Cofunction Identities

Name: ____________	Date: ____________	Score: ______ / 34

Quick Review

Complementary angles are angles that add to 90° (or $\frac{\pi}{2}$ radians). The **cofunction identities** connect trig functions at complementary angles:

$\sin(90^\circ - \theta) = \cos\theta$ and $\cos(90^\circ - \theta) = \sin\theta$.

$\tan(90^\circ - \theta) = \cot\theta$ and $\cot(90^\circ - \theta) = \tan\theta$.

$\sec(90^\circ - \theta) = \csc\theta$ and $\csc(90^\circ - \theta) = \sec\theta$.

Why this works. In a right triangle, swap which acute angle you focus on. The leg that *was* opposite to θ becomes adjacent to $90^\circ - \theta$ (the other acute angle). So sine and cosine literally trade places. The same swap works for tan/cot and sec/csc.

Pairing pattern. Each cofunction pair has exactly one function with a "co" prefix: sine/cosine, tangent/cotangent, secant/cosecant. That's the source of the name – "co" means "complementary."

Solving cofunction equations. If $\sin\alpha = \cos\beta$ and both angles are acute, then $\alpha + \beta = 90^\circ$ (they're complementary). This is the fast path to solving equations like $\sin(2x + 5^\circ) = \cos(3x - 10^\circ)$.

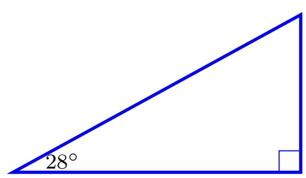
Radian version. In radians, the complement of θ is $\frac{\pi}{2} - \theta$, so $\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$, and so on.

Common slips. Confusing complementary (90° sum) with supplementary (180° sum) – the identity uses complements only. Adding a stray negative sign: $\sin(90^\circ - \theta) = +\cos\theta$, no minus. Skipping the check that the angles are acute when solving $\sin\alpha = \cos\beta$ for general angles – the complement rule is the simplest case but not the only solution.

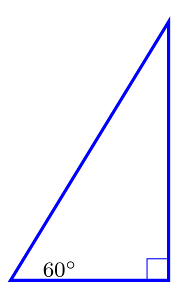
PRACTICE

Use cofunction identities to rewrite or solve. Express complements in the same unit as the original.

1. In the right triangle below the acute angle $\theta = 28^\circ$ is marked; the other acute angle is its complement. Rewrite $\sin 28^\circ$ in terms of a cofunction of that complement. ________



2. Rewrite $\cos 75^\circ$ in terms of its cofunction. ________
3. Rewrite $\tan 10^\circ$ in terms of its cofunction. ________
4. In the triangle below the marked acute angle is 60°. Complete: $\sin 60^\circ = \cos\,?$ ________



5. Rewrite $\cos\left(\frac{\pi}{6}\right)$ in terms of its cofunction. ________
6. $\sec(90^\circ - \theta) = ?$ ________
7. Solve for x (acute): $\sin(2x + 5^\circ) = \cos(3x - 10^\circ)$. ________

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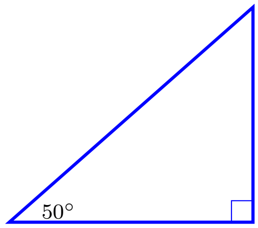
8. Solve for x (acute): $\tan(4x + 6^\circ) = \cot(2x + 12^\circ)$. ________

9. Rewrite $\sin\left(\frac{\pi}{8}\right)$ in terms of its cofunction. ________

10. True or False: $\sin 50^\circ = \cos 40^\circ$ (the marked angle below is 50°). ________



11. True or False: $\sin 30^\circ = \sin 60^\circ$. ________

12. $\cot 25^\circ =$? (cofunction form) ________

13. $\csc 80^\circ =$? (cofunction form) ________

14. Solve for x (acute): $\sin(x + 10^\circ) = \cos(x + 20^\circ)$. ________

15. $\cos\left(\frac{\pi}{2} - \theta\right) =$? ________

16. $\sin\left(\frac{\pi}{2} - \frac{\pi}{5}\right) =$? ________

17. $\tan 75^\circ = \cot$? ________

18. True or False: $\sec\theta \cdot \csc(90^\circ - \theta) = \sec^2\theta$. ________

19. Solve for x (acute): $\sec(3x + 9^\circ) = \csc(2x + 11^\circ)$. ________

20. $\cos 89^\circ \cdot \sec 1^\circ =$? ________

♦ Word Problems

21. In a right triangle, the two acute angles are α and β. If $\sin\alpha = \frac{3}{5}$, what is $\cos\beta$? Explain in one sentence. ____

22. Solve for the acute angle x if $\sin(3x - 5^\circ) = \cos(x + 15^\circ)$. ____

23. A surveyor measures one acute angle of a right triangle as 32°. Use a cofunction identity to express $\tan 58^\circ$ in terms of a function of 32°. ____

24. Simplify $\frac{\sin 38^\circ}{\cos 52^\circ}$ using a cofunction identity. ____

Additional Practice

25. Find $\sin\theta$ if opposite $= 5$, hypotenuse $= 13$. ________

26. Find $\cos\theta$ if adjacent $= 12$, hypotenuse $= 13$. ________

27. Find $\tan\theta$ if opposite $= 7$, adjacent $= 4$. ________

28. Find $\sin 30^\circ$. ________

29. Find $\cos 60^\circ$. ________

30. Find $\tan 45^\circ$. ________

31. Convert 180° to radians. ________

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32. Convert $\frac{\pi}{3}$ radians to degrees. ________

33. Find a coterminal angle with $70°$. ________

34. Reference angle for $150°$. ________

Answer Keys

1. $\boxed{\cos 62^\circ}$
2. $\boxed{\sin 15^\circ}$
3. $\boxed{\cot 80^\circ}$
4. $\boxed{30^\circ}$
5. $\boxed{\sin\left(\frac{\pi}{3}\right)}$
6. $\boxed{\csc\theta}$
7. $\boxed{x = 19^\circ}$
8. $\boxed{x = 12^\circ}$
9. $\boxed{\cos\left(\frac{3\pi}{8}\right)}$
10. True
11. False
12. $\boxed{\tan 65^\circ}$
13. $\boxed{\sec 10^\circ}$
14. $\boxed{x = 30^\circ}$
15. $\boxed{\sin\theta}$
16. $\boxed{\cos\left(\frac{\pi}{5}\right)}$
17. $\boxed{15^\circ}$
18. True
19. $\boxed{x = 14^\circ}$
20. $\boxed{\sin 1^\circ \cdot \sec 1^\circ = \tan 1^\circ}$
21. $\boxed{\frac{3}{5}}$
22. $\boxed{x = 20^\circ}$
23. $\boxed{\cot 32^\circ}$
24. $\boxed{1}$

Additional Practice Answers

25. $\boxed{\frac{5}{13}}$
26. $\boxed{\frac{12}{13}}$
27. $\boxed{\frac{7}{4}}$
28. $\boxed{\frac{1}{2}}$
29. $\boxed{\frac{1}{2}}$
30. $\boxed{1}$
31. $\boxed{\pi}$
32. $\boxed{60^\circ}$
33. $\boxed{430^\circ}$
34. $\boxed{30^\circ}$

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. A careful way to see it: The complement of 28° is $90^\circ - 28^\circ = 62^\circ$ (the top acute angle). $\sin 28^\circ = \cos 62^\circ$. That gives a quick check on the answer.

2. Cosine of an angle equals sine of its complement. The complement of 75° is $90^\circ - 75^\circ = 15^\circ$, so $\cos 75^\circ = \sin 15^\circ$.

3. Tangent and cotangent are cofunctions, so $\tan\theta$ equals $\cot$ of the complement. The complement of 10° is $90^\circ - 10^\circ = 80^\circ$, giving $\tan 10^\circ = \cot 80^\circ$.

4. The complement of 60° is 30° (the top acute angle). So $\sin 60^\circ = \cos 30^\circ$. (Both equal $\frac{\sqrt{3}}{2}$.)

5. A careful way to see it: Complement (radians) is $\frac{\pi}{2} - \frac{\pi}{6} = \frac{\pi}{3}$. So $\cos\frac{\pi}{6} = \sin\frac{\pi}{3}$. This is the part to check before moving on, because it keeps the answer tied to the original question.

6. Keep the rule visible: Secant and cosecant are cofunctions: $\sec(90^\circ - \theta) = \csc\theta$ wherever both are defined. That gives a quick check on the answer.

7. Complement: $(2x+5)+(3x-10) = 90$, so $5x-5 = 90$, $5x = 95$, $x = 19^\circ$. Check: angles become 43° and 47°, which sum to 90° ✓.

8. Start with the key idea: Cofunction sum: $(4x+6)+(2x+12) = 90$, so $6x+18 = 90$, $6x = 72$, $x = 12^\circ$. That gives a quick check on the answer.

9. A careful way to see it: Complement is $\frac{\pi}{2} - \frac{\pi}{8} = \frac{4\pi - \pi}{8} = \frac{3\pi}{8}$. This is the part to check before moving on, because it keeps the answer tied to the original question.

10. Keep the rule visible: $50^\circ + 40^\circ = 90^\circ$, so they're complements and the cofunction identity gives equality. That gives a quick check on the answer.

11. One steady path is: $\sin 30^\circ = \frac{1}{2}$ and $\sin 60^\circ = \frac{\sqrt{3}}{2}$ – different. The cofunction identity says $\sin 30^\circ = \cos 60^\circ$, not $\sin 60^\circ$. That gives a quick check on the answer.

12. Cotangent of an angle equals tangent of its complement. The complement of 25° is $90^\circ - 25^\circ = 65^\circ$, so $\cot 25^\circ = \tan 65^\circ$.

13. Cosecant and secant are cofunctions. The complement of 80° is $90^\circ - 80^\circ = 10^\circ$, so $\csc 80^\circ = \sec 10^\circ$.

14. Keep the rule visible: $(x+10)+(x+20) = 90$, so $2x+30 = 90$, $2x = 60$, $x = 30^\circ$. Check: $\sin 40^\circ = \cos 50^\circ$ ✓. That gives a quick check on the answer.

15. This is the cofunction identity written in radians: the complement of θ is $\frac{\pi}{2} - \theta$, and cosine of the complement equals sine of the angle. So $\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$.

16. Start with the key idea: By cofunction identity, $\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$ with $\theta = \frac{\pi}{5}$. This is the part to check before moving on, because it keeps the answer tied to the original question.

17. A careful way to see it: Complement of 75° is 15°, so $\tan 75^\circ = \cot 15^\circ$. This is the part to check before moving on, because it keeps the answer tied to the original question.

18. Keep the rule visible: $\csc(90^\circ - \theta) = \sec\theta$, so the product is $\sec\theta \cdot \sec\theta = \sec^2\theta$. This is the part to check before moving on, because it keeps the answer tied to the original question.

19. One steady path is: Cofunction: $(3x+9)+(2x+11) = 90$, $5x+20 = 90$, $5x = 70$, $x = 14^\circ$. That gives a quick check on the answer.

20. Start with the key idea: $\cos 89^\circ = \sin 1^\circ$ (cofunction). And $\sin 1^\circ \cdot \sec 1^\circ = \frac{\sin 1^\circ}{\cos 1^\circ} = \tan 1^\circ$. This is the part to check before moving on, because it keeps the answer tied to the original question.

21. Since α and β are the two acute angles of a right triangle, they sum to 90°. By the cofunction identity, $\cos\beta = \sin(90^\circ - \beta) = \sin\alpha = \frac{3}{5}$.

22. When $\sin\alpha = \cos\beta$ with both acute, $\alpha + \beta = 90^\circ$. So $(3x-5)+(x+15) = 90$, giving $4x+10 = 90$, $4x = 80$, $x = 20^\circ$. Check: the two angles become 55° and 35°, which sum to 90° ✓.

23. One steady path is: 58° is the complement of 32°. By cofunction, $\tan 58^\circ = \tan(90^\circ - 32^\circ) = \cot 32^\circ$. (Equivalently, $\tan 58^\circ = \frac{1}{\tan 32^\circ}$.) That gives a quick check on the answer.

24. Start with the key idea: $\cos 52^\circ = \sin(90^\circ - 52^\circ) = \sin 38^\circ$. So the quotient is $\frac{\sin 38^\circ}{\sin 38^\circ} = 1$. (Whenever the two angles in a sin/cos ratio sum to 90°, the result is 1.) That gives a quick check on the answer.

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