figure. Conjugate: flip the imaginary sign, giving $5 + 12i$ (reflection across the real axis).
 - A careful way to see it: $|Z| = \sqrt{8^2 + 6^2} = \sqrt{64 + 36} = \sqrt{100} = 10$ ohms. Physically, $|Z|$ is the total *apparent* impedance — the ratio of peak voltage to peak current. The real and imaginary parts split that into resistive and reactive components, and the modulus combines them via Pythagoras. That gives a quick check on the answer.
 - Keep the rule visible: $|P - (3 + i)|$ is the distance from P to $3 + i$ on the Argand plane (think of P and $3 + i$ as points). Setting that distance equal to 5 traces out a circle of radius 5 centered at $3 + i$ — the complex-number version of $(x - 3)^2 + (y - 1)^2 = 25$. That gives a quick check on the answer.
 - Left side: $|z| = \sqrt{1 + 1} = \sqrt{2}$, so $|z|^2 = 2$. Right side: $z\bar{z} = (1 + i)(1 - i) = 1 - i^2 = 1 + 1 = 2$. Equal — exactly as the identity promises. This pattern is why $(c + di)(c - di)$ shows up so often when you rationalize complex denominators.
 - By the triangle inequality, $|z + w| \leq |z| + |w| = 14$, with equality when z and w point the same direction. For the smallest, $|z + w| \geq ||z| - |w|| = |6 - 8| = 2$, with equality when they point opposite directions. So the range is $2 \leq |z + w| \leq 14$.



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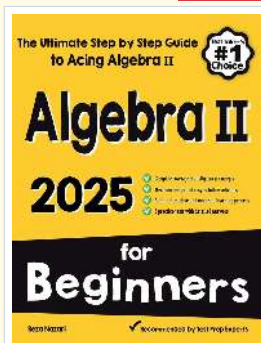
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