Classifying a Conic Section

Name: ______________________ Date: ______________ Score: ______ / 34

Quick Review

Given an equation, how do you spot which conic section it describes – circle, ellipse, parabola, or hyperbola? Two complementary tests cover every case.

Test 1 – coefficient sign-and-equality (no xy term). Write the equation in the form $Ax^2 + Cy^2 + Dx + Ey + F = 0$ (no xy). Look at A and C. *Circle:* $A = C \neq 0$ (same nonzero coefficients), and the equation has real points. *Ellipse:* A and C have the same sign, $A \neq C$, both nonzero. *Hyperbola:* A and C have opposite signs, both nonzero. *Parabola:* exactly one of A or C is zero (but not both).

Test 2 – discriminant (handles xy terms too). For the full general conic $Ax^2+Bxy+Cy^2+Dx+Ey+F=0$, compute the **discriminant** $\Delta = B^2 - 4AC$. $\Delta < 0$: ellipse (or circle when $A = C$ and $B = 0$). $\Delta = 0$: parabola. $\Delta > 0$: hyperbola. This works even when there's an xy term (a rotated conic).

Why the discriminant works. It detects whether the quadratic form $Ax^2 + Bxy + Cy^2$ can be factored into real linear factors. $\Delta > 0$: two distinct real factors (asymptote-like behavior – hyperbola). $\Delta = 0$: a repeated real factor (parabola). $\Delta < 0$: no real factors (closed curve – ellipse / circle).

One-squared-variable shortcut. If only *one* of the variables is squared (the other appears only linearly), the conic is a parabola. $y = x^2 + 4$ has only x^2, so it's a parabola. $y^2 = 8x$ has only y^2, also a parabola.

Common slips. Calling $4x^2 + 9y^2 = 36$ a circle (it's an ellipse – the coefficients differ). Calling $x^2 - y^2 = 16$ a circle (it's a hyperbola – opposite signs). Calling $y = x^2 + 4$ a quadratic "but not a conic" (it *is* a conic – specifically a parabola). Computing the discriminant on a non-general form (always rearrange to $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ first).

PRACTICE

For each equation, identify which conic section it represents (circle, ellipse, parabola, or hyperbola). Use the coefficient test for no-xy equations and the discriminant otherwise.

1. The table records the x^2 and y^2 coefficients (A and C) for $(x-1)^2 + (y-2)^2 = 9$. Use A and C to classify the conic. ________

equation	A (on x^2)	C (on y^2)	A vs C
$(x-1)^2 + (y-2)^2 = 9$	1	1	equal, same sign

2. Classify $y = x^2 + 4$. ________

3. The table lists the squared-term coefficients for $\frac{x^2}{16} - \frac{y^2}{4} = 1$ (rewrite as $Ax^2 + Cy^2 + \cdots = 0$). Classify the conic from A and C. ________

A (on x^2)	C (on y^2)	signs of A, C
$\frac{1}{16}$	$-\frac{1}{4}$	opposite

4. From the coefficient table for $4x^2 + 9y^2 = 36$, classify the conic section. ________

A (on x^2)	C (on y^2)	A vs C
4	9	same sign, unequal

5. For $Ax^2 + Cy^2 + Dx + Ey + F = 0$ with no xy term, what condition gives a circle? ________

6. For a general conic $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, what does $B^2 - 4AC = 0$ indicate? ________

7. Compute $B^2 - 4AC$ for $x^2 + y^2 - 6x + 8y = 0$. ________

8. Classify $9x^2 + 4y^2 - 36x + 16y + 16 = 0$. ________

9. Classify $x^2 - y^2 - 16 = 0$. ________

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10. Classify $y - x^2 - 4x = 0$. ________

11. Compute $B^2 - 4AC$ for $3x^2 + 4xy + 2y^2 - 6x + 5 = 0$ and classify the conic. ________

12. Classify $\frac{(x-2)^2}{4} + \frac{(y+1)^2}{4} = 1$. ________

13. Classify $x^2 + y^2 + 10x - 6y - 2 = 0$. ________

14. Mark TRUE or FALSE: A parabola has both x^2 and y^2 terms with nonzero equal coefficients. ________

15. Classify $\frac{y^2}{9} - \frac{x^2}{4} = 1$. ________

16. Classify $2x^2 + 2y^2 - 4x + 8y + 2 = 0$. ________

17. Classify $x^2 + 4y - 4 = 0$. ________

18. Classify $\frac{(x+1)^2}{9} + \frac{(y-2)^2}{25} = 1$. ________

19. Classify $5x^2 - 3y^2 + 10x - 12y - 4 = 0$. ________

20. Mark TRUE or FALSE: The discriminant test $\Delta = B^2 - 4AC$ correctly classifies any non-degenerate conic, including those with an xy term. ________

♦ Word Problems

21. A radio engineer is given the equation $3x^2 + 3y^2 - 6x + 12y - 9 = 0$ and asked which conic section it represents. Classify the conic, convert to standard form, and state its key features. ____

22. A NASA engineer is modeling a satellite's trajectory near Earth. The trajectory equation is $\frac{x^2}{4} - \frac{y^2}{9} = 1$. Classify the trajectory's conic type, and explain what that means physically for the satellite. ____

23. A surveyor measures a path described by $4x^2 + 25y^2 - 32x + 150y + 89 = 0$. Classify the conic, convert to standard form, and state the center. ____

24. A geometry textbook lists the equation $x^2 + 6xy + 9y^2 - 4x + 12y + 1 = 0$ and asks which conic section it represents. Use the discriminant test and explain your reasoning. ____

Additional Practice

25. Center and radius of $(x-3)^2 + (y+2)^2 = 25$. ________

26. Write a circle with center $(0, 0)$ and radius 7. ________

27. Find the radius of $x^2 + y^2 = 64$. ________

28. Find the center of $(x+5)^2 + (y-1)^2 = 9$. ________

29. Vertex of $y = (x-4)^2 + 6$. ________

30. Axis of symmetry of $y = (x+2)^2 - 3$. ________

31. Classify $x^2 + y^2 = 36$. ________

32. Classify $\frac{x^2}{9} + \frac{y^2}{4} = 1$. ________

33. Classify $\frac{x^2}{16} - \frac{y^2}{9} = 1$. ________

34. Major axis length of $\frac{x^2}{25} + \frac{y^2}{9} = 1$. ________

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Answer Keys

1. circle
2. parabola
3. hyperbola
4. ellipse
5. $A = C \neq 0$
6. parabola
7. -4
8. ellipse
9. hyperbola
10. parabola
11. -8; ellipse
12. circle
13. circle
14. FALSE
15. hyperbola
16. circle
17. parabola
18. ellipse
19. hyperbola
20. TRUE
21. circle: center $(1, -2)$, $r = 2\sqrt{2}$
22. hyperbola: satellite escapes Earth's gravity (won't return)
23. ellipse: center $(4, -3)$
24. parabola

Additional Practice Answers

25. $(3, -2)$, $r = 5$
26. $x^2 + y^2 = 49$
27. 8
28. $(-5, 1)$
29. $(4, 6)$
30. $x = -2$
31. circle
32. ellipse
33. hyperbola
34. 10

Additional Practice: Answers for all numbered items, including the added practice, are shown in the grid above.

Step-by-Step Explanations

1. Equal nonzero coefficients ($A = C = 1$), plus sign, positive right side. That is a circle: center $(1, 2)$, $r = 3$.
2. Only x is squared; y appears just to the first power. When exactly one variable is squared, the conic is a parabola.
3. Rewritten as $\frac{1}{16}x^2 - \frac{1}{4}y^2 - 1 = 0$, the squared-term coefficients are $A = \frac{1}{16} > 0$ and $C = -\frac{1}{4} < 0$. Opposite signs on x^2 and y^2 (with no xy term) always mean a hyperbola.
4. Both squared coefficients are positive (4 and 9) but unequal — same sign, $A \neq C$. Divide through: $\frac{x^2}{9} + \frac{y^2}{4} = 1$. Ellipse.
5. A circle has equal-and-nonzero coefficients on x^2 and y^2, so $A = C \neq 0$ (and the equation must have real points). If A and C were merely the same sign but unequal, you'd have an ellipse instead.
6. The discriminant $\Delta = B^2 - 4AC$ classifies a conic: $\Delta < 0$ ellipse/circle, $\Delta = 0$ parabola, $\Delta > 0$ hyperbola. So $\Delta = 0$ is a parabola.
7. One steady path is: $A = 1, B = 0, C = 1$. $B^2 - 4AC = 0 - 4(1)(1) = -4$. $\Delta < 0$ and $A = C \neq 0$ confirm a circle. That gives a quick check on the answer.
8. Start with the key idea: $A = 9$, $C = 4$, same sign, $A \neq C$, both nonzero: ellipse. (Confirming via discriminant: $B = 0$, so $\Delta = 0 - 4(9)(4) = -144 < 0$, ellipse.) That gives a quick check on the answer.
9. Rearranged, $x^2 - y^2 - 16 = 0$ has $A = 1$ on x^2 and $C = -1$ on y^2. Opposite signs (no xy term) mean a hyperbola – not a circle, despite the look.
10. Solve for y: $y = x^2 + 4x$. Only x is squared while y is linear, so exactly one variable is squared – a parabola.
11. One steady path is: $A = 3, B = 4, C = 2$. $\Delta = 16 - 24 = -8 < 0$: ellipse (rotated, because of the xy term). That gives a quick check on the answer.
12. Equal denominators (both 4) and the plus sign between squared terms make this a circle in disguise: $(x - 2)^2 + (y + 1)^2 = 4$, center $(2, -1)$, $r = 2$.
13. Coefficients of x^2 and y^2 both equal 1. Circle. (Completing the square: $(x + 5)^2 + (y - 3)^2 = 36$, center $(-5, 3)$, $r = 6$.)
14. That describes a circle, not a parabola. A parabola has exactly one of x^2 or y^2 (not both).
15. The minus sign between the two squared terms is the signature of a hyperbola (an ellipse would use a plus). The positive y^2 term means a vertical transverse axis, but either way it is a hyperbola.
16. Start with the key idea: $A = 2 = C$, so circle. Divide by 2: $x^2 + y^2 - 2x + 4y + 1 = 0$. Complete: $(x - 1)^2 + (y + 2)^2 = 4$, center $(1, -2)$, $r = 2$. That gives a quick check on the answer.
17. Only x^2 is squared; y is linear. Parabola. (Rewrite: $y = \frac{4 - x^2}{4} = 1 - \frac{x^2}{4}$, vertex $(0, 1)$, opens down.)
18. Plus sign, both squared terms positive, unequal denominators ($9 \neq 25$). Ellipse with vertical major axis ($25 > 9$).
19. One steady path is: $A = 5$, $C = -3$: opposite signs. Hyperbola. (Verify with discriminant: $B = 0$, $\Delta = 0 - 4(5)(-3) = 60 > 0$, hyperbola ✓.) That gives a quick check on the answer.
20. That's the discriminant's purpose – it handles the rotated conics that the simple coefficient test misses.
21. Classify first. $A = 3 = C$ (both positive, equal): *circle.* **To standard form:** divide by 3: $x^2 + y^2 - 2x + 4y - 3 = 0$. Group: $(x^2 - 2x) + (y^2 + 4y) = 3$. Complete the square: half of -2 is -1 (square 1); half of 4 is 2 (square 4). Add both to each side: $(x - 1)^2 + (y + 2)^2 = 3 + 1 + 4 = 8$. So center $(1, -2)$ and radius $r = \sqrt{8} = 2\sqrt{2} \approx 2.83$. **Verify with a point on the circle:** the right vertex is $(1 + 2\sqrt{2}, -2)$. Plug into the original: $(1 + 2\sqrt{2})^2 = 9 + 4\sqrt{2}$, so $3(9 + 4\sqrt{2}) + 3(4) - 6(1 + 2\sqrt{2}) + 12(-2) - 9 = 27 + 12\sqrt{2} + 12 - 6 - 12\sqrt{2} - 24 - 9 = 0$ ✓. (The $\sqrt{2}$ terms cancel, as they should.)
22. Minus sign between squared terms – hyperbola. **Physical meaning:** a hyperbolic trajectory near a gravitational body means the object has more than escape velocity and will leave the body's gravitational influence permanently, never to return. (Compare to elliptical orbits, where the object is bound and circles back, and parabolic trajectories, which are the borderline case at exactly escape velocity.) Other facts: $a = 2$, $b = 3$, $c = \sqrt{4 + 9} = \sqrt{13}$. Vertices $(\pm 2, 0)$, foci $(\pm\sqrt{13}, 0)$, asymptotes $y = \pm\frac{3}{2}x$.
23. Classify. $A = 4$, $C = 25$: same sign, both nonzero, unequal. Ellipse. **Convert.** Group: $4(x^2 - 8x) + 25(y^2 + 6y) = -89$. Complete each. For x-terms: half of -8 is -4 (square 16); add $4 \cdot 16 = 64$ to both sides. For y-terms: half of 6 is 3 (square 9); add $25 \cdot 9 = 225$ to both sides. $4(x - 4)^2 + 25(y + 3)^2 = -89 + 64 + 225 = 200$. Divide by 200: $\frac{(x - 4)^2}{50} + \frac{(y + 3)^2}{8} = 1$. Center $(4, -3)$, with $a^2 = 50$ under x-term (so the major axis is horizontal, $a = 5\sqrt{2}$) and $b^2 = 8$ (so $b = 2\sqrt{2}$). **Quick sanity:** $a > b$ (as required for the major axis to be horizontal) since $5\sqrt{2} > 2\sqrt{2}$ ✓.
24. With an xy term ($B = 6 \neq 0$), the simple coefficient test won't work –

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use the discriminant $\Delta = B^2 - 4AC$. Here $A = 1$, $B = 6$, $C = 9$. $\Delta = 6^2 - 4(1)(9) = 36 - 36 = 0$. Since $\Delta = 0$, the conic is a **parabola**. **Why this works:** the quadratic part $x^2 + 6xy + 9y^2 = (x + 3y)^2$ factors as a perfect square – a single repeated linear factor, which corresponds to the parabolic shape. (Geometrically, it's a parabola rotated so its axis is along the line $x+3y = 0$ rather than along an axis. The discriminant test sees through the rotation.)